# CHAPTER I

# INTRODUCTION

Let $H$ be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$, the variational inequality problem is to find $u \in K$ such that

$$\langle Au, v - u \rangle \geq 0, \tag{1.0.1}$$

for all $v \in K$, where $K$ is a nonempty subset of $H$, and $A : K \to H$ is a mapping. This type of problem was firstly introduced in the 1964 by Stampacchia [1], has applied in a variety of diverse fields of pure and applied sciences and proved to productive and innovative. It has been shown that the variational inequalities theory provides the most natural, direct, simple, unified and efficient framework for a general treatment of a wide class of linear and nonlinear problems. The development of variational inequality theory can be extended and generalized in several directions for studying a wide class of equilibrium problems arising in financial, economics, transportation, elasticity, optimization, pure and applied sciences. An important and useful generalization of variational inequalities is called the *general variational inequality* introduced by Noor [2] in 1988, which is a problem of finding $u^* \in H, g(u^*) \in K$ such that

$$\langle A(u^*), g(v) - g(u^*) \rangle \geq 0, \quad \forall v \in H : g(v) \in K, \tag{1.0.2}$$

where $K$ is a nonempty subset of $H$, and $A, g : H \to H$ are mappings. It is known that a class of nonsymmetric and odd-order obstacle, unilateral and moving boundary value problems arising in pure and applied can be studied in the unified framework of general variational inequalities, see [3] and the references therein. Observe that to guarantee the existence and uniqueness of a solution of the problem (1.0.2) one has to impose conditions on the operator $A$ and $g$, see [4] for examples in a more general case. By the way, it is worth to noting that, if $A$ fails to be

Lipschitz continuous or strongly monotone, then the solution set of the problem (1.0.2) may be an empty set.

Related to the variational inequalities, the problem of finding the fixed points of the nonlinear mappings is the subject of current interest in functional analysis. Motivated and inspired by the research going in this direction, in this thesis, we present a regularization and its induced iteration method for the problem (1.0.2) in a general case. That is, we will consider a method for finding a solution of the problem (1.0.2), in the following sense: Find $u^* \in H, g(u^*) \in S$ such that

$$\langle A(u^*), g(v) - g(u^*) \rangle \geq 0, \quad \forall v \in H : g(v) \in K, \tag{1.0.3}$$

where $A$ is a kind generalized monotone mapping, $\{A_i\}_{i=1}^N$ is a finite family of $\lambda_i$-inverse strongly monotone mappings from $K$ into $H$ and $S = \bigcap_{i=1}^N S_i$, $S_i = \{x \in K : A_i(x) = 0\}$. Observe that, if $A_i =: 0$, $\forall i = 1, 2, ..., N$ the zero operator, then the problem (1.0.3) reduces to (1.0.2). Moreover, we also would like to notice that, although many authors have proved results for finding the solution of the variational inequality problem and the solution set of a finite inverse strongly monotone mapping, see [5, 6, 7] for examples, it is clear that it cannot be directly applied to the problem (1.0.3) due to the presence of $g$.

This thesis is divided into 4 chapters. Chapter I is an introduction to the research problem. Chapter II is dealing with some preliminaries and give some useful results that will be duplicated in later chapter. Chapter III is the main results of this research. The conclusion of research is in Chapter IV.