

ห้องสมุดงานวิจัย สำนักงานคณะกรรมการการวิจัยแห่งชาติ



E42122

METHODS FOR FINDING SOLUTIONS OF GENERALIZED  
NON-STRONGLY MONOTONE VARIATIONAL  
INEQUALITY PROBLEMS

PHAYOONGSAK KAEWDAM

A Thesis Submitted to the Graduate School of Naresuan University

in Partial Fulfillment of the Requirements

for the Master of Science Degree in Mathematics

January 2012

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This thesis entitled “Methods for Finding Solutions of Generalized Non-strongly Monotone Variational Inequality Problems” submitted by Phayoongsak Kaewdam in partial fulfillment of the requirements for the Master of Science Degree in Mathematics is hereby approved.

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#### ABSTRACT

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Without the strong monotonicity assumption of the considered mappings, we provide a regularization method for general variational inequality problem, when its solutions set is related to a solution set of a finite family of inverse strongly monotone mappings. Consequently, an iterative algorithm for finding such a solution is constructed and its convergent theorem is proved. It is worth to point out that, since the strong monotonicity assumption is not assumed, our results improve and extend many well-known results in the literature.

# LIST OF CONTENTS

Chapter	Page
I INTRODUCTION.....	1
II PRELIMINARIES .....	3
Normed spaces and Hilberts spaces.....	3
Nonlinear mappings .....	5
Useful lemmas.....	10
III METHODS FOR FINDING SOLUTIONS OF GENERALIZED NON-STRONGLY MONOTONE VARIATIONAL INEQUALITY PROBLEMS .....	11
Regularization for general variational inequality problem in Hilbert spaces .....	11
Regularization inertial proximal point algorithm for general variational inequality problem in Hilbert spaces .....	21
IV CONCLUSION .....	29
REFERENCES .....	34
BIOGRAPHY .....	37