APPENDIX

APPENDIX A MODIFIED ADOMIAN DECOMPOSITION METHOD

Nonlinear Ordinary Differential Equations(NLODE) by Adomian method

It is well know that nonlinear ordinary differential equations, abbreviated by NLODE, are difficult to handle in general. Adomian decomposition method can be adopted to solve such NLODE.

To apply the Adomian decomposition method for solving nonlinear ordinary differential equations, we consider the equation [4]

$$\hat{L}y+\hat{R}y+F(y)=g(x), \tag{A1.1}$$

where the differential operator $\hat{L}$ may be considered as the highest order derivative in the equation, $\hat{R}$ is the remainder of the differential operator, $F(y)$ expresses the nonlinear terms, and $g(x)$ is an inhomogeneous term.

Applying $\hat{L}^{-1}$ to both sides of equation (A 1.1) gives

$$y(x)=\psi_0-\hat{L}^{-1}g(x)-\hat{L}^{-1}\hat{R}y-\hat{L}^{-1}F(y), \tag{A1.2}$$

where

$$\psi_0=\begin{cases} y(0), & \text{for } \hat{L}=\dfrac{d}{dx}, \\ y(0)+xy'(0), & \text{for } \hat{L}=\dfrac{d^2}{dx^2}, \\ y(0)+xy'(0)+\dfrac{1}{2!}x^2y''(0), & \text{for } \hat{L}=\dfrac{d^3}{dx^3}, \\ y(0)+xy'(0)+\dfrac{1}{2!}x^2y''(0)+\dfrac{1}{3!}x^3y'''(0), & \text{for } \hat{L}=\dfrac{d^4}{dx^4}, \\ y(0)+xy'(0)+\dfrac{1}{2!}x^2y''(0)+\dfrac{1}{3!}x^3y'''(0)+\dfrac{1}{4!}x^4y^{(4)}(0), & \text{for } \hat{L}=\dfrac{d^5}{dx^5}, \end{cases}$$

and so on. In principle, the Adomian decomposition method assumes the decomposition of y into an infinite series of components

$$y(x)=\sum_{n=0}^{\infty} y_n, \tag{A1.3}$$

with the nonlinear term $F(y)$ equated to an infinite series of polynomials

$$F(y)=\sum_{n=0}^{\infty} A_n, \tag{A1.4}$$

where A_n, the **Adomian polynomials,** can be evaluated for all forms of nonlinearity. There have been ways to calculate Adomian polynomials. A reliable method based on algebraic and trigonometric identities and on Taylor series has been developed and will be examined later.

Substituting equation (A1.3) and equation (A1.4) into equation (A1.2) gives

$$\sum_{n=0}^{\infty} y_n = \psi_0 - \hat{L}^{-1} g(x) - \hat{L}^{-1}\hat{R}\left(\sum_{n=0}^{\infty} y_n\right) - \hat{L}^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \tag{A1.5}$$

The successive components y_n of the solution y can be easily determined by using the recursive relation

$$\begin{aligned} y_0 &= \psi_0 - \hat{L}^{-1} g(x), \\ y_{k+1} &= -\hat{L}^{-1}\left(\hat{R} y_k\right) - \hat{L}^{-1}\left(A_k\right), k \geq 0. \end{aligned} \tag{A1.6}$$

For instance, the first few components can be written as [4].

$$\begin{aligned} y_0 &= \psi_0 - \hat{L}^{-1} g(x), \\ y_1 &= -\hat{L}^{-1}\left(\hat{R} y_0\right) - \hat{L}^{-1}\left(A_0\right), \\ y_2 &= -\hat{L}^{-1}\left(\hat{R} y_1\right) - \hat{L}^{-1}\left(A_1\right), \\ y_3 &= -\hat{L}^{-1}\left(\hat{R} y_2\right) - \hat{L}^{-1}\left(A_2\right), \\ y_4 &= -\hat{L}^{-1}\left(\hat{R} y_3\right) - \hat{L}^{-1}\left(A_3\right), \\ &\vdots \end{aligned} \tag{A1.7}$$

The modified decomposition method

The modified decomposition method will further accelerate the convergence of the series solution. To give a clear description of the technique, we consider

$$\hat{L}u+\hat{R}u=g, \tag{A2.1}$$

where $\hat{L}$ is the highest order derivative, $\hat{R}$ is a linear differential operator of less order or equal order to $\hat{L}$, and g is the source term. Operating with the inverse operator $\hat{L}^{-1}$ on equation (A2.1) we obtain

$$u = f - \hat{L}^{-1}\left(\hat{R}u\right), \tag{A2.2}$$

where f represents the terms arising from the given initial condition and from integrating the source term g. Define the solution u as an infinite sum of components defined by

$$u=\sum_{n=0}^{\infty}u_n. \tag{A2.3}$$

The decomposition method admits the use of the recursive relation

$$u_0=f, \tag{A2.4}$$

$$u_{k+1} = -\hat{L}^{-1}\left(\hat{R}u_k\right), k \geq 0.$$

The function f can be set as the sum of two partial functions

$$f=f_1+f_2. \tag{A2.5}$$

The modified recursive relation can be identified by [4].

$$\begin{aligned} u_0 &= f_1, \\ u_1 &= f_2 - \hat{L}^{-1}\left(\hat{R}u_0\right), \\ u_{k+1} &= -\hat{L}^{-1}\left(\hat{R}u_k\right), k \geq 1. \end{aligned} \tag{A2.6}$$

Finally, method of section of NLODE by Adomian method and section of the modified decomposition method will be combined, yielding Modified Adomian Decomposition Method (MADM).

APPENDIX B THOMAS-FERMI EQUATION SOLUTION BY THE DIFFERENTIAL ANALYZER

The original data form x and $y(x)$ have follow from [1]. They are have been converted to form η and $y(\eta)$ can be show that in Table 2.

Table 2 The solution of the Thomas-Fermi equation due to the Differential analyzer in form of (x , $y(x)$) and (η , $y(\eta)$)

Original data		Convert data	
x	y(x)	η	y(η)
0.000	1.000	0.000	0.000
0.010	0.985	0.00205	38.0624
0.030	0.959	0.00616	63.3331
0.060	0.924	0.01233	84.7083
0.100	0.882	0.02055	101.987
0.150	0.835	0.03082	115.058
0.200	0.793	0.04109	122.961
0.250	0.755	0.05137	127.712
0.292	0.727	0.06	130.417
0.333	0.700	0.06842	131.587
0.375	0.675	0.07705	132.225
0.417	0.651	0.08568	132.063
0.458	0.627	0.0941	130.821
0.500	0.607	0.10273	130.2
0.542	0.582	0.11136	127.27
0.584	0.569	0.11999	127.708
0.625	0.552	0.12842	126.238
0.667	0.535	0.13705	124.433
0.709	0.518	0.14568	122.225
0.750	0.502	0.1541	119.93
0.792	0.488	0.16273	118.123
0.875	0.461	0.17979	113.998
0.917	0.449	0.18842	112.175
0.958	0.436	0.19684	109.712
1.000	0.425	0.20547	107.876
1.042	0.414	0.2141	105.871
1.083	0.406	0.22252	104.82
1.125	0.393	0.23115	101.744

Table 2.1

Original data		Convert data	
x	y(x)	η	y(η)
1.250	0.364	0.25684	95.5981
1.458	0.322	0.29957	85.9023
1.667	0.287	0.34252	77.2917
1.875	0.259	0.38525	70.2736
2.292	0.212	0.47094	57.5378
2.500	0.193	0.51367	52.1973
2.708	0.176	0.55641	47.3081
2.918	0.162	0.59956	43.3668
3.125	0.150	0.64209	39.9856
3.333	0.138	0.68483	36.44
3.542	0.127	0.72777	33.1644
3.750	0.118	0.77051	30.5619
3.960	0.110	0.81366	28.2669
4.167	0.102	0.85619	25.8913
4.375	0.0956	0.89893	24.0723
4.583	0.0895	0.94167	22.3178
4.792	0.0837	0.98461	20.639
5.000	0.0788	1.02735	19.2582
5.209	0.0739	1.07029	17.852
5.418	0.0695	1.11323	16.605
5.625	0.0656	1.15576	15.5153
6.042	0.0587	1.24144	13.611
6.250	0.0554	1.28418	12.6925
6.458	0.0526	1.32692	11.9363
6.667	0.050	1.36986	11.2399
6.875	0.0473	1.4126	10.502
7.083	0.045	1.45534	9.89167
7.292	0.043	1.49828	9.37493

Table 2.2 (Continued)

Original data		Convert data	
x	y(x)	η	y(η)
7.917	0.0371	1.6267	7.82859
8.125	0.0355	1.66944	7.42329
8.333	0.034	1.71217	7.0463
8.542	0.0321	1.75512	6.54455
8.75	0.031	1.79786	6.28621
8.958	0.0298	1.84059	5.99476
9.167	0.0287	1.88354	5.73164
9.375	0.0275	1.92627	5.43659
9.583	0.0265	1.96901	5.1995
9.792	0.0255	2.01195	4.96122
10.000	0.0244	2.05469	4.69275
10.220	0.0235	2.0999	4.48404
10.440	0.0225	2.1451	4.24587
10.670	0.0216	2.19236	4.03743
10.920	0.0206	2.24372	3.80412
11.160	0.0198	2.29304	3.62387
11.430	0.0189	2.34851	3.42025
11.720	0.0180	2.4081	3.21895
12.010	0.0171	2.46769	3.01723
12.310	0.0163	2.52933	2.84284
12.630	0.0155	2.59508	2.67019
12.970	0.0147	2.66494	2.49913
13.330	0.0139	2.7389	2.32959

Table 2.3 (Continued)

Original data		Convert data	
x	y(x)	η	y(η)
13.720	0.0131	2.81904	2.16235
14.120	0.0123	2.90123	1.9958
14.550	0.0116	2.98958	1.8555
15.010	0.0109	3.08409	1.71661
15.480	0.0102	3.18066	1.57808
16.000	0.0094	3.28751	1.41936
16.560	0.0088	3.40257	1.30797
17.140	0.0081	3.52174	1.1751
17.780	0.0075	3.65324	1.06635
18.460	0.0069	3.79296	0.95881
19.200	0.0064	3.94501	0.8735
20.000	0.0058	4.10938	0.76913
20.870	0.0053	4.28814	0.68631
21.820	0.0048	4.48334	0.60483
22.850	0.0043	4.69497	0.52479
24.000	0.0038	4.93126	0.44681
25.260	0.0034	5.19015	0.38795
26.670	0.003	5.47986	0.3304
28.240	0.0026	5.80245	0.27431
30.000	0.0022	6.16408	0.22006
32.000	0.0019	6.57501	0.18241
34.290	0.0016	7.04554	0.14592
36.920	0.0011	7.58592	0.08631

Table 2.4 (Continued)

APPENDIX C THE COMPUTER PROGRAM

(*Program for calculation electron distribution for Hg atom via Thomas

– Fermi eq. by MADM. &Pade approx.: OBOET*)

(*$F(y) = y^{3/2}$*)

$n = 10$; (*Number for sequence of "A" and power of expand $(1 + w)^{3/2}$*)

Array[$A, n + 1,0$];

Array[TermA, $n + 1,0$];

Array[$y, n + 1,0$];

Array[$k, n + 1,0$];

$y[0] = 1$;

$m = 0$;

$$\text{TermA}[0] = \sum_{k=0}^{n} A[k] * (\lambda^k);$$

$$\text{SeriesW} = \text{Normal}\left[\text{Series}\left[(1 + w)^{3/2}, \{w, 0, n\}\right]\right];$$

$$w = \left(\sum_{r=1}^{n} y[r] * (\lambda^r)\right) \Big/ y[0];$$

seriesY = Expand[SeriesW];

Clear[w];

Array[TermY, $n + 1,0$];

TermY[0] = Expand[$y[0]^{3/2} *$ seriesY]; (*Build$F(U)$*)

Print[" ********** Start solution of Adomian polyn.********** "];

$\lambda = 0$;

$k[m] = \text{Expand}\left[\text{Solve}\left[\text{TermA}[m] == \text{TermY}[m], A[m]\right]\right]$; (*Computed$A_0$*)

Clear[λ];

$A[0] = A[0]/.k[m]$;

Print["Solution of A"[m], "=", $A[0]$];

For[$m = 1, m < n + 1, m++$,

TermA[m] = Expand[(TermA[$m - 1$] – $A[m - 1]$)/λ];

TermY[m] = Expand[(TermY[$m-1$] − $A[m-1]$)/λ];

$\lambda = 0$;

$k[m]$ = Expand [Solve[TermA[m] == TermY[m], $A[m]$]]; (*ComputedA_n*)

Clear[λ];

$A[m] = A[m]/.k[m]$;

Print["Solution of A"[m], "=", $A[m]$];

]

Print[" ********** Finish solution of Adomian polyn.********** "];

********** Start solution of Adomian polyn.**********

Solution of A[0]={1}

Solution of A[1] = $\{\frac{3y[1]}{2}\}$

Solution of A[2] = $\{\frac{3y[1]^2}{8} + \frac{3y[2]}{2}\}$

Solution of A[3] = $\{-\frac{1}{16}y[1]^3 + \frac{3}{4}y[1]y[2] + \frac{3y[3]}{2}\}$

Solution of A[4] = $\{\frac{3y[1]^4}{128} - \frac{3}{16}y[1]^2y[2] + \frac{3y[2]^2}{8} + \frac{3}{4}y[1]y[3] + \frac{3y[4]}{2}\}$

Solution of A[5]

$$= \{-\frac{3}{256}y[1]^5 + \frac{3}{32}y[1]^3y[2] - \frac{3}{16}y[1]y[2]^2 - \frac{3}{16}y[1]^2y[3] + \frac{3}{4}y[2]y[3] + \frac{3}{4}y[1]y[4] + \frac{3y[5]}{2}\}$$

Solution of A[6]

$$= \left\{\frac{7y[1]^6}{1024} - \frac{15}{256}y[1]^4y[2] + \frac{9}{64}y[1]^2y[2]^2 - \frac{y[2]^3}{16} + \frac{3}{32}y[1]^3y[3] - \frac{3}{8}y[1]y[2]y[3] + \frac{3y[3]^2}{8} - \frac{3}{16}y[1]^2y[4] + \frac{3}{4}y[2]y[4] + \frac{3}{4}y[1]y[5] + \frac{3y[6]}{2}\right\}$$

Solution of A[7]

$$= \left\{ -\frac{9y[1]^7}{2048} + \frac{21}{512}y[1]^5y[2] - \frac{15}{128}y[1]^3y[2]^2 + \frac{3}{32}y[1]y[2]^3 \right.$$
$$- \frac{15}{256}y[1]^4y[3] + \frac{9}{32}y[1]^2y[2]y[3] - \frac{3}{16}y[2]^2y[3] - \frac{3}{16}y[1]y[3]^2$$
$$+ \frac{3}{32}y[1]^3y[4] - \frac{3}{8}y[1]y[2]y[4] + \frac{3}{4}y[3]y[4] - \frac{3}{16}y[1]^2y[5]$$
$$\left. + \frac{3}{4}y[2]y[5] + \frac{3}{4}y[1]y[6] + \frac{3y[7]}{2} \right\}$$

Solution of A[8]

$$= \{ \frac{99y[1]^8}{32768} - \frac{63y[1]^6y[2]}{2048} + \frac{105y[1]^4y[2]^2}{1024} - \frac{15}{128}y[1]^2y[2]^3$$
$$+ \frac{3y[2]^4}{128} + \frac{21}{512}y[1]^5y[3] - \frac{15}{64}y[1]^3y[2]y[3] + \frac{9}{32}y[1]y[2]^2y[3]$$
$$+ \frac{9}{64}y[1]^2y[3]^2 - \frac{3}{16}y[2]y[3]^2 - \frac{15}{256}y[1]^4y[4]$$
$$+ \frac{9}{32}y[1]^2y[2]y[4] - \frac{3}{16}y[2]^2y[4] - \frac{3}{8}y[1]y[3]y[4] + \frac{3y[4]^2}{8}$$
$$+ \frac{3}{32}y[1]^3y[5] - \frac{3}{8}y[1]y[2]y[5] + \frac{3}{4}y[3]y[5] - \frac{3}{16}y[1]^2y[6]$$
$$+ \frac{3}{4}y[2]y[6] + \frac{3}{4}y[1]y[7] + \frac{3y[8]}{2} \}$$

Solution of A[9]

$$= \left\{ -\frac{143y[1]^9}{65536} + \frac{99y[1]^7y[2]}{4096} - \frac{189y[1]^5y[2]^2}{2048} + \frac{35}{256}y[1]^3y[2]^3 \right.$$
$$- \frac{15}{256}y[1]y[2]^4 - \frac{63y[1]^6y[3]}{2048} + \frac{105}{512}y[1]^4y[2]y[3]$$
$$- \frac{45}{128}y[1]^2y[2]^2y[3] + \frac{3}{32}y[2]^3y[3] - \frac{15}{128}y[1]^3y[3]^2$$
$$+ \frac{9}{32}y[1]y[2]y[3]^2 - \frac{y[3]^3}{16} + \frac{21}{512}y[1]^5y[4] - \frac{15}{64}y[1]^3y[2]y[4]$$
$$+ \frac{9}{32}y[1]y[2]^2y[4] + \frac{9}{32}y[1]^2y[3]y[4] - \frac{3}{8}y[2]y[3]y[4]$$
$$- \frac{3}{16}y[1]y[4]^2 - \frac{15}{256}y[1]^4y[5] + \frac{9}{32}y[1]^2y[2]y[5] - \frac{3}{16}y[2]^2y[5]$$

$$-\frac{3}{8}y[1]y[3]y[5]+\frac{3}{4}y[4]y[5]+\frac{3}{32}y[1]^3y[6]-\frac{3}{8}y[1]y[2]y[6]+\frac{3}{4}y[3]y[6]$$
$$-\frac{3}{16}y[1]^2y[7]+\frac{3}{4}y[2]y[7]+\frac{3}{4}y[1]y[8]+\frac{3y[9]}{2}\Big\}$$

Solution of A[10]

$$=\{\frac{429y[1]^{10}}{262144}-\frac{1287y[1]^8y[2]}{65536}+\frac{693y[1]^6y[2]^2}{8192}-\frac{315y[1]^4y[2]^3}{2048}$$
$$+\frac{105y[1]^2y[2]^4}{1024}-\frac{3y[2]^5}{256}+\frac{99y[1]^7y[3]}{4096}-\frac{189y[1]^5y[2]y[3]}{1024}$$
$$+\frac{105}{256}y[1]^3y[2]^2y[3]-\frac{15}{64}y[1]y[2]^3y[3]+\frac{105y[1]^4y[3]^2}{1024}$$
$$-\frac{45}{128}y[1]^2y[2]y[3]^2+\frac{9}{64}y[2]^2y[3]^2+\frac{3}{32}y[1]y[3]^3$$
$$-\frac{63y[1]^6y[4]}{2048}+\frac{105}{512}y[1]^4y[2]y[4]-\frac{45}{128}y[1]^2y[2]^2y[4]$$
$$+\frac{3}{32}y[2]^3y[4]-\frac{15}{64}y[1]^3y[3]y[4]+\frac{9}{16}y[1]y[2]y[3]y[4]$$
$$-\frac{3}{16}y[3]^2y[4]+\frac{9}{64}y[1]^2y[4]^2-\frac{3}{16}y[2]y[4]^2+\frac{21}{512}y[1]^5y[5]$$
$$-\frac{15}{64}y[1]^3y[2]y[5]+\frac{9}{32}y[1]y[2]^2y[5]+\frac{9}{32}y[1]^2y[3]y[5]$$
$$-\frac{3}{8}y[2]y[3]y[5]-\frac{3}{8}y[1]y[4]y[5]+\frac{3y[5]^2}{8}-\frac{15}{256}y[1]^4y[6]$$
$$+\frac{9}{32}y[1]^2y[2]y[6]-\frac{3}{16}y[2]^2y[6]-\frac{3}{8}y[1]y[3]y[6]+\frac{3}{4}y[4]y[6]$$
$$+\frac{3}{32}y[1]^3y[7]-\frac{3}{8}y[1]y[2]y[7]+\frac{3}{4}y[3]y[7]-\frac{3}{16}y[1]^2y[8]$$
$$+\frac{3}{4}y[2]y[8]+\frac{3}{4}y[1]y[9]+\frac{3y[10]}{2}\}$$

**********Finish solution of Adomianpolyn.**********

(*Calculated $y_n(x)$*)

Print[" ********** Start solution of $y_n(x)$ ********** "];

$y[1] = (B * x) + \text{Integrate}[\text{Integrate}[x^{-1/2} * A[0], x], x]$;

Print["Solution of y"[0], "=", $y[0]$];

Print["Solution of y"[1], "=", $y[1]$];

$i = 2$; While$[i \leq n + 1,$

$y[i] =$ Expand $\left[\text{Integrate}\left[\text{Integrate}\left[x^{-1/2} * A[i-1], x\right], x\right]\right];$

Print["Solution of y"$[i]$, "=", $y[i]$];

$i++$];

Print[" ********** Finish solution of $y_n(x)$ ********** "];

********** Start solution of $y_n(x)$ **********

Solution of y[0] $= 1$

Solution of y[1] $= \{Bx + \frac{4x^{3/2}}{3}\}$

Solution of y[2] $= \{\{\frac{2}{5}Bx^{5/2} + \frac{x^3}{3}\}\}$

Solution of y[3] $= \{\{\{\frac{3}{70}B^2x^{7/2} + \frac{2Bx^4}{15} + \frac{2x^{9/2}}{27}\}\}\}$

Solution of y[4] $= \{\{\{\{-\frac{1}{252}B^3x^{9/2} + \frac{B^2x^5}{175} + \frac{31Bx^{11/2}}{1485} + \frac{4x^6}{405}\}\}\}\}$

Solution of y[5] $= \{\{\{\{\{\frac{B^4x^{11/2}}{1056} + \frac{4B^3x^6}{1575} + \frac{557B^2x^{13/2}}{100100} + \frac{4Bx^7}{693} + \frac{101x^{15/2}}{52650}\}\}\}\}\}$

Solution of y[6]

$$= \{\{\{\{\{\{-\frac{3B^5x^{13/2}}{9152} - \frac{29B^4x^7}{24255} - \frac{623B^3x^{15/2}}{351000} - \frac{46B^2x^8}{45045} - \frac{113Bx^{17/2}}{1178100} + \frac{23x^9}{473850}\}\}\}\}\}\}$$

Solution of y[7]

$$= \{\{\{\{\{\{\{\frac{7B^6x^{15/2}}{49920} + \frac{68B^5x^8}{105105} + \frac{153173B^4x^{17/2}}{116424000} + \frac{1046B^3x^9}{675675} + \frac{799399B^2x^{19/2}}{698377680} + \frac{51356Bx^{10}}{103378275} + \frac{35953x^{21/2}}{378132300}\}\}\}\}\}\}\}$$

Solution of y[8]

$$= \{\{\{\{\{\{\{-\frac{3B^7x^{17/2}}{43520} - \frac{4B^6x^9}{10395} - \frac{1232941B^5x^{19/2}}{1278076800} - \frac{99856B^4x^{10}}{70945875} - \frac{33232663B^3x^{21/2}}{25881055200} - \frac{250054B^2x^{11}}{342953325} - \frac{22773977Bx^{23/2}}{95108013000} - \frac{823x^{12}}{23108085}\}\}\}\}\}\}\}$$

Solution of y[9]

$$= \{\{\{\{\{\{\{\{\frac{99B^8x^{19/2}}{2646016} + \frac{256B^7x^{10}}{1044225} + \frac{705965027B^6x^{21/2}}{966226060800} + \frac{43468B^5x^{11}}{33622875} + \frac{1861464749B^4x^{23/2}}{1253187936000} + \frac{27134428B^3x^{12}}{23880381525} + \frac{17319117797B^2x^{25/2}}{30580884180000} + \frac{494880923Bx^{13}}{2936459901375} + \frac{172159489x^{27/2}}{7487019540000}\}\}\}\}\}\}\}\}$$

Solution of y[10]

$$= \{\{\{\{\{\{\{\{\{-\frac{143B^9x^{21/2}}{6537216} - \frac{6272B^8x^{11}}{38105925} - \frac{4524629159B^7x^{23/2}}{7953566100480} - \frac{14756758B^6x^{12}}{12455257815} - \frac{2383837819589B^5x^{25/2}}{1455090436800000} - \frac{96201013897B^4x^{13}}{61665657928875} - \frac{308663609101B^3x^{27/2}}{301554110088000} - \frac{303011686294B^2x^{14}}{678322237217625} - \frac{1487118494129Bx^{29/2}}{12613018163803200} - \frac{50500903x^{15}}{3509540409375}\}\}\}\}\}\}\}\}\}$$

Solution of y[11]

$$= \{\{\{\{\{\{\{\{\{\{\frac{143B^{10}x^{23/2}}{10551296} + \frac{2048B^{9}x^{12}}{17782765} + \frac{144926432597B^{8}x^{25/2}}{319825938432000}$$
$$+ \frac{3474669398B^{7}x^{13}}{3197478559275} + \frac{50468746588277B^{6}x^{27/2}}{28696914005760000}$$
$$+ \frac{8824737028748B^{5}x^{14}}{4396533019003125} + \frac{605561615787857B^{4}x^{29/2}}{369981866138227200}$$
$$+ \frac{3016967993986B^{3}x^{15}}{3201870700153125} + \frac{31823185257584653B^{2}x^{31/2}}{86728773867896160000}$$
$$+ \frac{15076074226306Bx^{16}}{172443607708246875} + \frac{3220476338281x^{33/2}}{331899576208200000}\}\}\}\}\}\}\}\}\}\}$$

********** Finish solution of $y_n(x)$ **********

(*calculatedapprox. for$y(x)$*)

Print[" ********** Start solution of y(x) ********** "];

Clear[x]; Expand[SeriesY = Sum[$y[i]$, $\{i, 0, n+1\}$]];

Print["Solution of y(x) =", SeriesY];

Print[" ********** Finish solution of y(x) ********** "];

********** Start solution of y(x) **********

Solution of y(x)

$$= \{\{\{\{\{\{\{\{\{\{\{1 + Bx + \frac{4x^{3/2}}{3} + \frac{2}{5}Bx^{5/2} + \frac{x^3}{3} + \frac{3}{70}B^2x^{7/2} + \frac{2Bx^4}{15}$$
$$+ \frac{2x^{9/2}}{27} - \frac{1}{252}B^3x^{9/2} + \frac{B^2x^5}{175} + \frac{31Bx^{11/2}}{1485} + \frac{B^4x^{11/2}}{1056} + \frac{4x^6}{405}$$
$$+ \frac{4B^3x^6}{1575} + \frac{557B^2x^{13/2}}{100100} - \frac{3B^5x^{13/2}}{9152} + \frac{4Bx^7}{693} - \frac{29B^4x^7}{24255} + \frac{101x^{15/2}}{52650}$$
$$- \frac{623B^3x^{15/2}}{351000} + \frac{7B^6x^{15/2}}{49920} - \frac{46B^2x^8}{45045} + \frac{68B^5x^8}{105105} - \frac{113Bx^{17/2}}{1178100}$$
$$+ \frac{153173B^4x^{17/2}}{116424000} - \frac{3B^7x^{17/2}}{43520} + \frac{23x^9}{473850} + \frac{1046B^3x^9}{675675} - \frac{4B^6x^9}{10395}$$
$$+ \frac{799399B^2x^{19/2}}{698377680} - \frac{1232941B^5x^{19/2}}{1278076800} + \frac{99B^8x^{19/2}}{2646016} + \frac{51356Bx^{10}}{103378275}$$

$$-\frac{99856B^4x^{10}}{70945875}+\frac{256B^7x^{10}}{1044225}+\frac{35953x^{21/2}}{378132300}-\frac{33232663B^3x^{21/2}}{25881055200}$$
$$+\frac{705965027B^6x^{21/2}}{966226060800}-\frac{143B^9x^{21/2}}{6537216}-\frac{250054B^2x^{11}}{342953325}$$
$$+\frac{43468B^5x^{11}}{33622875}-\frac{6272B^8x^{11}}{38105925}-\frac{22773977Bx^{23/2}}{95108013000}$$
$$+\frac{1861464749B^4x^{23/2}}{1253187936000}-\frac{4524629159B^7x^{23/2}}{7953566100480}+\frac{143B^{10}x^{23/2}}{10551296}$$
$$-\frac{823x^{12}}{23108085}+\frac{27134428B^3x^{12}}{23880381525}-\frac{14756758B^6x^{12}}{12455257815}+\frac{2048B^9x^{12}}{17782765}$$
$$+\frac{17319117797B^2x^{25/2}}{30580884180000}-\frac{2383837819589B^5x^{25/2}}{1455090436800000}$$
$$+\frac{144926432597B^8x^{25/2}}{319825938432000}+\frac{494880923Bx^{13}}{2936459901375}$$
$$-\frac{96201013897B^4x^{13}}{61665657928875}+\frac{3474669398B^7x^{13}}{3197478559275}+\frac{172159489x^{27/2}}{7487019540000}$$
$$-\frac{308663609101B^3x^{27/2}}{301554110088000}+\frac{50468746588277B^6x^{27/2}}{28696914005760000}$$
$$-\frac{303011686294B^2x^{14}}{678322237217625}+\frac{8824737028748B^5x^{14}}{4396533019003125}$$
$$-\frac{1487118494129Bx^{29/2}}{12613018163803200}+\frac{605561615787857B^4x^{29/2}}{369981866138227200}$$
$$-\frac{50500903x^{15}}{3509540409375}+\frac{3016967993986B^3x^{15}}{3201870700153125}$$
$$+\frac{31823185257584653B^2x^{31/2}}{867287738678961600000}+\frac{150760742263068x^{16}}{172443607708246875}$$
$$+\frac{3220476338281x^{33/2}}{331899576208200000}\}\}\}\}\}\}\}\}\}\}$$

********** Finish solution of y(x) **********

(*calculated convert from $y(x)$ to $y(t)$*)

Print[" ********** Start solution of y(t) ********** "];

$x = t\text{^}2;$

seriesT = Normal[Series[SeriesY, {t, 0,35}]];

Print["Solution of y(t) =", seriesT];

Print[" ********** Finish solution of y(t) ********** "];

********** Start solution of y(t) **********

Solution of y(t)

$$= \{\{\{\{\{\{\{\{\{\{1 + Bt^2 + \frac{4t^3}{3} + \frac{2Bt^5}{5} + \frac{t^6}{3} + \frac{3B^2t^7}{70} + \frac{2Bt^8}{15} + (\frac{2}{27} - \frac{B^3}{252})t^9 + \frac{B^2t^{10}}{175} + (\frac{31B}{1485} + \frac{B^4}{1056})t^{11} + (\frac{4}{405} + \frac{4B^3}{1575})t^{12} + (\frac{557B^2}{100100} - \frac{3B^5}{9152})t^{13} + (\frac{4B}{693} - \frac{29B^4}{24255})t^{14} + (\frac{101}{52650} - \frac{623B^3}{351000} + \frac{7B^6}{49920})t^{15} + (-\frac{46B^2}{45045} + \frac{68B^5}{105105})t^{16} + (-\frac{113B}{1178100} + \frac{153173B^4}{116424000} - \frac{3B^7}{43520})t^{17} + (\frac{23}{473850} + \frac{1046B^3}{675675} - \frac{4B^6}{10395})t^{18} + (\frac{799399B^2}{698377680} - \frac{1232941B^5}{1278076800} + \frac{99B^8}{2646016})t^{19} + (\frac{51356B}{103378275} - \frac{99856B^4}{70945875} + \frac{256B^7}{1044225})t^{20} + (\frac{35953}{378132300} - \frac{33232663B^3}{25881055200} + \frac{705965027B^6}{966226060800} - \frac{143B^9}{6537216})t^{21} + (-\frac{250054B^2}{342953325} + \frac{43468B^5}{33622875} - \frac{6272B^8}{38105925})t^{22} + (-\frac{22773977B}{95108013000} + \frac{1861464749B^4}{1253187936000} - \frac{4524629159B^7}{7953566100480} + \frac{143B^{10}}{10551296})t^{23} + (-\frac{823}{23108085} + \frac{27134428B^3}{23880381525} - \frac{14756758B^6}{12455257815} + \frac{2048B^9}{17782765})t^{24} + (\frac{17319117797B^2}{30580884180000} - \frac{2383837819589B^5}{1455090436800000} + \frac{144926432597B^8}{319825938432000})t^{25}$$

$$+\left(\frac{494880923B}{2936459901375}-\frac{96201013897B^4}{61665657928875}+\frac{3474669398B^7}{3197478559275}\right)t^{26}$$
$$+\left(\frac{172159489}{7487019540000}-\frac{308663609101B^3}{301554110088000}\right.$$
$$\left.+\frac{50468746588277B^6}{28696914005760000}\right)t^{27}+\left(-\frac{303011686294B^2}{678322237217625}\right.$$
$$\left.+\frac{8824737028748B^5}{4396533019003125}\right)t^{28}+\left(-\frac{1487118494129B}{12613018163803200}\right.$$
$$\left.+\frac{605561615787857B^4}{369981866138227200}\right)t^{29}+\left(-\frac{50500903}{3509540409375}\right.$$
$$\left.+\frac{3016967993986B^3}{3201870700153125}\right)t^{30}+\frac{31823185257584653B^2t^{31}}{86728773867896160000}$$
$$+\frac{15076074226306Bt^{32}}{172443607708246875}+\frac{3220476338281t^{33}}{331899576208200000}\}\}\}\}\}\}\}\}\}\}$$

********** Finish solution of y(t) **********

$$\text{seriesT1} = 1 + Bt^2 + \frac{4t^3}{3} + \frac{2Bt^5}{5} + \frac{t^6}{3} + \frac{3B^2t^7}{70} + \frac{2Bt^8}{15} + (\frac{2}{27} - \frac{B^3}{252})t^9 + \frac{B^2t^{10}}{175}$$
$$+ (\frac{31B}{1485} + \frac{B^4}{1056})t^{11} + (\frac{4}{405} + \frac{4B^3}{1575})t^{12} + (\frac{557B^2}{100100} - \frac{3B^5}{9152})t^{13}$$
$$+ (\frac{4B}{693} - \frac{29B^4}{24255})t^{14} + (\frac{101}{52650} - \frac{623B^3}{351000} + \frac{7B^6}{49920})t^{15} + (-\frac{46B^2}{45045}$$
$$+ \frac{68B^5}{105105})t^{16} + (-\frac{113B}{1178100} + \frac{153173B^4}{116424000} - \frac{3B^7}{43520})t^{17}$$
$$+ (\frac{23}{473850} + \frac{1046B^3}{675675} - \frac{4B^6}{10395})t^{18} + (\frac{799399B^2}{698377680} - \frac{1232941B^5}{1278076800}$$
$$+ \frac{99B^8}{2646016})t^{19} + (\frac{51356B}{103378275} - \frac{99856B^4}{70945875} + \frac{256B^7}{1044225})t^{20}$$
$$+ (\frac{35953}{378132300} - \frac{33232663B^3}{25881055200} + \frac{705965027B^6}{966226060800} - \frac{143B^9}{6537216})t^{21}$$
$$+ (-\frac{250054B^2}{342953325} + \frac{43468B^5}{33622875} - \frac{6272B^8}{38105925})t^{22}$$
$$+ (-\frac{22773977B}{95108013000} + \frac{1861464749B^4}{1253187936000} - \frac{4524629159B^7}{7953566100480}$$
$$+ \frac{143B^{10}}{10551296})t^{23} + (-\frac{823}{23108085} + \frac{27134428B^3}{23880381525} - \frac{14756758B^6}{12455257815}$$
$$+ \frac{2048B^9}{17782765})t^{24} + (\frac{17319117797B^2}{30580884180000} - \frac{2383837819589B^5}{1455090436800000}$$
$$+ \frac{144926432597B^8}{319825938432000})t^{25} + (\frac{494880923B}{2936459901375} - \frac{96201013897B^4}{61665657928875}$$
$$+ \frac{3474669398B^7}{3197478559275})t^{26} + (\frac{172159489}{7487019540000} - \frac{308663609101B^3}{301554110088000}$$
$$+ \frac{504687465882 77B^6}{28696914005760000})t^{27} + (-\frac{303011686294B^2}{678322237217625}$$
$$+ \frac{8824737028748B^5}{4396533019003125})t^{28} + (-\frac{1487118494129B}{12613018163803200}$$
$$+ \frac{605561615787857B^4}{369981866138227200})t^{29} + (-\frac{50500903}{3509540409375}$$
$$+ \frac{3016967993986B^3}{3201870700153125})t^{30} + \frac{31823185257584653B^2t^{31}}{86728773867896160000}$$
$$+ \frac{15076074226306Bt^{32}}{172443607708246875} + \frac{3220476338281t^{33}}{331899576208200000};$$

```
Array[padeSoln, 20,2];
Array[padeSolnOFx, 20,2];
(*calculation for Padeapprox.*)
j = 2;
Print[" ********** Start solution of Pade approx. [2/2] ********** "];
padeSoln[j] = PadeApproximant[seriesT1, {t, 0, {j, j}}](*Padeapprox. [2/2]*)
Print["Solution in term of t: [", j, "/", j, "]", " = ", padeSoln[j]];
r = Numerator[padeSoln[j]];
s = Expand[r/t^j];
t = ∞;
bb = FindRoot[s == 0, {B, −1.588071}]; (*comparison root by Kobayashi*)
B = B/. bb;
Print["Initial slope B for[", j, "/", j, "]", "=", B];
errB
= Abs[(B − (−1.588071))/(−1.588071)]100; (*comparison by Kobayashi*)
Print["Error(%) of Initial slope B for[", j, "/", j, "]", "=", errB];
Clear[errB];

Clear[t, x];
padeSolnOFx[j] = Expand[padeSoln[j]/. t → x^(1/2)];
Print["Solution in term of x: [", j, "/", j, "]", " = ", padeSolnOFx[j]];
Clear[t, r, s, j, B, bb];
Print[" ********** Finish solution of Pade approx. [2/2] ********** "];
```

********** Start solution of Pade approx. [2/2] **********

Solution in term of t: $[2/2] = \dfrac{1 - \frac{4t}{3B} + \frac{(16+9B^3)t^2}{9B^2}}{1 - \frac{4t}{3B} + \frac{16t^2}{9B^2}}$

Initial slope B for[2/2] = −1.2114137285547597

Error(%) of Initial slope B for[2/2] = 23.717911317897013

Solution in term of x: [2/2]

$$= \frac{1}{1 + 1.100642416298209\sqrt{x} + 1.21141372855476x} + \frac{1.100642416298209\sqrt{x}}{1 + 1.100642416298209\sqrt{x} + 1.21141372855476x} + \frac{2.689878827577008 \times 10^{-16}x}{1 + 1.100642416298209\sqrt{x} + 1.21141372855476x}$$

********** Finish solution of Pade approx. [2/2] **********

$j = 4;$

Print[" ********** Start solution of Pade approx. [4/4] ********** "];

padeSoln[j] = PadeApproximant[seriesT1, $\{t, 0, \{j, j\}\}$]; (*Padeapprox. [4/4]*)

Print["Solution in term of t: [",j,"/",j,"]"," = ", padeSoln[j]];

r = Numerator[padeSoln[j]];

s = Expand[r/t^j];

$t = \infty;$

bb = FindRoot[$s == 0, \{B, -1.588071\}$]; (*comparisonroot by Kobayashi*)

$B = B/.$ bb;

Print["Initial slope B for[",j,"/",j,"]","=",B];

errB

= Abs[$(B - (-1.588071))/(-1.588071)$]100; (*comparison by Kobayashi*)

Print["Error(%) of Initial slope B for[",j,"/",j,"]","=", errB];

Clear[errB];

Clear[t, x];

padeSolnOFx[j] = Expand[padeSoln[j]/. $t \to x$^(1/2)];

Print["Solution in term of x: [",j,"/",j,"]"," = ", padeSolnOFx[j]];

Clear[t, r, s, j, B, bb];

Print[" ********** Finish solution of Pade approx. [4/4] ********** "];

********** Start solution of Pade approx. [4/4] **********

Solution in term of t: [4/4] =

$$\frac{1+\left(-\frac{35}{18B}-\frac{-6125-1674B^3}{30B(97+27B^3)}\right)t+\frac{B(12236+3375B^3)t^2}{140(97+27B^3)}+\frac{(14980+4077B^3)t^3}{126(97+27B^3)}}{1+\left(-\frac{35}{18B}-\frac{-6125-1674B^3}{30B(97+27B^3)}\right)t-\frac{3B(448+135B^3)t^2}{140(97+27B^3)}+\frac{(-910-243B^3)t^3}{35(97+27B^3)}}$$

$$+\frac{\frac{(318500-118368B^3-54675B^6)t^4}{18900B(97+27B^3)}}{+\frac{(-6125-1674B^3)t^4}{1575B(97+27B^3)}}$$

Initial slope B for[4/4] = −1.5505259190109406

Error(%) of Initial slope B for[4/4] = 2.3641941065014977

********** Finish solution of Pade approx. [4/4] **********

```
j = 7;
Print[" ********** Start solution of Pade approx. [7/7] ********** "];
padeSoln[j] = PadeApproximant[seriesT1, {t, 0, {j, j}}]; (*Padeapprox. [7/7]*)
r = Numerator[padeSoln[j]];
s = Expand[r/t^j];
t = ∞;
bb = FindRoot[s == 0, {B, −1.588071}]; (*comparisonroot by Kobayashi*)
B = B/. bb;
Print["Initial slope B for[", j, "/", j, "]", "=", B];
errB
= Abs[(B − (−1.588071))/(−1.588071)]100; (*comparison by Kobayashi*)
Print["Error(%) of Initial slope B for[", j, "/", j, "]", "=", errB];
Clear[errB];

Clear[t, x];
padeSolnOFx[j] = Expand[padeSoln[j]/. t → x^(1/2)];
Print["Solution in term of x: [", j, "/", j, "]", " = ", padeSolnOFx[j]];
Clear[t, r, s, j, B, bb];
  Print[" ********** Finish solution of Pade approx. [7/7] ********** "];
```

********** Start solution of Pade approx. [7/7] **********

```
Initial slope B for[7/7] = −1.5860210348569346
Error(%) of Initial slope B for[7/7] = 0.12908523252835893
********** Finish solution of Pade approx. [7/7] **********


j = 8;
Print[" ********** Start solution of Pade approx. [8/8] ********** "];
padeSoln[j] = PadeApproximant[seriesT1, {t, 0, {j, j}}]; (*Padeapprox. [8/8]*)
r = Numerator[padeSoln[j]];
s = Expand[r/t^j];
t = ∞;
bb = FindRoot[s == 0, {B, −1.588071}]; (*comparisonroot by Kobayashi*)
B = B/. bb;
Print["Initial slope B for[", j, "/", j, "]", "=", B];
errB
= Abs[(B − (−1.588071))/(−1.588071)]100; (*comparison by Kobayashi*)
Print["Error(%) of Initial slope B for[", j, "/", j, "]", "=", errB];
Clear[errB];

Clear[t, x];
padeSolnOFx[j] = Expand[padeSoln[j]/. t → x^(1/2)];
Print["Solution in term of x: [", j, "/", j, "]", " = ", padeSolnOFx[j]];
Clear[t, r, s, j, B, bb];
Print[" ********** Finish solution of Pade approx. [8/8] ********** "];

********** Start solution of Pade approx. [8/8] **********
Initial slope B for[8/8] = −1.5880768204351274
Error(%) of Initial slope B for[8/8] = 0.00036650975475427774
********** Finish solution of Pade approx. [8/8] **********
```

$j = 10;$

Array$[a, j + 1,0];$

Array$[b, j];$

$$\text{Terma} = \sum_{k=0}^{j} a[k]t\text{^}k;$$

$$\text{Termb} = 1 + \sum_{k=1}^{j} b[k]t\text{^}k;$$

ExpandSeriesT1 = Normal[Series[seriesT1, $\{t, 0,2j\}$]];

Array[TermT, $2j + 1,0];$

Array$[T, 2j + 1,0];$

TermT[0] = Expand[ExpandSeriesT1 * Termb];

For$[v = 0, v < 2j + 1, v + +,$

$t = 0;$

$T[v] =$ TermT$[v];$

Clear$[t];$

TermT$[v + 1]$ = Expand[(TermT$[v] - T[v])/t];$

]

$a[0] =$ a0; $a[1] =$ a1; $a[2] =$ a2; $a[3] =$ a3; $a[4] =$ a4; $a[5] =$ a5; $a[6] =$ a6; $a[7]$ = a7; $a[8] =$ a8; $a[9] =$ a9; $a[10] =$ a10;

$b[1] =$ b1; $b[2] =$ b2; $b[3] =$ b3; $b[4] =$ b4; $b[5] =$ b5; $b[6] =$ b6; $b[7] =$ b7; $b[8]$ = b8; $b[9] =$ b9; $b[10] =$ b10;

For[v1 = 0, v1 < $2j + 1$, v1 + +,

Print["coefficient of t^", v1, " = ", T[v1]];

]

Clear$[a, b];$

coefficient of t^0 = 1

coefficient of t^1 = b1

coefficient of t^2 = B + b2

coefficient of t^3 = $\frac{4}{3}$ + Bb1 + b3

coefficient of t^4 $= \frac{4b1}{3} + Bb2 + b4$

coefficient of t^5 $= \frac{2B}{5} + \frac{4b2}{3} + Bb3 + b5$

coefficient of t^6 $= \frac{1}{3} + \frac{2Bb1}{5} + \frac{4b3}{3} + Bb4 + b6$

coefficient of t^7 $= \frac{3B^2}{70} + \frac{b1}{3} + \frac{2Bb2}{5} + \frac{4b4}{3} + Bb5 + b7$

coefficient of t^8 $= \frac{2B}{15} + \frac{3B^2b1}{70} + \frac{b2}{3} + \frac{2Bb3}{5} + \frac{4b5}{3} + Bb6 + b8$

coefficient of t^9 $= \frac{2}{27} - \frac{B^3}{252} + \frac{2Bb1}{15} + \frac{3B^2b2}{70} + \frac{b3}{3} + \frac{2Bb4}{5} + \frac{4b6}{3} + Bb7 + b9$

coefficient of t^10

$$= \frac{B^2}{175} + \frac{2b1}{27} - \frac{B^3b1}{252} + b10 + \frac{2Bb2}{15} + \frac{3B^2b3}{70} + \frac{b4}{3} + \frac{2Bb5}{5} + \frac{4b7}{3} + Bb8$$

coefficient of t^11

$$= \frac{31B}{1485} + \frac{B^4}{1056} + \frac{B^2b1}{175} + \frac{2b2}{27} - \frac{B^3b2}{252} + \frac{2Bb3}{15} + \frac{3B^2b4}{70} + \frac{b5}{3} + \frac{2Bb6}{5} + \frac{4b8}{3} + Bb9$$

coefficient of t^12

$$= \frac{4}{405} + \frac{4B^3}{1575} + \frac{31Bb1}{1485} + \frac{B^4b1}{1056} + Bb10 + \frac{B^2b2}{175} + \frac{2b3}{27} - \frac{B^3b3}{252} + \frac{2Bb4}{15} + \frac{3B^2b5}{70} + \frac{b6}{3} + \frac{2Bb7}{5} + \frac{4b9}{3}$$

coefficient of t^13

$$= \frac{557B^2}{100100} - \frac{3B^5}{9152} + \frac{4b1}{405} + \frac{4B^3b1}{1575} + \frac{4b10}{3} + \frac{31Bb2}{1485} + \frac{B^4b2}{1056} + \frac{B^2b3}{175} + \frac{2b4}{27} - \frac{B^3b4}{252} + \frac{2Bb5}{15} + \frac{3B^2b6}{70} + \frac{b7}{3} + \frac{2Bb8}{5}$$

coefficient of t^14

$$= \frac{4B}{693} - \frac{29B^4}{24255} + \frac{557B^2b1}{100100} - \frac{3B^5b1}{9152} + \frac{4b2}{405} + \frac{4B^3b2}{1575} + \frac{31Bb3}{1485} + \frac{B^4b3}{1056} + \frac{B^2b4}{175} + \frac{2b5}{27} - \frac{B^3b5}{252} + \frac{2Bb6}{15} + \frac{3B^2b7}{70} + \frac{b8}{3} + \frac{2Bb9}{5}$$

coefficient of t^15

$$= \frac{101}{52650} - \frac{623B^3}{351000} + \frac{7B^6}{49920} + \frac{4B\mathrm{b}1}{693} - \frac{29B^4\mathrm{b}1}{24255} + \frac{2B\mathrm{b}10}{5}$$
$$+ \frac{557B^2\mathrm{b}2}{100100} - \frac{3B^5\mathrm{b}2}{9152} + \frac{4\mathrm{b}3}{405} + \frac{4B^3\mathrm{b}3}{1575} + \frac{31B\mathrm{b}4}{1485} + \frac{B^4\mathrm{b}4}{1056} + \frac{B^2\mathrm{b}5}{175}$$
$$+ \frac{2\mathrm{b}6}{27} - \frac{B^3\mathrm{b}6}{252} + \frac{2B\mathrm{b}7}{15} + \frac{3B^2\mathrm{b}8}{70} + \frac{\mathrm{b}9}{3}$$

coefficient of t^16

$$= -\frac{46B^2}{45045} + \frac{68B^5}{105105} + \frac{101\mathrm{b}1}{52650} - \frac{623B^3\mathrm{b}1}{351000} + \frac{7B^6\mathrm{b}1}{49920} + \frac{\mathrm{b}10}{3} + \frac{4B\mathrm{b}2}{693}$$
$$- \frac{29B^4\mathrm{b}2}{24255} + \frac{557B^2\mathrm{b}3}{100100} - \frac{3B^5\mathrm{b}3}{9152} + \frac{4\mathrm{b}4}{405} + \frac{4B^3\mathrm{b}4}{1575} + \frac{31B\mathrm{b}5}{1485} + \frac{B^4\mathrm{b}5}{1056}$$
$$+ \frac{B^2\mathrm{b}6}{175} + \frac{2\mathrm{b}7}{27} - \frac{B^3\mathrm{b}7}{252} + \frac{2B\mathrm{b}8}{15} + \frac{3B^2\mathrm{b}9}{70}$$

coefficient of t^17

$$= -\frac{113B}{1178100} + \frac{153173B^4}{116424000} - \frac{3B^7}{43520} - \frac{46B^2\mathrm{b}1}{45045} + \frac{68B^5\mathrm{b}1}{105105}$$
$$+ \frac{3B^2\mathrm{b}10}{70} + \frac{101\mathrm{b}2}{52650} - \frac{623B^3\mathrm{b}2}{351000} + \frac{7B^6\mathrm{b}2}{49920} + \frac{4B\mathrm{b}3}{693} - \frac{29B^4\mathrm{b}3}{24255}$$
$$+ \frac{557B^2\mathrm{b}4}{100100} - \frac{3B^5\mathrm{b}4}{9152} + \frac{4\mathrm{b}5}{405} + \frac{4B^3\mathrm{b}5}{1575} + \frac{31B\mathrm{b}6}{1485} + \frac{B^4\mathrm{b}6}{1056} + \frac{B^2\mathrm{b}7}{175}$$
$$+ \frac{2\mathrm{b}8}{27} - \frac{B^3\mathrm{b}8}{252} + \frac{2B\mathrm{b}9}{15}$$

coefficient of t^18

$$= \frac{23}{473850} + \frac{1046B^3}{675675} - \frac{4B^6}{10395} - \frac{113B\mathrm{b}1}{1178100} + \frac{153173B^4\mathrm{b}1}{116424000} - \frac{3B^7\mathrm{b}1}{43520}$$
$$+ \frac{2B\mathrm{b}10}{15} - \frac{46B^2\mathrm{b}2}{45045} + \frac{68B^5\mathrm{b}2}{105105} + \frac{101\mathrm{b}3}{52650} - \frac{623B^3\mathrm{b}3}{351000} + \frac{7B^6\mathrm{b}3}{49920}$$
$$+ \frac{4B\mathrm{b}4}{693} - \frac{29B^4\mathrm{b}4}{24255} + \frac{557B^2\mathrm{b}5}{100100} - \frac{3B^5\mathrm{b}5}{9152} + \frac{4\mathrm{b}6}{405} + \frac{4B^3\mathrm{b}6}{1575} + \frac{31B\mathrm{b}7}{1485}$$
$$+ \frac{B^4\mathrm{b}7}{1056} + \frac{B^2\mathrm{b}8}{175} + \frac{2\mathrm{b}9}{27} - \frac{B^3\mathrm{b}9}{252}$$

coefficient of t^19

$$= \frac{799399B^2}{698377680} - \frac{1232941B^5}{1278076800} + \frac{99B^8}{2646016} + \frac{23\text{b1}}{473850} + \frac{1046B^3\text{b1}}{675675}$$
$$- \frac{4B^6\text{b1}}{10395} + \frac{2\text{b10}}{27} - \frac{B^3\text{b10}}{252} - \frac{113B\text{b2}}{1178100} + \frac{153173B^4\text{b2}}{116424000} - \frac{3B^7\text{b2}}{43520}$$
$$- \frac{46B^2\text{b3}}{45045} + \frac{68B^5\text{b3}}{105105} + \frac{101\text{b4}}{52650} - \frac{623B^3\text{b4}}{351000} + \frac{7B^6\text{b4}}{49920} + \frac{4B\text{b5}}{693}$$
$$- \frac{29B^4\text{b5}}{24255} + \frac{557B^2\text{b6}}{100100} - \frac{3B^5\text{b6}}{9152} + \frac{4\text{b7}}{405} + \frac{4B^3\text{b7}}{1575} + \frac{31B\text{b8}}{1485} + \frac{B^4\text{b8}}{1056}$$
$$+ \frac{B^2\text{b9}}{175}$$

coefficient of t^20

$$= \frac{51356B}{103378275} - \frac{99856B^4}{70945875} + \frac{256B^7}{1044225} + \frac{799399B^2\text{b1}}{698377680}$$
$$- \frac{1232941B^5\text{b1}}{1278076800} + \frac{99B^8\text{b1}}{2646016} + \frac{B^2\text{b10}}{175} + \frac{23\text{b2}}{473850} + \frac{1046B^3\text{b2}}{675675}$$
$$- \frac{4B^6\text{b2}}{10395} - \frac{113B\text{b3}}{1178100} + \frac{153173B^4\text{b3}}{116424000} - \frac{3B^7\text{b3}}{43520} - \frac{46B^2\text{b4}}{45045} + \frac{68B^5\text{b4}}{105105}$$
$$+ \frac{101\text{b5}}{52650} - \frac{623B^3\text{b5}}{351000} + \frac{7B^6\text{b5}}{49920} + \frac{4B\text{b6}}{693} - \frac{29B^4\text{b6}}{24255} + \frac{557B^2\text{b7}}{100100}$$
$$- \frac{3B^5\text{b7}}{9152} + \frac{4\text{b8}}{405} + \frac{4B^3\text{b8}}{1575} + \frac{31B\text{b9}}{1485} + \frac{B^4\text{b9}}{1056}$$

Solve[1 == a0&&

b1 == a1&&

$B + \text{b2} == \text{a2}\&\&$

$\frac{4}{3} + B\text{b1} + \text{b3} == \text{a3}\&\&$

$\frac{4\text{b1}}{3} + B\text{b2} + \text{b4} == \text{a4}\&\&$

$\frac{2B}{5} + \frac{4\text{b2}}{3} + B\text{b3} + \text{b5} == \text{a5}\&\&$

$\frac{1}{3} + \frac{2B\text{b1}}{5} + \frac{4\text{b3}}{3} + B\text{b4} + \text{b6} == \text{a6}\&\&$

$$\frac{3B^2}{70}+\frac{\mathrm{b1}}{3}+\frac{2B\mathrm{b2}}{5}+\frac{4\mathrm{b4}}{3}+B\mathrm{b5}+\mathrm{b7}==\mathrm{a7}\&\&$$

$$\frac{2B}{15}+\frac{3B^2\mathrm{b1}}{70}+\frac{\mathrm{b2}}{3}+\frac{2B\mathrm{b3}}{5}+\frac{4\mathrm{b5}}{3}+B\mathrm{b6}+\mathrm{b8}==\mathrm{a8}\&\&$$

$$\frac{2}{27}-\frac{B^3}{252}+\frac{2B\mathrm{b1}}{15}+\frac{3B^2\mathrm{b2}}{70}+\frac{\mathrm{b3}}{3}+\frac{2B\mathrm{b4}}{5}+\frac{4\mathrm{b6}}{3}+B\mathrm{b7}+\mathrm{b9}==\mathrm{a9}\&\&$$

$$\frac{B^2}{175}+\frac{2\mathrm{b1}}{27}-\frac{B^3\mathrm{b1}}{252}+\mathrm{b10}+\frac{2B\mathrm{b2}}{15}+\frac{3B^2\mathrm{b3}}{70}+\frac{\mathrm{b4}}{3}+\frac{2B\mathrm{b5}}{5}+\frac{4\mathrm{b7}}{3}+B\mathrm{b8}=$$
$$=\mathrm{a10}\&\&$$

$$\frac{31B}{1485}+\frac{B^4}{1056}+\frac{B^2\mathrm{b1}}{175}+\frac{2\mathrm{b2}}{27}-\frac{B^3\mathrm{b2}}{252}+\frac{2B\mathrm{b3}}{15}+\frac{3B^2\mathrm{b4}}{70}+\frac{\mathrm{b5}}{3}+\frac{2B\mathrm{b6}}{5}+\frac{4\mathrm{b8}}{3}+B\mathrm{b9}$$
$$==0\&\&$$

$$\frac{4}{405}+\frac{4B^3}{1575}+\frac{31B\mathrm{b1}}{1485}+\frac{B^4\mathrm{b1}}{1056}+B\mathrm{b10}+\frac{B^2\mathrm{b2}}{175}+\frac{2\mathrm{b3}}{27}-\frac{B^3\mathrm{b3}}{252}+\frac{2B\mathrm{b4}}{15}+\frac{3B^2\mathrm{b5}}{70}$$
$$+\frac{\mathrm{b6}}{3}+\frac{2B\mathrm{b7}}{5}+\frac{4\mathrm{b9}}{3}==0\&\&$$

$$\frac{557B^2}{100100}-\frac{3B^5}{9152}+\frac{4\mathrm{b1}}{405}+\frac{4B^3\mathrm{b1}}{1575}+\frac{4\mathrm{b10}}{3}+\frac{31B\mathrm{b2}}{1485}+\frac{B^4\mathrm{b2}}{1056}+\frac{B^2\mathrm{b3}}{175}+\frac{2\mathrm{b4}}{27}-\frac{B^3\mathrm{b4}}{252}$$
$$+\frac{2B\mathrm{b5}}{15}+\frac{3B^2\mathrm{b6}}{70}+\frac{\mathrm{b7}}{3}+\frac{2B\mathrm{b8}}{5}==0\&\&$$

$$\frac{4B}{693}-\frac{29B^4}{24255}+\frac{557B^2\mathrm{b1}}{100100}-\frac{3B^5\mathrm{b1}}{9152}+\frac{4\mathrm{b2}}{405}+\frac{4B^3\mathrm{b2}}{1575}+\frac{31B\mathrm{b3}}{1485}+\frac{B^4\mathrm{b3}}{1056}+\frac{B^2\mathrm{b4}}{175}$$
$$+\frac{2\mathrm{b5}}{27}-\frac{B^3\mathrm{b5}}{252}+\frac{2B\mathrm{b6}}{15}+\frac{3B^2\mathrm{b7}}{70}+\frac{\mathrm{b8}}{3}+\frac{2B\mathrm{b9}}{5}==0\&\&$$

$$\frac{101}{52650}-\frac{623B^3}{351000}+\frac{7B^6}{49920}+\frac{4B\mathrm{b1}}{693}-\frac{29B^4\mathrm{b1}}{24255}+\frac{2B\mathrm{b10}}{5}+\frac{557B^2\mathrm{b2}}{100100}-\frac{3B^5\mathrm{b2}}{9152}$$
$$+\frac{4\mathrm{b3}}{405}+\frac{4B^3\mathrm{b3}}{1575}+\frac{31B\mathrm{b4}}{1485}+\frac{B^4\mathrm{b4}}{1056}+\frac{B^2\mathrm{b5}}{175}+\frac{2\mathrm{b6}}{27}-\frac{B^3\mathrm{b6}}{252}+\frac{2B\mathrm{b7}}{15}$$
$$+\frac{3B^2\mathrm{b8}}{70}+\frac{\mathrm{b9}}{3}==0\&\&$$

$$-\frac{46B^2}{45045}+\frac{68B^5}{105105}+\frac{101\mathrm{b1}}{52650}-\frac{623B^3\mathrm{b1}}{351000}+\frac{7B^6\mathrm{b1}}{49920}+\frac{\mathrm{b10}}{3}+\frac{4B\mathrm{b2}}{693}-\frac{29B^4\mathrm{b2}}{24255}$$
$$+\frac{557B^2\mathrm{b3}}{100100}-\frac{3B^5\mathrm{b3}}{9152}+\frac{4\mathrm{b4}}{405}+\frac{4B^3\mathrm{b4}}{1575}+\frac{31B\mathrm{b5}}{1485}+\frac{B^4\mathrm{b5}}{1056}+\frac{B^2\mathrm{b6}}{175}$$
$$+\frac{2\mathrm{b7}}{27}-\frac{B^3\mathrm{b7}}{252}+\frac{2B\mathrm{b8}}{15}+\frac{3B^2\mathrm{b9}}{70}==0\&\&$$

$$-\frac{113B}{1178100}+\frac{153173B^4}{116424000}-\frac{3B^7}{43520}-\frac{46B^2\text{b1}}{45045}+\frac{68B^5\text{b1}}{105105}+\frac{3B^2\text{b10}}{70}+\frac{101\text{b2}}{52650}$$
$$-\frac{623B^3\text{b2}}{351000}+\frac{7B^6\text{b2}}{49920}+\frac{4B\text{b3}}{693}-\frac{29B^4\text{b3}}{24255}+\frac{557B^2\text{b4}}{100100}-\frac{3B^5\text{b4}}{9152}+\frac{4\text{b5}}{405}$$
$$+\frac{4B^3\text{b5}}{1575}+\frac{31B\text{b6}}{1485}+\frac{B^4\text{b6}}{1056}+\frac{B^2\text{b7}}{175}+\frac{2\text{b8}}{27}-\frac{B^3\text{b8}}{252}+\frac{2B\text{b9}}{15}==0\&\&$$

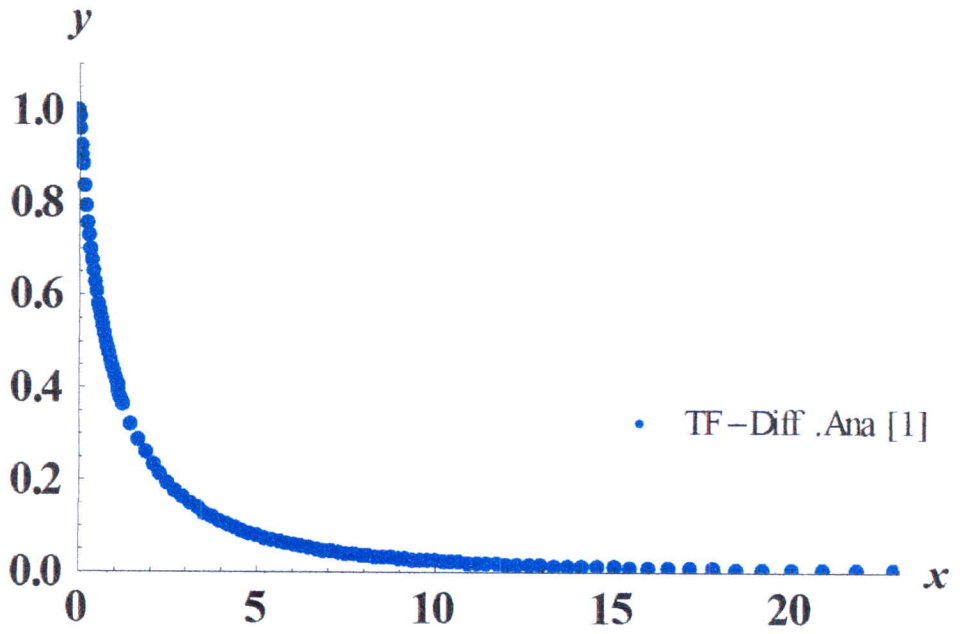
$$\frac{23}{473850}+\frac{1046B^3}{675675}-\frac{4B^6}{10395}-\frac{113B\text{b1}}{1178100}+\frac{153173B^4\text{b1}}{116424000}-\frac{3B^7\text{b1}}{43520}+\frac{2B\text{b10}}{15}$$
$$-\frac{46B^2\text{b2}}{45045}+\frac{68B^5\text{b2}}{105105}+\frac{101\text{b3}}{52650}-\frac{623B^3\text{b3}}{351000}+\frac{7B^6\text{b3}}{49920}+\frac{4B\text{b4}}{693}$$
$$-\frac{29B^4\text{b4}}{24255}+\frac{557B^2\text{b5}}{100100}-\frac{3B^5\text{b5}}{9152}+\frac{4\text{b6}}{405}+\frac{4B^3\text{b6}}{1575}+\frac{31B\text{b7}}{1485}+\frac{B^4\text{b7}}{1056}$$
$$+\frac{B^2\text{b8}}{175}+\frac{2\text{b9}}{27}-\frac{B^3\text{b9}}{252}==0\&\&$$

$$\frac{799399B^2}{698377680}-\frac{1232941B^5}{1278076800}+\frac{99B^8}{2646016}+\frac{23\text{b1}}{473850}+\frac{1046B^3\text{b1}}{675675}-\frac{4B^6\text{b1}}{10395}+\frac{2\text{b10}}{27}$$
$$-\frac{B^3\text{b10}}{252}-\frac{113B\text{b2}}{1178100}+\frac{153173B^4\text{b2}}{116424000}-\frac{3B^7\text{b2}}{43520}-\frac{46B^2\text{b3}}{45045}+\frac{68B^5\text{b3}}{105105}$$
$$+\frac{101\text{b4}}{52650}-\frac{623B^3\text{b4}}{351000}+\frac{7B^6\text{b4}}{49920}+\frac{4B\text{b5}}{693}-\frac{29B^4\text{b5}}{24255}+\frac{557B^2\text{b6}}{100100}$$
$$-\frac{3B^5\text{b6}}{9152}+\frac{4\text{b7}}{405}+\frac{4B^3\text{b7}}{1575}+\frac{31B\text{b8}}{1485}+\frac{B^4\text{b8}}{1056}+\frac{B^2\text{b9}}{175}==0\&\&$$

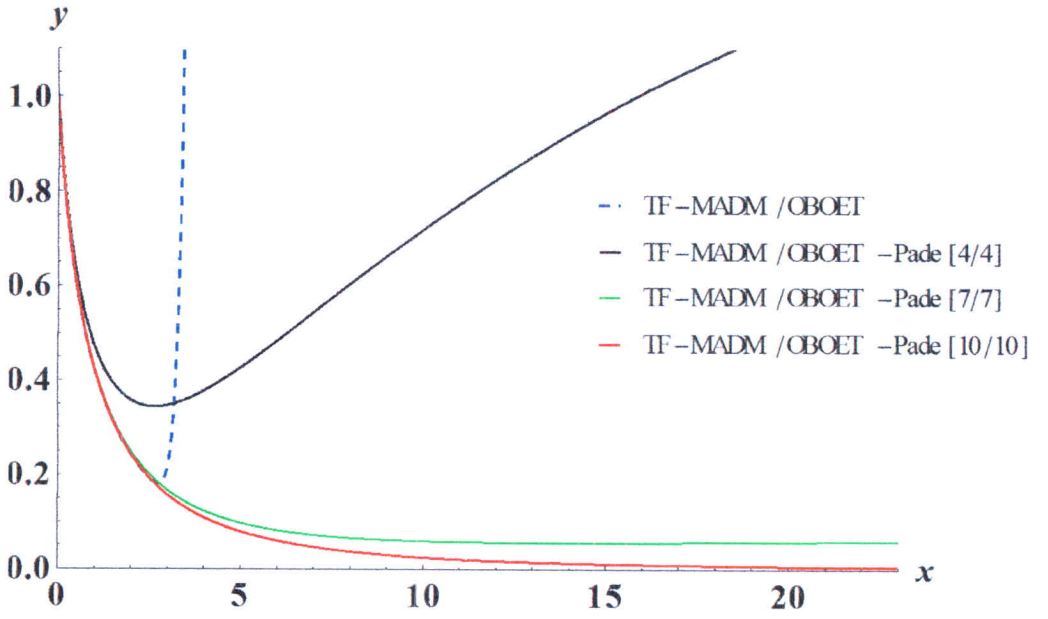
$$\frac{51356B}{103378275}-\frac{99856B^4}{70945875}+\frac{256B^7}{1044225}+\frac{799399B^2\text{b1}}{698377680}-\frac{1232941B^5\text{b1}}{1278076800}+\frac{99B^8\text{b1}}{2646016}$$
$$+\frac{B^2\text{b10}}{175}+\frac{23\text{b2}}{473850}+\frac{1046B^3\text{b2}}{675675}-\frac{4B^6\text{b2}}{10395}-\frac{113B\text{b3}}{1178100}$$
$$+\frac{153173B^4\text{b3}}{116424000}-\frac{3B^7\text{b3}}{43520}-\frac{46B^2\text{b4}}{45045}+\frac{68B^5\text{b4}}{105105}+\frac{101\text{b5}}{52650}-\frac{623B^3\text{b5}}{351000}$$
$$+\frac{7B^6\text{b5}}{49920}+\frac{4B\text{b6}}{693}-\frac{29B^4\text{b6}}{24255}+\frac{557B^2\text{b7}}{100100}-\frac{3B^5\text{b7}}{9152}+\frac{4\text{b8}}{405}+\frac{4B^3\text{b8}}{1575}$$
$$+\frac{31B\text{b9}}{1485}+\frac{B^4\text{b9}}{1056}==0$$

```
,{a0,a1,a2,a3,a4,a5,a6,a7,a8,a9,a10
,b1,b2,b3,b4,b5,b6,b7,b8,b9,b10}]

a0 = a0/.sol;  b1 = b1/.sol;
a1 = a1/.sol;  b2 = b2/.sol;
a2 = a2/.sol;  b3 = b3/.sol;
a3 = a3/.sol;  b4 = b4/.sol;
a4 = a4/.sol;  b5 = b5/.sol;
a5 = a5/.sol;  b6 = b6/.sol;
a6 = a6/.sol;  b7 = b7/.sol;
a7 = a7/.sol;  b8 = b8/.sol;
a8 = a8/.sol;  b9 = b9/.sol;
a9 = a9/.sol;  b10 = b10/.sol;
a10 = a10/.sol;

a[0] = a0; a[1] = a1; a[2] = a2; a[3] = a3; a[4] = a4; a[5] = a5; a[6] = a6; a[7]
        = a7; a[8] = a8; a[9] = a9; a[10] = a10;
b[1] = b1; b[2] = b2; b[3] = b3; b[4] = b4; b[5] = b5; b[6] = b6; b[7] = b7; b[8]
        = b8; b[9] = b9; b[10] = b10;

PadeTF = Terma/Termb;
padeSoln[j] = PadeApproximant[seriesT1,{t,0,{j,j}}]; (*Padeapprox.[10/10]*)
Print[" ********** Start solution of Pade approx.[10/10] ********** "];
r = Numerator[padeSoln[j]];
s = Expand[r/t^j];
t = ∞;
bb = FindRoot[s == 0,{B,-1.588071}]; (*comparisonroot by Kobayashi*)
B = B/.bb;
Print["Initial slope B for[",j,"/",j,"]","=",B];
errB
= Abs[(B - (-1.588071))/(-1.588071)]100; (*comparison by Kobayashi*)
```

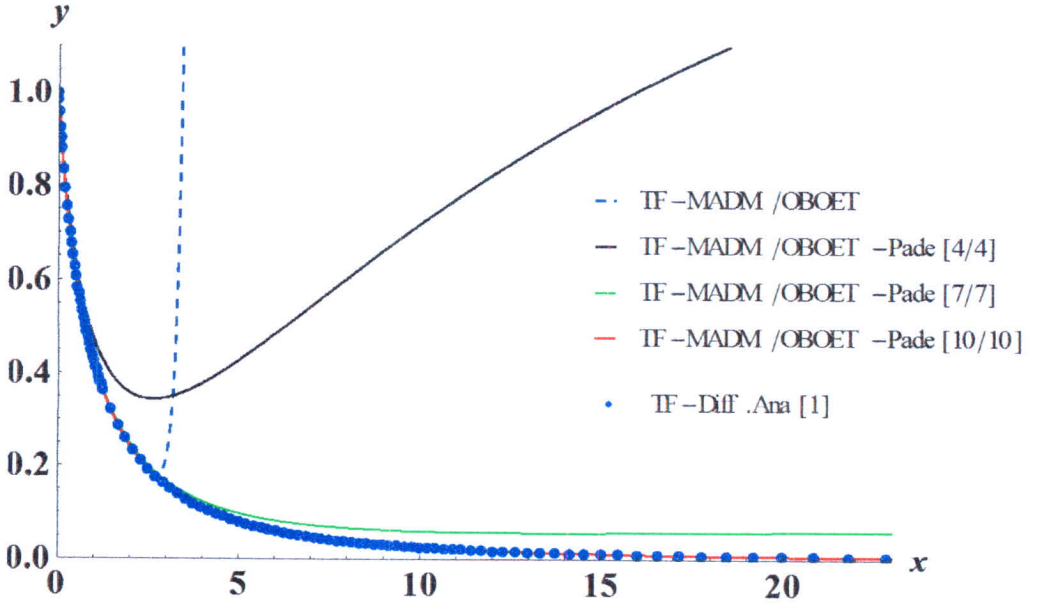
```
Print["Error(%) of Initial slope B for[",j,"/",j,"]","=",errB];
Clear[errB];

Clear[t,x];
padeSolnOFx[j] = Expand[padeSoln[j]/.t → x^(1/2)];
Clear[t,r,s,j,B,bb];
Print[" ********** Finish solution of Pade approx. [10/10] ********** "];

********** Start solution of Pade approx. [10/10] **********
Initial slope B for[10/10] = −1.5880696574833328
Error(%) of Initial slope B for[10/10] = 0.0000845375721342925
********** Finish solution of Pade approx. [10/10] **********

GraphRange = 23;
<< PlotLegends`
DiaLessTF = Import[D:\\Thesis W.Srikom\\analytic data\\phiX data.xls];
(*Imported Data File for dim. lessof T − Feq.*)
  GraDiaLessTF = ListPlot[DiaLessTF, PlotStyle
                → {Blue, AbsolutePointSize[5]}, PlotRange
                → {{0, GraphRange}, {0,1.1}}, PlotLegend
                → "TF − Diff. Ana[1]", LegendPosition → {0.11, −0.3}, AxesLabel
                → {x, y}, LabelStyle → Directive[Black, Bold, FontSize
                → 15], LegendShadow → None, LegendBackground
                → White, LegendBorder → White, LegendSize
                → 0.8, LegendTextSpace → 4.5]
```




```
Clear[t, x];
B = −1.588071;
MADMTFx = seriesT1/. t → x^(1/2);
Clear[B];
    Graphdata
    = Plot[{MADMTFx, padeSolnOFx[4], padeSolnOFx[7], padeSolnOFx[10]},
 {x, 0, GraphRange}, PlotRange → {{0, GraphRange}, {0,1.1}}, PlotStyle
                → {Dashed, Black, Green, Red}, AxesLabel → {x, y}, LabelStyle
                → Directive[Black, Bold, FontSize → 15], PlotLegend
                → {Style["TF − MADM/OBOET", 12], Style["TF − MADM/OBOET
                − Pade[4/4]", 12], Style["TF − MADM/OBOET
                − Pade[7/7]", 12], Style["TF − MADM/OBOET
                − Pade[10/10]", 12]}, LegendShadow → None, LegendSize
                → 0.9, LegendPosition → {0.16, −0.13}, LegendBackground
                → White, LegendBorder → White, LegendBorderSpace
                → 0.1, LegendTextSpace → 10]
```



Show[Graphdata, GraDiaLessTF]



$Z = 80$; (*Atomic number of Hg*)

$x = Z\text{^}(1/3)\,\eta/0.88534$;

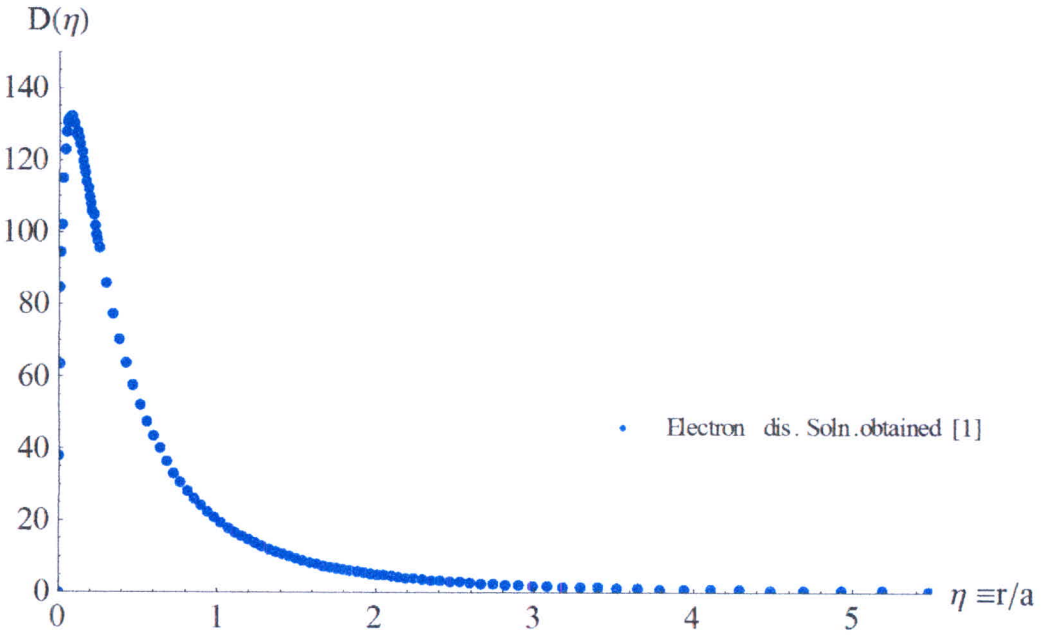
DistrOfTauMADM = $2\text{^}(7/2)Z\text{^}(3/2)/(3\pi)(\eta\text{^}(1/2))$(MADMTFx^(3/2));

DistrOfTau44 = $2\text{^}(7/2)Z\text{^}(3/2)/(3\pi)(\eta\text{^}(1/2))$(padeSolnOFx[4]^(3/2));

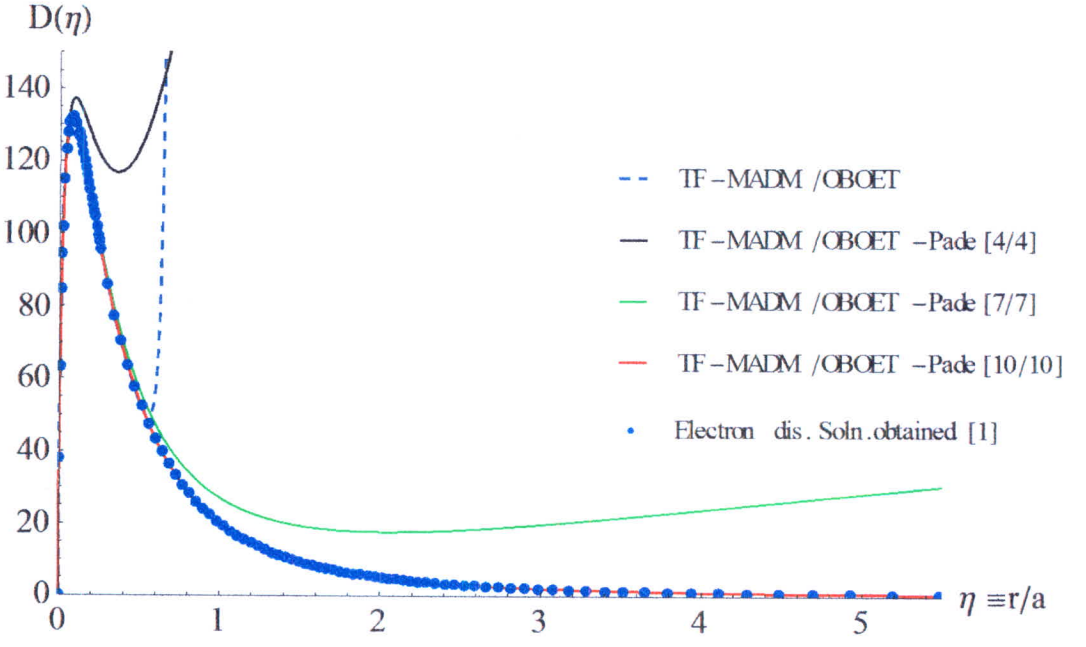
DistrOfTau77 = $2\text{^}(7/2)Z\text{^}(3/2)/(3\pi)(\eta\text{^}(1/2))$(padeSolnOFx[7]^(3/2));

DistrOfTau1010 = $2\text{^}(7/2)Z\text{^}(3/2)/(3\pi)(\eta\text{^}(1/2))$(padeSolnOFx[10]^(3/2));

```
graphDist
= Plot[{DistrOfTauMADM, DistrOfTau44, DistrOfTau77, DistrOfTau1010}, {η, 0,8},
, PlotStyle → {Dashed, Black, Green, Red}, PlotRange →
{{0,5.5}, {0,150}}, AxesLabel → {"η ≡ r/a", "D(η)"}, LabelStyle →
Directive[Black, FontSize → 15], PlotLegend →
{Style["TF − MADM/OBOET", 12], Style["TF − MADM/OBOET − Pade[4/
4]", 12], Style["TF − MADM/OBOET − Pade[7/7]", 12], Style["TF − MADM/
OBOET − Pade[10/10]", 12]}, LegendShadow → None, LegendSize →
0.9, LegendPosition → {0.16, −0.15}, LegendBackground →
White, LegendBorder → White, LegendBorderSpace → 0.1, LegendTextSpace →
7.5];

    eDistAna = Import[D:\\Thesis W.Srikom\\analytic data \\eDistAna.xls];
                (*Imported Data File for e distribution analytic*)
  GeDistAna = ListPlot[eDistAna, PlotStyle
                → {Blue, AbsolutePointSize[5]}, PlotRange
                → {{0,5.5}, {0,150}}, AxesLabel → {"η ≡ r/a", "D(η)"}, LabelStyle
                → Directive[Black, FontSize → 15], PlotLegend
                → {Style["Electron dis. Soln. obtained[1]", 12], LegendSpacing
                → 0.01}, LegendPosition → {0.11, −0.3}, LegendShadow
                → None, LegendBackground → White, LegendBorder
                → White, LegendSize → 0.8, LegendTextSpace → 4.6]
```

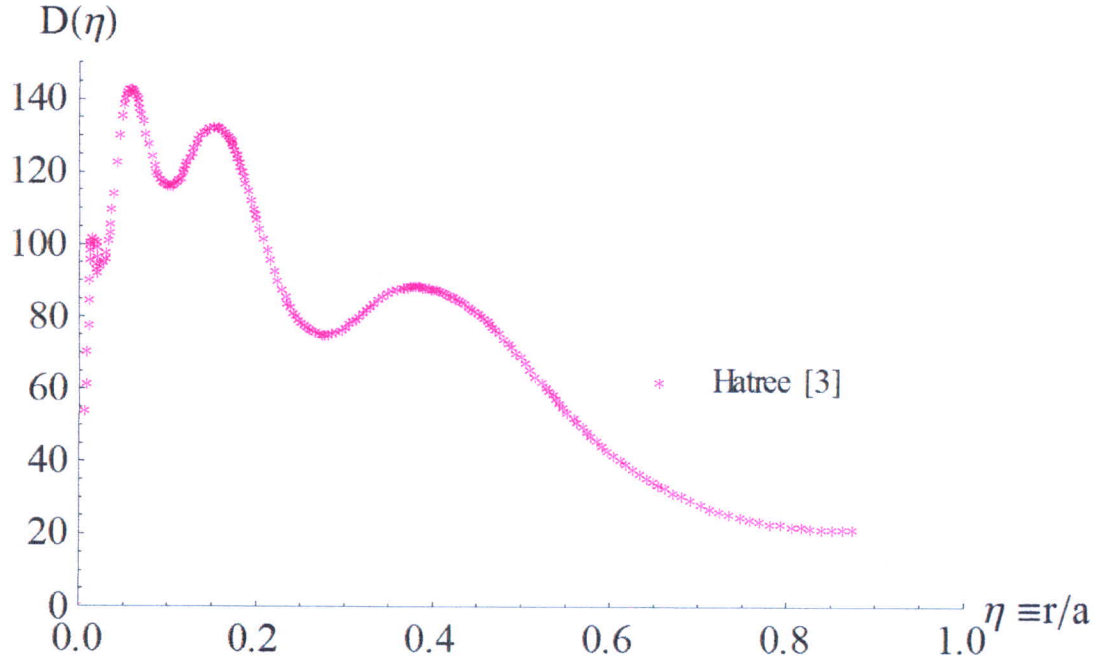


Show[graphDist, GeDistAna]



```
graphDistVsHatree = Plot[DistrOfTau1010, {η, 0,8}, PlotStyle
                → {Red, Thickness[0.002]}, PlotRange → {{0,1}, {0,150}}, AxesLabel
                → {"η ≡ r/a", "D(η)"}, LabelStyle → Directive[Black, FontSize
                → 15], PlotLegend
                → {Style["TF − MADM/OBOET
                − Pade[10/10]", 12], LegendSpacing → 0.01}, LegendPosition
                → {0.1,0.2}, LegendShadow → None, LegendBackground
                → White, LegendBorder → White, LegendSize
                → 0.9, LegendTextSpace → 4.6]GeDistAnaVsHatree
                = ListPlot[eDistAna, PlotStyle
                → {Blue, AbsolutePointSize[5]}, PlotRange
                → {{0,1}, {0,150}}, AxesLabel → {η ≡r/a,D(η)}, LabelStyle
                → Directive[Black, FontSize → 15], PlotLegend
                → {Style[Electron dis. Soln.[1], 12], LegendSpacing
                → 0.01}, LegendPosition → {0.1,0}, LegendShadow
                → None, LegendBackground → White, LegendBorder
                → White, LegendSize → 0.9, LegendTextSpace → 4.6];
   GeDistAnaVsHatree = ListPlot[eDistAna, PlotStyle
                → {Blue, AbsolutePointSize[5]}, PlotRange
                → {{0,1}, {0,150}}, AxesLabel → {"η ≡ r/a", "D(η)"}, LabelStyle
                → Directive[Black, FontSize → 15], PlotLegend
                → {Style["Electron dis. Soln. [1]", 12], LegendSpacing
                → 0.01}, LegendPosition → {0.1,0}, LegendShadow
                → None, LegendBackground → White, LegendBorder
                → White, LegendSize → 0.9, LegendTextSpace → 4.6];
Hartree = Import[D:\\Thesis W.Srikom\\analytic data\\Hartree.xls];
(*Imported Data File for Hatree*)
GraphHartree = ListPlot[Hartree, PlotStyle →
{Magenta, Thickness[0.004]}, PlotRange → {{0,1}, {0,150}}, PlotMarkers → " *
", AxesLabel → {"η ≡ r/a", "D(η)"}, LabelStyle → Directive[Black, FontSize →
15], PlotLegend → {Style["Hatree[3]", 12], LegendSpacing →
```

0.01}, LegendPosition → {0.1, −0.2}, LegendShadow →
None, LegendBackground → White, LegendBorder → White, LegendSize →
0.9, LegendTextSpace → 4.6]



Show[graphDistVsHatree, GeDistAnaVsHatree, GraphHartree]