CHAPTER IV

CONCLUSION

The following results are all main theorems of this thesis:

1. Let K be a closed convex subset of a real Hilbert space H and $g : H \to H$ be a mapping such that $K \subset g(H)$. Let $A : H \to H$ be a hemicontinuous and g-monotone mapping. Let A_i be a λ_i-inverse strongly monotone mapping of K into H, for each $i = 1, 2, ..., N$. If g is an expanding affine continuous mapping and $GVI_K(A, g, S) \neq \emptyset$, then the following conclusions are true:

(a) For each $\alpha \in (0, 1)$, the problem

$$\langle A(u_\alpha) + \alpha^\mu \sum_{i=1}^{N} (A_i \circ g)(u_\alpha) + \alpha g(u_\alpha), g(v) - g(u_\alpha) \rangle \geq 0,$$

$\forall v \in H,\ g(v) \in K, 0 < \mu < 1$, has the unique solution u_α.

(b) $\lim_{\alpha \to 0^+} g(u_\alpha) = g(u^*)$, for some $u^* \in GVI_K(A, g, S)$.

(c) There exists a positive constant M such that

$$\|g(u_\alpha) - g(u_\beta)\|^2 \leq \frac{M|\alpha - \beta|}{\alpha^2}, \quad \text{for all } \alpha, \beta \in (0, 1).$$

2. Let K be a closed convex subset of a real Hilbert space H and $g : H \to H$ be an expanding affine continuous mapping such that $K \subset g(H)$. Let $A : H \to H$ be a hemicontinuous and g-monotone mapping. Let A_i be a λ_i-inverse strongly monotone mapping of K into H, for each $i = 1, 2, ..., N$. If $GVI_K(A, g, S) \neq \emptyset$ and the parameters c_n and α_n are chosen positive real numbers such that

(i) $\lim_{n \to \infty} \alpha_n = 0$,

(ii) $\lim_{n \to \infty} \frac{\alpha_n - \alpha_{n+1}}{\alpha_{n+1}^2} = 0$,

(iii) $\liminf_{n \to \infty} c_n \alpha_n > 0$.

Then the sequence $\{g(z_n)\}$ defined by, starting with an element $z_1 \in H$ such that $g(z_1) \in K$, we will consider the following processes:

$$\langle c_n[A(z_{n+1})+\alpha_n^{\mu}\sum_{i=1}^{N}(A_i o g)(z_{n+1})+\alpha_n g(z_{n+1})]+g(z_{n+1})-g(z_n), g(v)-g(z_{n+1})\rangle \geq 0,$$

for all $v \in H, g(v) \in K$, where $\{c_n\}$ and $\{\alpha_n\}$ are sequences of positive real numbers,converges strongly to the element $g(u^*)$ as $n \to +\infty$.

3. Let K be a closed convex subset of a real Hilbert space H. Let $A : K \to H$ be a hemicontinuous and monotone mapping. Let A_i be a λ_i-inverse strongly monotone mapping of K into H, for each $i = 1, 2, ..., N$. If $VI_K(A) \bigcap S \neq \emptyset$, where $VI_K(A)$ is denoted for the solution set of the problem (1.0.1), then the following conclusions are true:

(a) For each $\alpha \in (0, 1)$, the problem

$$\langle A(u_\alpha) + \alpha^{\mu}\sum_{i=1}^{N} A_i(u_\alpha) + \alpha u_\alpha, v - u_\alpha\rangle \geq 0, \forall v \in H, 0 < \mu < 1,$$

has the unique solution u_α.

(b) $\lim_{\alpha\to 0+} u_\alpha = u^*$, for some $u^* \in VI_K(A)\bigcap S$.

(c) There exists a positive constant M such that

$$\|u_\alpha - u_\beta\|^2 \leq \frac{M|\alpha-\beta|}{\alpha^2}, \quad \text{for all } \alpha, \beta \in (0, 1). \tag{3.2.13}$$

4. Let K be a closed convex subset of a real Hilbert space H. Let $A : K \to H$ be a hemicontinuous and monotone mapping. Let A_i be a λ_i-inverse strongly monotone mapping of K into H, for each $i = 1, 2, ..., N$. If $VI_K(A) \bigcap S \neq \emptyset$ and the parameters c_n and α_n are chosen positive real numbers such that

$$(i) \quad \lim_{n\to\infty} \alpha_n = 0,$$

$$(ii) \quad \lim_{n\to\infty} \tfrac{\alpha_n - \alpha_{n+1}}{\alpha_{n+1}^2} = 0,$$

$$(iii) \quad \liminf_{n\to\infty} c_n\alpha_n > 0. \tag{3.2.14}$$

Then the sequence $\{z_n\}$ defined by, starting with an element $z_1 \in K$, we will consider the following processes:

$$\langle c_n[A(z_{n+1}) + \alpha_n^{\mu} \sum_{i=1}^{N} A_i(z_{n+1}) + \alpha_n z_{n+1}] + z_{n+1} - z_n, v - z_{n+1}\rangle \geq 0,$$

for all $v \in K$, where $\{c_n\}$ and $\{\alpha_n\}$ are sequences of positive real numbers, converges strongly to the element $u^* \in H$ as $n \to +\infty$.

5. Let K be a closed convex subset of a real Hilbert space H and $g : H \to H$ be a mapping such that $K \subset g(H)$. Let $A : H \to H$ be a hemicontinuous and g-monotone mapping. If g is an expanding affine continuous mapping and $GVI_K(A, g) \neq \emptyset$, where $GVI_K(A, g)$ is denoted for the solution set of the problem 1.0.2, then the following conclusions are true:

(a) For each $\alpha \in (0, 1)$, the problem

$$\langle A(u_\alpha) + \alpha g(u_\alpha), g(v) - g(u_\alpha)\rangle \geq 0,$$

$\forall v \in H,\ g(v) \in K, 0 < \mu < 1$, has the unique solution u_α.

(b) $\lim_{\alpha \to 0^+} g(u_\alpha) = g(u^*)$, for some $u^* \in GVI_K(A, g)$.

(c) There exists a positive constant M such that

$$\|g(u_\alpha) - g(u_\beta)\|^2 \leq \frac{M|\alpha - \beta|}{\alpha^2}, \quad \text{for all } \alpha, \beta \in (0, 1). \tag{3.2.15}$$

6. Let K be a closed convex subset of a real Hilbert space H and $g : H \to H$ be an expanding affine continuous mapping such that $K \subset g(H)$. Let $A : H \to H$ be a hemicontinuous and g-monotone mapping. If $GVI_K(A, g) \neq \emptyset$, where $GVI_K(A, g)$ is denoted for the solution set of the problem 1.0.2 and the parameters c_n and α_n are chosen positive real numbers such that

(i) $\lim_{n\to\infty} \alpha_n = 0,$

(ii) $\lim_{n\to\infty} \frac{\alpha_n - \alpha_{n+1}}{\alpha_{n+1}^2} = 0,$

(iii) $\liminf_{n\to\infty} c_n \alpha_n > 0.$ (3.2.16)

Then the sequence $\{g(z_n)\}$ defined by, starting with an element $z_1 \in H$ such that $g(z_1) \in K$, we will consider the following processes:

$$\langle c_n[A(z_{n+1}) + \alpha_n g(z_{n+1})] + g(z_{n+1}) - g(z_n), g(v) - g(z_{n+1})\rangle \geq 0,$$

for all $v \in H, g(v) \in K$, where $\{c_n\}$ and $\{\alpha_n\}$ are sequences of positive real numbers, converges strongly to the element $g(u^*)$ as $n \to +\infty$.

7. Let K be a closed convex subset of a real Hilbert space H. Let $A : K \to H$ be a hemicontinuous and monotone mapping. If $VI_K(A) \neq \emptyset$, then the following conclusions are true:

(a) For each $\alpha \in (0, 1)$, the problem

$$\langle A(u_\alpha) + \alpha u_\alpha, v - u_\alpha\rangle \geq 0, \forall v \in H, 0 < \mu < 1,$$

has the unique solution u_α.

(b) $\lim_{\alpha\to 0+} u_\alpha = u^*$, for some $u^* \in VI_K(A)$.

(c) There exists a positive constant M such that

$$\|u_\alpha - u_\beta\|^2 \leq \frac{M|\alpha - \beta|}{\alpha^2}, \quad \text{for all } \alpha, \beta \in (0, 1). \tag{3.2.17}$$

8. Let K be a closed convex subset of a real Hilbert space H. Let $A : K \to H$ be a hemicontinuous and monotone mapping. If $VI_K(A) \neq \emptyset$ and the parameters c_n and α_n are chosen positive real numbers such that

(i) $\lim_{n \to \infty} \alpha_n = 0,$

(ii) $\lim_{n \to \infty} \frac{\alpha_n - \alpha_{n+1}}{\alpha_{n+1}^2} = 0,$

(iii) $\liminf_{n \to \infty} c_n \alpha_n > 0.$ (3.2.18)

Then the sequence $\{z_n\}$ defined by, starting with an element $z_1 \in K$, we will consider the following processes:

$$\langle c_n[A(z_{n+1}) + \alpha_n z_{n+1}] + z_{n+1} - z_n, v - z_{n+1}\rangle \geq 0,$$

for all $v \in K$, where $\{c_n\}$ and $\{\alpha_n\}$ are sequences of positive real numbers, converges strongly to the element $u^* \in H$ as $n \to +\infty$.