

## CHAPTER V

### THIN-LAYER DRYING EMPIRICAL MATHEMATICAL MODELING DEVELOPMENT

#### 1. Mathematical models and new model development used to analyze the relationship of moisture ratio with drying time

The researchers have used experimental data from the relationship of moisture ratio with drying time in the drying temperature range, 50, 60, 70, 80, 90 and 100 °C and drying air velocity 4, 5 and 6 m/s into statistical analysis using nonlinear regression. And determine the ability of mathematical models to predict the results the most accurate equations, 6 equations are equations of Henderson, Logarithmic, Lewis, Wang and Singh, Two-term and Aghabashlo et al.,. The following 6 equation were used in the statistical analysis of experimental results using a computer program and the results are to develop or create a new equation to predict changes in moisture ratio of the experiment--each equation has the form of characteristic equation as follows.

Henderson's model ;

$$MR = a \exp(-kt) \quad (5.1)$$

Logarithmic's model ;

$$MR = a \exp(-kt) + c \quad (5.2)$$

Lewis's model ;

$$MR = \exp(-kt) \quad (5.3)$$

Wang and Singh's model ;

$$MR = 1 + at + bt^2 \quad (5.4)$$



Two-term's model ;

$$MR = a \exp(-kt) + b \exp(-ct) \quad (5.5)$$

Aghabashlo et al.,'s model ;

$$MR = \exp(-(k_1 t)/(1 + k_2 t)) \quad (5.6)$$

The results obtained from the analysis of nonlinear regression which is used to detect precise with coefficient of decisions ( $R^2$ : Coefficient of determination) coefficient penalty decision (Adj.  $R^2$ : Adjusted coefficient of determination) index indicate Point (SSE: Summation of Square Error). When the experimental data were analyzed by the analysis of the nonlinear regression using the six mathematical model for determine a constant of equation of each variable. From the analysis of the accuracy with the decision and the coefficient index as shown in Table 5.1. This chapter presents only the drying air velocity 5 meters per second only. Because a lot of information and analysis on drying air velocity of 4 and 6 m/s--tended in the same direction. (more information is available in Appendix D)

**Table 5.1** shows the empirical constant,  $R^2$ , Adj.  $R^2$  and SSE of the six model at drying air velocity of 5 m/s

Model	Temp. (°C)	Model constant	$R^2$ (Model)	Adj. $R^2$ (Model)	SSE (Model)
Henderson  $MR = ae^{-kt}$	50	a = 1.181 and k = 0.011	0.911	0.910	1.356
	60	a = 1.159 and k = 0.023	0.933	0.931	0.545
	70	a = 1.176 and k = 0.054	0.913	0.908	0.346
	80	a = 1.160 and k = 0.076	0.879	0.867	0.267
	90	a = 1.169 and k = 0.112	0.904	0.893	0.213
	100	a = 1.125 and k = 0.147	0.959	0.954	0.075

**Table 5.1** Shows the empirical constant  $R^2$ , Adj.  $R^2$  and SSE of the six model at drying air velocity of 5 m/s(Cont.)

Model	Temp. (°C)	Model and constant	$R^2$ (Model)	Adj. $R^2$ (Model)	SSE (Model)
Logarithmic $MR = ae^{(-kt) + n}$	50	a = 205.791, k = 0.00003 and n = -204.715	0.988	0.988	0.185
	60	a = 4.994, k = 0.003 and n = -3.941	0.986	0.986	0.111
	70	a = 50.502, k = 0.001 and n = -49.442	0.986	0.986	0.054
	80	a = 279.411, k = 0.000 and n = -278.318	0.976	0.975	0.053
	90	a = 4.809, k = 0.015 and n = -3.734	0.971	0.970	0.065
	100	a = 1.537, k = 0.074 and n = -0.472	0.985	0.984	0.027
Lewis $MR = e^{-kt}$	50	k = 0.009	0.864	0.863	2.078
	60	k = 0.020	0.898	0.897	0.830
	70	k = 0.045	0.872	0.868	0.510
	80	k = 0.063	0.836	0.828	0.361
	90	k = 0.095	0.867	0.860	0.295
	100	k = 0.130	0.939	0.935	0.111
Wang and Singh $MR = 1 + at + bt^2$	50	a = -0.005 and b = -0.000013	0.987	0.987	0.201
	60	a = -0.0120 and b = -0.000002	0.981	0.981	0.153
	70	a = -0.024 and b = 0.000	0.982	0.982	0.071
	80	a = 0.024 and b = -0.001	0.989	0.989	0.025
	90	a = -0.054 and b = 0.000	0.964	0.962	0.061
	100	a = -0.093 and b = 0.002	0.985	0.984	0.027

**Table 5.1** Shows the empirical constant,  $R^2$ , Adj.  $R^2$  and SSE of the six model at drying air velocity of 5 m/s(Cont.)

Model	Temp. (°C)	Model and constant	$R^2$ (Model)	Adj. $R^2$ (Model)	SSE (Model)
Two-term  $MR = a \exp(-kt) + b \exp(-ct)$	50	$a = 23.086, k = 0.000,$ $b = -22.034$ and $c = -0.001$	0.990	0.990	0.160
	60	$a = 120.022, k = 0.048,$ $b = -119.098$ and $c = 0.049$	0.986	0.986	0.111
	70	$a = 13.349, k = 0.001,$ $b = -12.289$ and $c = 0.000$	0.986	0.984	0.054
	80	$a = 10.009, k = -0.015,$ $b = -8.978$ and $c = -0.020$	0.988	0.985	0.025
	90	$a = 17.867, k = 0.010,$ $b = -16.792$ and $c = -0.006$	0.971	0.963	0.065
	100	$a = 0.958, k = 0.147,$ $b = 0.167$ and $c = 0.147$	0.959	0.946	0.075
Aghabashlo et al.,  $MR = e^{\left(\frac{k_1 t}{1+k_2 t}\right)}$	50	$k_1 = 0.004$ and $k_2 = -0.005$	0.994	0.994	0.090
	60	$k_1 = 0.010$ and $k_2 = -0.009$	0.992	0.992	0.067
	70	$k_1 = 0.020$ and $k_2 = -0.025$	0.995	0.995	0.019
	80	$k_1 = 0.025$ and $k_2 = -0.042$	0.995	0.995	0.010
	90	$k_1 = 0.038$ and $k_2 = -0.056$	0.990	0.989	0.029
	100	$k_1 = 0.075$ and $k_2 = -0.047$	0.994	0.994	0.011

The analysis of the  $R^2$ , adj. $R^2$  and SSE in Table 5.1 and appendix D(Table D.1 and D.2) was found that the  $R^2$  and adj. $R^2$  of Aghabaslo et al.,'s model valuable approach most 1.00. and the lowest SSE in all the hot air velocity and the temperature of drying when compared with Lewis's model, Henderson's model, Logarithmic's model and Two-term's model. Therefore, the researcher has chosen to Aghabaslo et al.'s model as the main equation and bring the remaining four models to analyzed together with Agbahaslo et al.,'s model for determine new model. The tests showed that standard error estimation(SE) of the equation arising from bring mixture to a replica of the Henderson equation is Aghabaslo value less than the other equations—investigate from the table D.3.in Appendix D. Also, the author has chosen to model Aghabaslo et al.,'s model with Henderson's model to be used in the

development of new model for explain the experimental results as shown in Equation 5.7.

The analysis of the decision coefficient( $R^2$ : Coefficient of determination) coefficient penalty decision(Adj. $R^2$ : Adjusted coefficient of determination) value index(SSE: Summation of square error) in Table 5.2. The author has used the 6 model to analyze and take the test each model are combined, reduce some terms, to add some terms and adding coefficients. By the way, by trial error to develop a new model until the results come out as the model equation 5.7.

$$MR = ae^{\left(\frac{-kt}{1+bt}\right)} \quad (5.7)$$

Equation 5.7 was a model developed from the application of a model of Henderson and Aghabashlo et al., combine remaining on the same constant. And increase the coefficients of express exponential same equation of Aghabashlo et al. The coefficient this changes the value of k and b, and instead took on the new model to test various statistical values can be displayed in Table 5.2.

**Table 5.2** Shows the empirical constant,  $R^2$ , Adj.  $R^2$  and SSE of the New model

Velocity(m/s)	Temp. (°C)	Model constant	$R^2$ (Model)	Adj. $R^2$ (Model)	SSE (Model)
4	50	a = 1.038, k = 0.005 and b = -0.005	0.994	0.994	0.094
	60	a = 1.076, k = 0.016 and b = -0.005	0.979	0.979	0.229
	70	a = 1.062, k = 0.028 and b = -0.012	0.984	0.983	0.075
	80	a = 1.022, k = 0.027 and b = -0.029	0.996	0.996	0.009
	90	a = 1.030, k = 0.043 and b = -0.035	0.994	0.993	0.013
	100	a = 1.040, k = 0.065 and b = -0.045	0.989	0.988	0.024

**Table 5.2** Shows the empirical constant,  $R^2$ , Adj.  $R^2$  and SSE of the New model  
(Cont.)

Velocity(m/s)	Temp. (°C)	Model constant	$R^2$ (Model)	Adj. $R^2$ (Model)	SSE (Model)
5	50	a = 1.033, k = 0.005 and b = -0.005	0.996	0.995	0.064
	60	a = 1.031, k = 0.011 and b = -0.009	0.993	0.992	0.057
	70	a = 1.028, k = 0.022 and b = -0.0240	0.996	0.995	0.015
	80	a = 1.016, k = 0.026 and b = -0.042	0.996	0.995	0.009
	90	a = 1.027, k = 0.041 and b = -0.054	0.990	0.989	0.023
	100	a = 1.036, k = 0.083 and b = -0.044	0.996	0.995	0.008
6	50	a = 0.926, k = 0.004 and b = -0.005	0.996	0.996	0.028
	60	a = 0.986, k = 0.008 and b = -0.008	0.999	0.999	0.006
	70	a = 1.019, k = 0.026 and b = -0.022	0.988	0.987	0.029
	80	a = 1.007, k = 0.032 and b = -0.035	0.996	0.996	0.006
	90	a = 0.990, k = 0.040 and b = -0.039	0.995	0.994	0.005
	100	a = 1.003, k = 0.079 and b = -0.034	0.996	0.995	0.004

The results in Table 5.2 shown the various constants and statistics equations developed. Empirical constants of each equation to be analyzed non-linear regression again. And in the form of the correlation equation as a function of drying temperature and drying air velocity. Finally, the function was organized equations in form the polynomial equation as shown in Equation 5.8 and 5.9.

$$f1=(A_0+A_1V+A_2V^2)+(B_0+B_1V+B_2V^2)T+(C_0+C_1V+C_2V^2)T^2+(D_0+D_1V+D_2V^2)T^3+(E_0+E_1V+E_2V^2)T^4 \quad (5.8)$$

$$f2=(A_0+A_1V+A_2V^2+A_3V^3)+(B_0+B_1V+B_2V^2+B_3V^3)T+(C_0+C_1V+C_2V^2+C_3V^3)T^2+(D_0+D_1V+D_2V^2+D_3V^3)T^3 \quad (5.9)$$

where ;  $f_1$  = correlation equation of a constant ( a )  
 $f_2$  = correlation equation of constants ( k, b )  
 $V$  = drying air velocity(m/s)  
 $T$  = drying temperature( $^{\circ}$ C)

$A_0, A_1, A_2, A_3, B_0, B_1, B_2, B_3, C_0, C_1, C_2, C_3, D_0, D_1, D_2, D_3, E_0, E_1$  and  $E_3$

were coefficients of correlation equation which showed in equation 5.10, 5.11 and 5.12

From Equation 5.7 (new model) can be determine constant and coefficient in the equation as the following equation.

$$\begin{aligned}
 a = & (-47.095 + 15.209v - 1.165v^2) + (2.077 - 0.597v + 0.039v^2)T \\
 & + (-0.036 + 0.0095v - 0.0005v^2)T^2 + (-0.0013 + 0.005v - 5.093E \\
 & -05v^2)T^3 + (-2.959E - 07 - 2.60E - 08v + 1.562E - 08v^2)T^4
 \end{aligned} \quad (5.10)$$

where :  $R^2 = 0.946$  ,  $Adj.R^2 = 0.733$  and  $SSE = 0.00000$

$$\begin{aligned}
 b = & (2.99133 - 0.74623v - 0.11406v^2 + 0.02417v^3) \\
 & + (-0.14213 + 0.03514v + 0.00539v^2 - 0.00114v^3)T \\
 & + (0.00217 - 0.00053v - 0.00008v^2 + 0.00002v^3)T^2 \\
 & + (-0.00001 + 2.55E - 06v + 3.9E - 06v^2 - 8E - 08v^3)T^3
 \end{aligned} \quad (5.11)$$

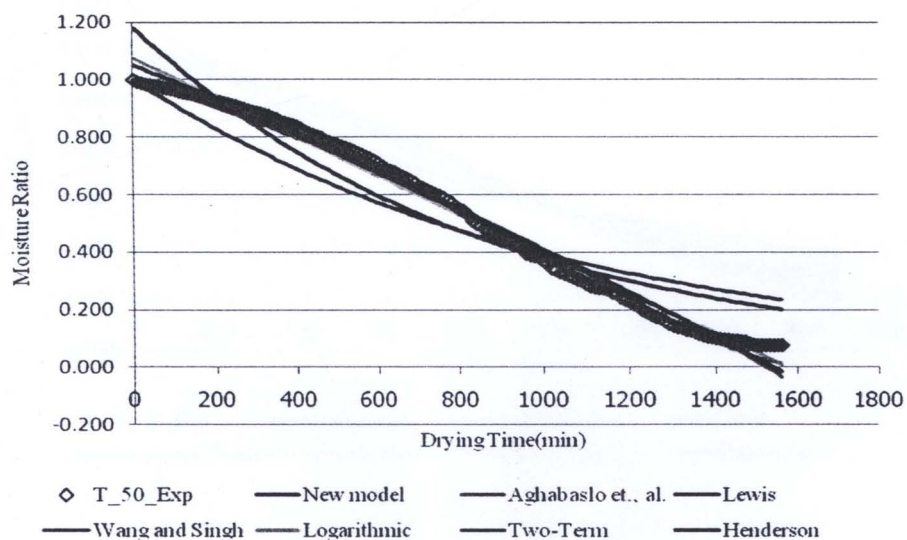
where :  $R^2 = 0.986$  ,  $Adj.R^2 = 0.966$  and  $SSE = 0.0000194$

$$\begin{aligned}
 k = & (0.45435 - 0.20537v - 0.02794v^2 + 0.00635v^3) \\
 & + (-0.03226 + 0.01168v + 0.00165v^2 - 0.00037v^3)T \\
 & + (0.00066 - 0.00021v - 0.00003v^2 + 6.66E - 06v^3)T^2 \\
 & + (-3.92E - 06 + 1.17E - 06v + 1.7E - 07v^2 - 4E - 08v^3)T^3
 \end{aligned} \quad (5.12)$$

where :  $R^2 = 0.987$  ,  $Adj.R^2 = 0.968$  and  $SSE = 0.0000289$

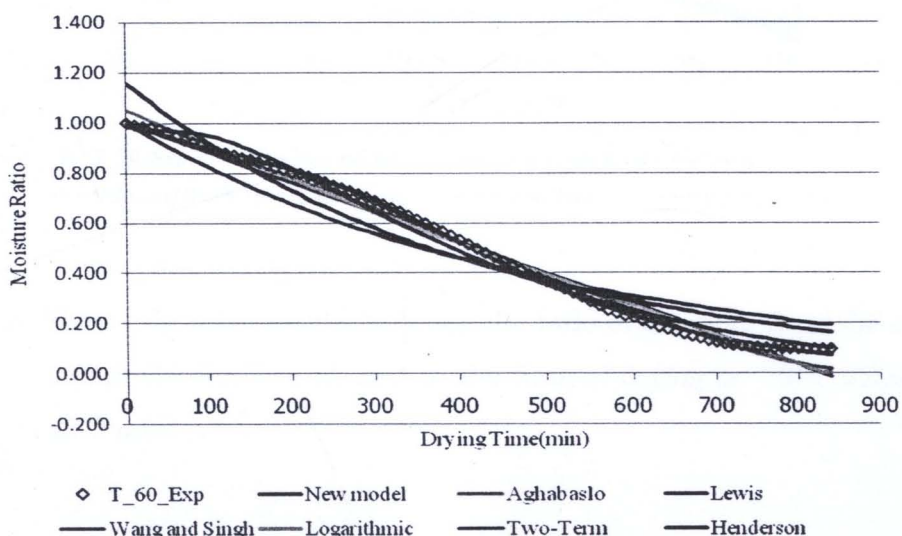
The  $R^2$ ,  $Adj.R^2$  and SSE in Equation 5.10, 5.11 and 5.12 were the coefficient constant--shows the accuracy in predicting a, k and b as shown in the relationship of 6 levels of drying temperature(50, 60, 70, 80, 90 and 100 °C) with hot air velocity of 4 m/s. While, The  $R^2$ ,  $Adj.R^2$  and SSE of constant a, k and b at velocity of hot air of 5 and 6 m/s with can be shown in Appendix D(Table D.6).

Then the moisture ratio of from the experiment with the 6 models were used in the analysis using equations 5.1-5.6 and results from the model developed (Equation 5.7) to write chart shows the relationship between moisture ratio and drying time as shown follows. These expressions can be used to estimate the moisture content ratio of red chili at any time during the drying process and this model can be describe the change of moisture content ratio at drying temperature 50, 60, 70, 80, 90 and 100°C and drying air velocity increase from 4, 5 and 6 m/s which were in good agreement with the experimental results.



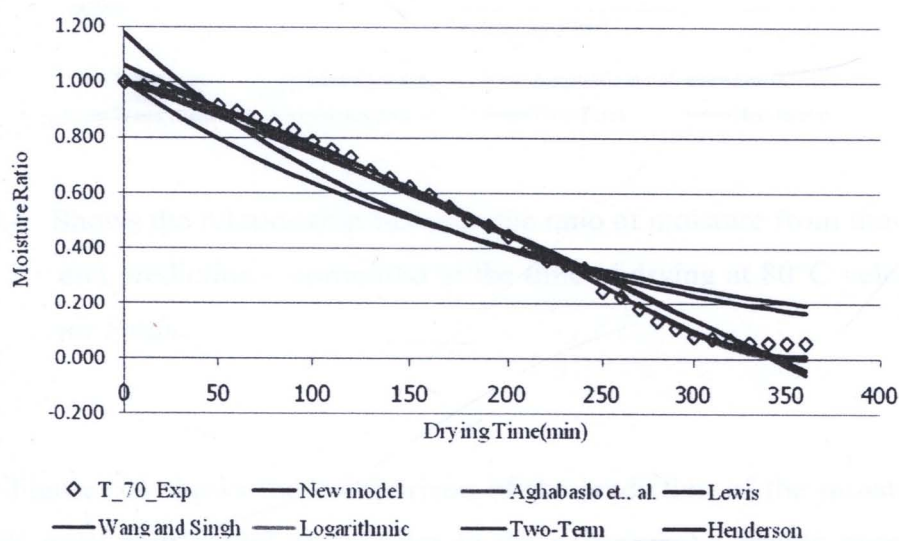
**Figure 5.1** Shows the relationship between the moisture ratio from the experiment and prediction—compared to the drying time of drying temperature of 50 °C and velocity of hot air of 5 m/s

The analysis results of the nonlinear regression of 6 models as shown in Table 5.1 and Table 5.2. It was found that the  $R^2$  and  $Adj.R^2$  from the analysis using New model equation are most valuable and the smallest SSE. Coefficients of each equation is used to calculate the moisture ratio of drying--to compare the results from the prediction of each equation with the experimental results as shown in Figure 5.1. Figure 5.1 shows the results of the prediction of moisture ratio, from 6 model. Trend changing of moisture ratio from the experiment and new model was similarity with the experimental results over the whole equation. When consider of the  $R^2$  with  $Adj.R^2$  and SSE from Table 5.2--at drying temperature of 50 °C and velocity of hot air of 5 m/s. New model showed that the equation coefficients to decide  $R^2 = 0.996$ . Coefficient decision  $Adj.R^2 = 0.995$  and the  $SSE = 0.064$ --there is less and closer to zero. Indicates the ability of the equation, New model can explain the experimental results very well.



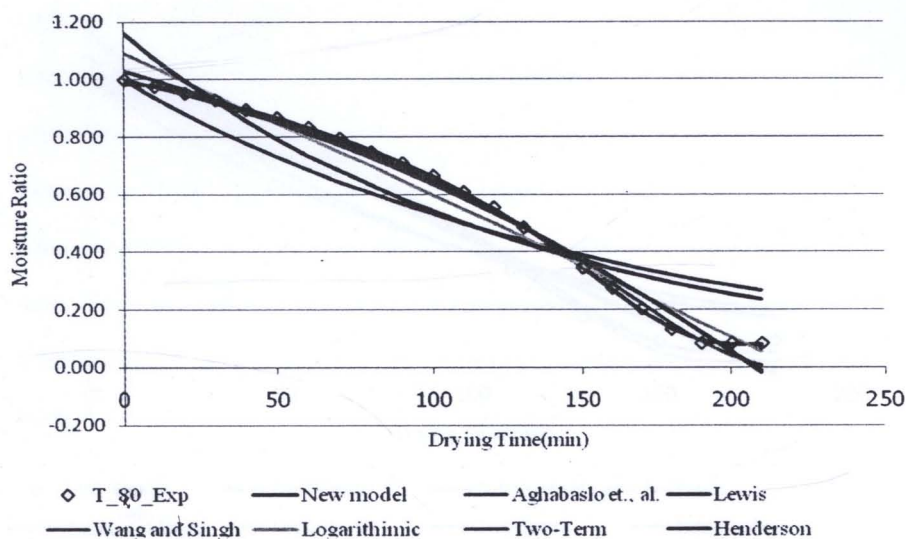
**Figure 5.2** Shows the relationship between the ratio of moisture from the experiment and prediction—compared to the time of drying at 60°C velocity of hot air 5 m/s.

From Figure 5.2 shows the comparison of the prediction of the moisture ratio of dried chili were new model are similarity to the experimental results over the whole equation. When considering of the  $R^2$  with Adj.  $R^2$  and SSE from Table 5.2--at drying temperature of 60 °C and velocity of hot air of 5 m/s. New model showed that the equation coefficients to decide  $R^2 = 0.993$ . Coefficient decision Adj. $R^2 = 0.992$  and the SSE = 0.057-- there is less and closer to zero. It indicates the ability of the equation the new model can be explained the experimental results very well.



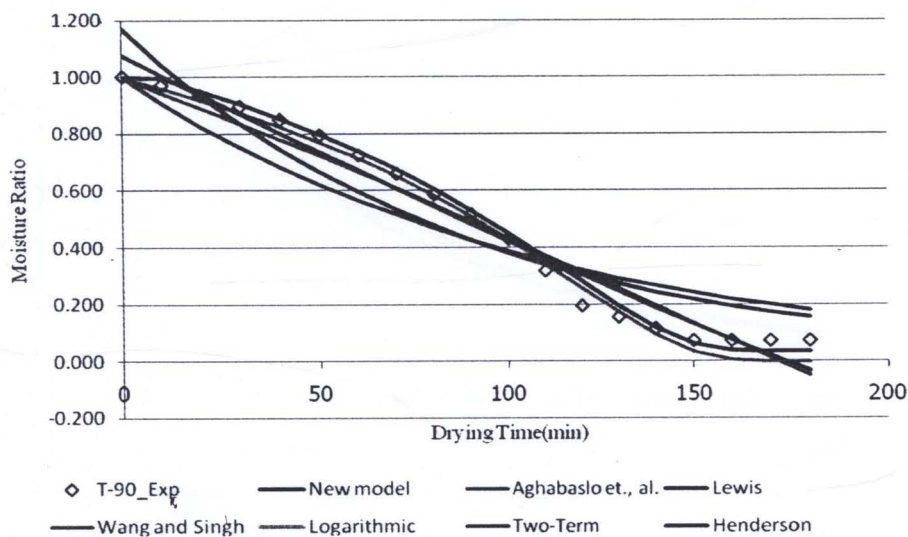
**Figure 5.3** Shows the relationship between the ratio of moisture from the experiment and prediction—compared to the time of drying at 70°C velocity of hot air 5 m/s.

From Figure 5.3 shows the comparison of the prediction of the moisture ratio of dried chili were new model and similar to the experimental results over the whole equation. When considering of the  $R^2$  with Adj.  $R^2$  and SSE from Table 5.2--at drying temperature of 70 °C and velocity of hot air of 5 m/s. New model showed that the equation coefficients to decide  $R^2 = 0.996$ . Coefficient decision Adj. $R^2 = 0.995$  and the SSE = 0.015--there is less and closer to zero. It indicates the ability of the equation the new model can be explained the experimental results very well.



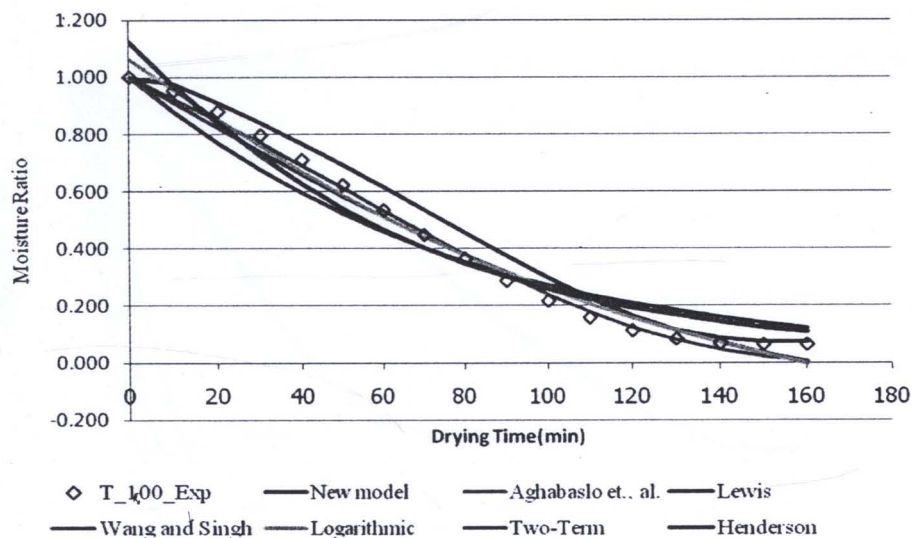
**Figure 5.4** Shows the relationship between the ratio of moisture from the experiment and prediction—compared to the time of drying at 80°C velocity of hot air 5 m/s.

Figure 5.4 shows the comparison of the prediction of the moisture ratio of dried chili were new model are similar to the experimental results over the whole equation. When consider of the  $R^2$  with Adj.  $R^2$  and SSE from Table 5.2--at drying temperature of 80 °C and velocity of hot air of 5 m/s. New model showed that the equation coefficients to decide  $R^2 = 0.996$ . Coefficient decision Adj. $R^2 = 0.995$  and the SSE = 0.009--there is less and closer to zero. It indicates the ability of the equation the new model can be explained the experimental results very well.



**Figure 5.5** Shows the relationship between the ratio of moisture from the experiment and prediction—compared to the time of drying at 90°C velocity of hot air 5 m/s.

Figure 5.5 it presented the comparison of the prediction of the moisture ratio of dried chili were new model are similarity to the experimental results over the whole equation. When consider of the  $R^2$  with Adj.  $R^2$  and SSE from Table 5.2--at drying temperature of 90 °C and velocity of hot air of 5 m/s. New model showed that the equation coefficients to decide  $R^2 = 0.990$ . Coefficient decision Adj. $R^2 = 0.989$  and the SSE = 0.023--there is less and closer to zero. It indicates the ability of the equation the new model can be explained the experimental results very well.

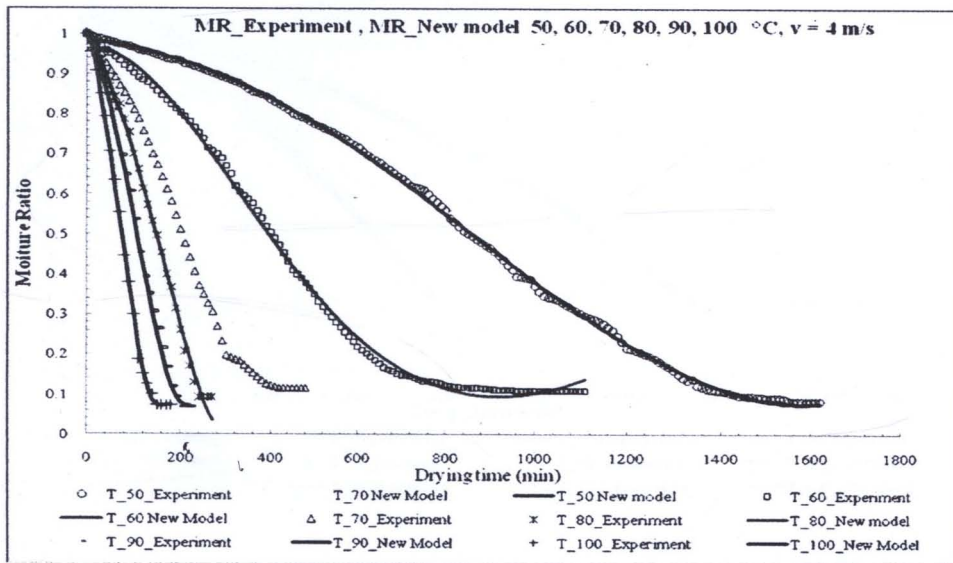


**Figure 5.6** Shows the relationship between the ratio of moisture from the experiment and prediction—compared to the time of drying at 100°C velocity of hot air 5 m/s.

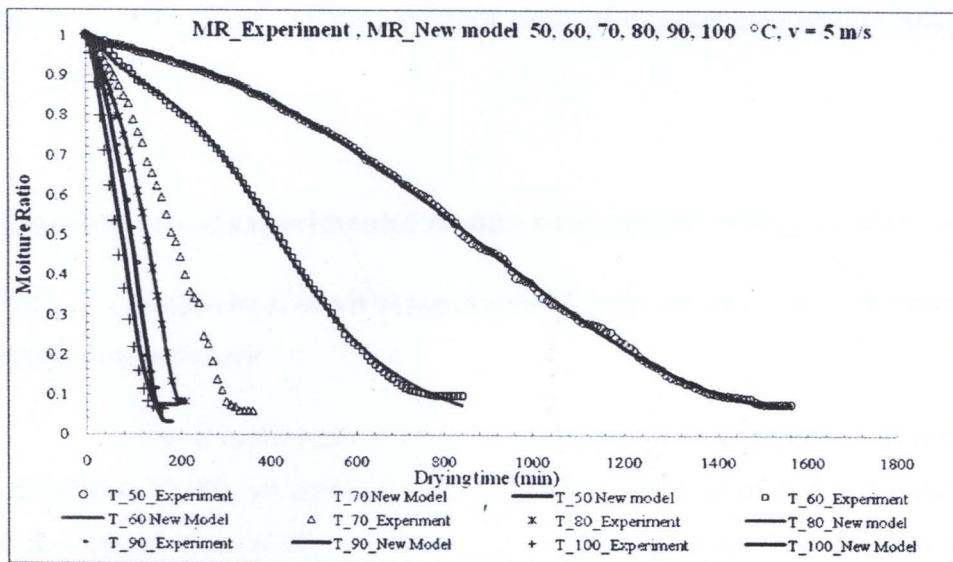
Figure 5.6 presents the comparison of the prediction of the moisture ratio of dried chili were new model are similarity to the experimental results over the whole equation. When consider of the  $R^2$  with Adj.  $R^2$  and SSE from Table 5.2--at drying temperature of 100 °C and velocity of hot air of 5 m/s. New model showed that the equation coefficients to decide  $R^2 = 0.996$ . Coefficient decision Adj. $R^2 = 0.995$  and the SSE = 0.008--there is less and closer to zero. It indicates the ability of the equation the new model can bed explained the experimental results very well.

## 2. Changes the moisture ratio between the experimental with new model

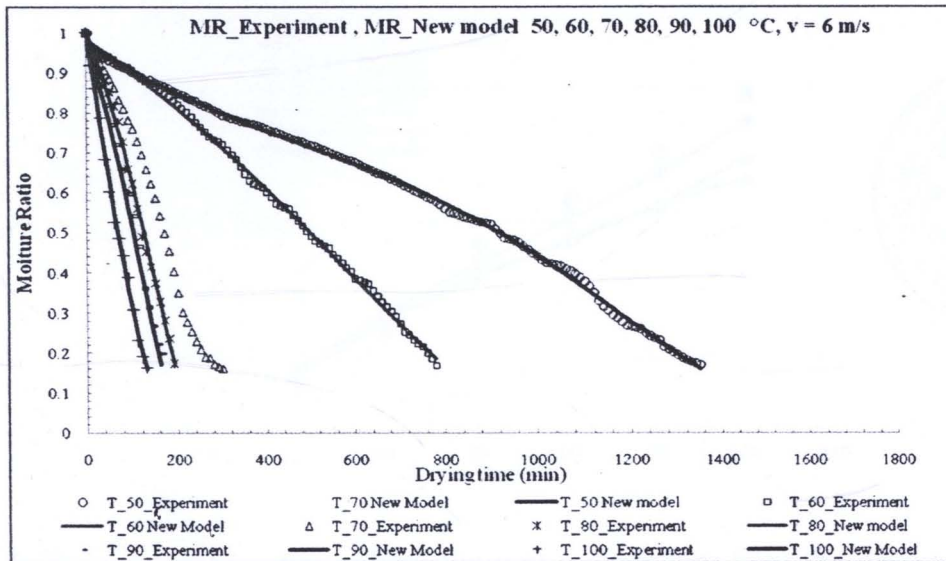
Effects of changes in moisture content ratio from the experimental and the prediction from new model--plot graph shown the relationship with drying time at temperature drying of 50, 60, 70, 80, 90 and 100 °C at velocity of hot air of 4, 5 and 6 m/s can be displayed in Figure 5.7, 5.8 and 5.9.



**Figure 5.7** Shows the relation between moisture versus drying time of experiments data with new model at velocity of hot air of 4 m/s



**Figure 5.8** Shows the relation between moisture versus drying time of experiments data with new model at velocity of hot air of 5 m/s



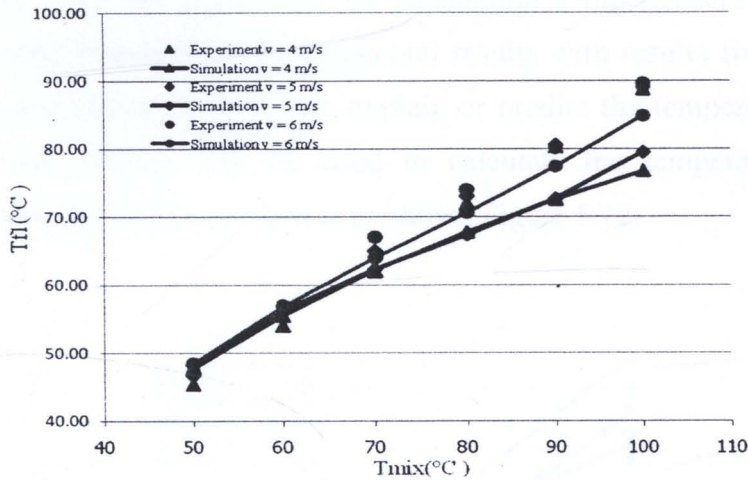
**Figure 5.9** Shows the relation between moisture versus drying time of experiments data with new model at velocity of hot air of 5 m/s

Figure 5.7, 5.8 and 5.9 presented that trend of the new model was similar to the experimental results. So, showing that the new model development can be described as the change of moisture content ratio of experiments results with sufficient accuracy.

### 3. Comparison of experimental results with results from computer program

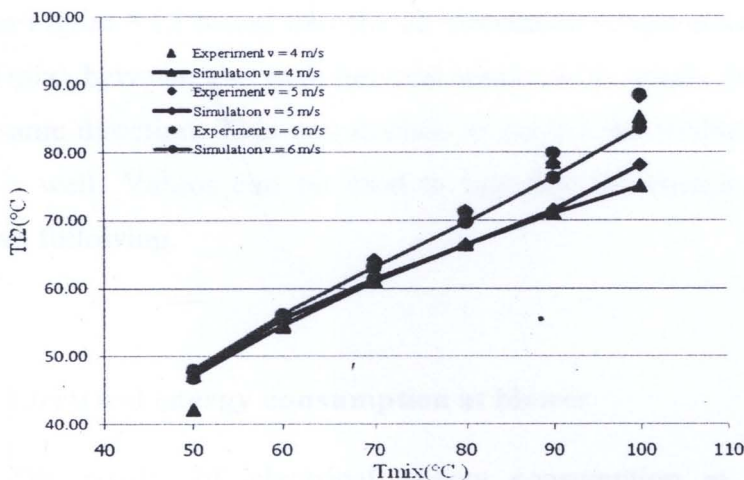
#### 3.1 Comparison of air temperature at the drying out of the chamber with drying temperature

From a mathematical model of drying chili continuous at temperature drying of 50, 60, 70, 80, 90 and 100 °C at velocity of hot air of 4, 5 and 6 m/s. On the steps of the calculation model of drying using trial error method for determine the moisture ratio of dry mixing air. And the values used to calculate the temperature outlet from chamber at the exit location drying as shown below.



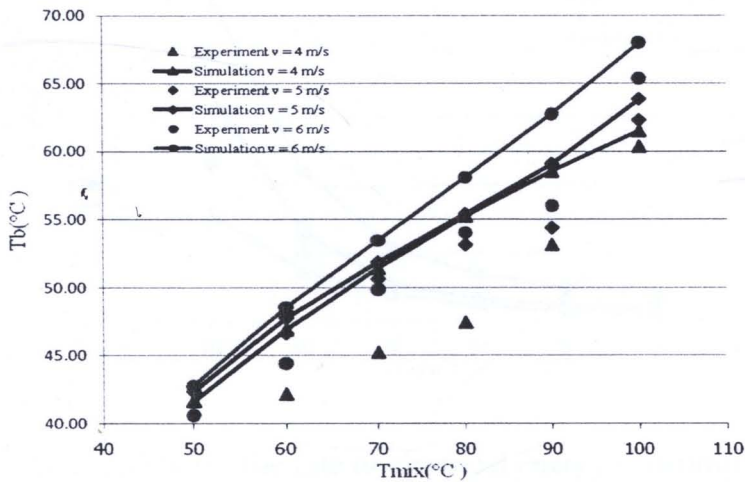
**Figure 5.10** Comparison chart where the exit temperature from the drying chamber ( $T_{f1}$ ) and drying temperature ( $T_{mix}$ ) between the experimental results with results from simulations.

Figure 5.10 showed the exit temperature from the drying chamber ( $T_{f1}$ ) and drying temperature ( $T_{mix}$ ) between the experimental results with results from simulations-tended in the same direction. This can explain or predict the temperature at the exit from the drying chamber, is well. Values can be used to calculate the temperature of the air temperature where the exit pipe recirculation as shown Figure 5.11:



**Figure 5.11** Graph comparing the air circulation temperature from the experimental results with simulation results.

Figure 5.11 presented the air circulation temperature ( $T_f$ ) and drying temperature ( $T_{mix}$ ) between the experimental results with results from simulations-tended in the same direction. This can explain or predict the temperature at the exit from blower well. Values can be used to calculate the temperature of the air temperature where the exit from blower as shown Figure 5.12.



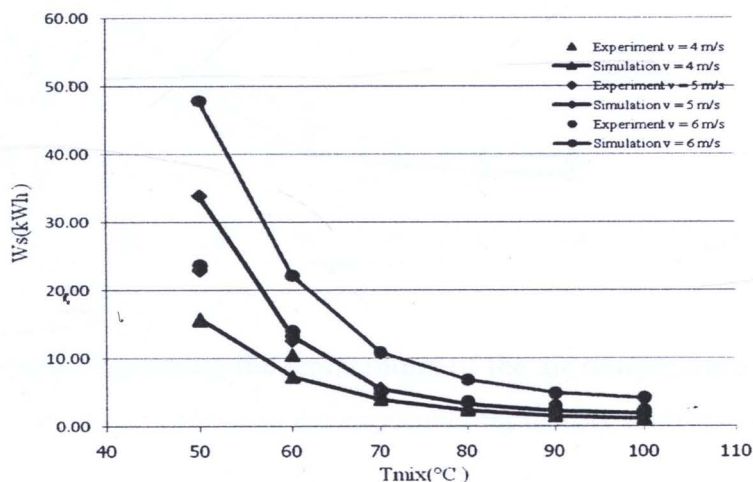
**Figure 5.12** Graph comparing the temperature of the air temperature where the exit from blower between the experimental results with simulation results

From Figure 5.12 found that the air circulation temperature ( $T_b$ ) and drying temperature ( $T_{mix}$ ) between the experimental results with results from simulations-tended in the same direction. This can explain or predict the temperature at the exit from blower, is well. Values can be used to calculate the energy consumption of blower as shown following.

### 3.2 Electrical energy consumption at blower

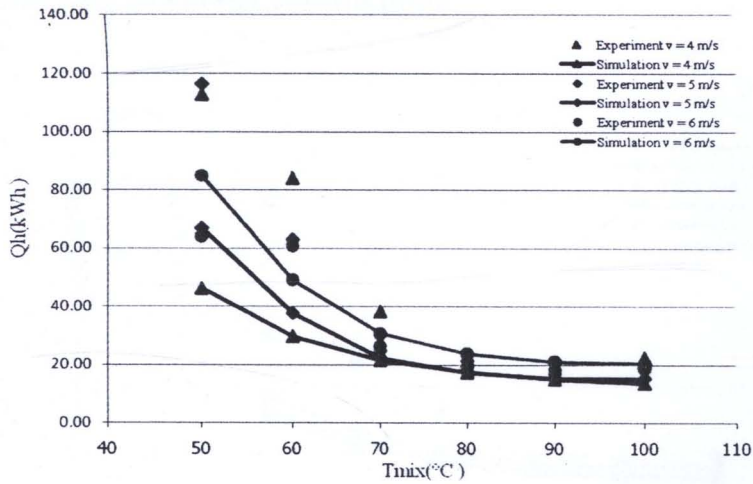
The results of electrical energy consumption at blower can be determined from computer program in Appendix B – at temperature drying of 50, 60, 70, 80, 90 and 100 °C at velocity of hot air of 4, 5 and 6 m/s. And drying time

decreased as the velocity of hot air increased. Also, electrical energy consumption of blower and heat coil can be shown as following.



**Figure 5.13** The graph shows the rate of electrical energy consumption at blower from experimental results and simulation model

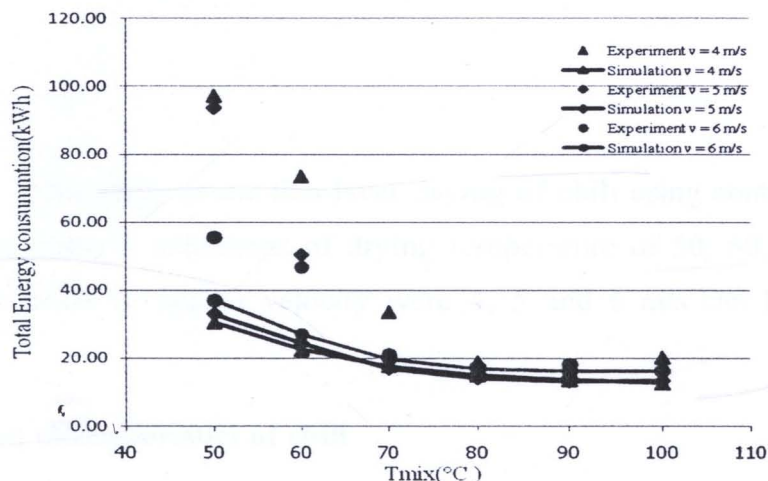
From Figure 5.13 it was found that consumption of energy from blower, the simulation was slightly higher than experimental. When the drying velocity increases effect to the energy consumption will also increase accordingly. Electrical energy consumption at the same velocity will be reduced by drying temperature. Due to higher temperatures effect to drying time decreased and trends are all the same velocity of hot air.



**Figure 5.14** Graph comparing the temperature of the air temperature where the exit from blower between the experimental results with simulation results

Graph 5.14 showed that the rate of heat energy consumption of drying increase when drying temperature increases. Because the rate of heat energy consumption depends on the drying temperature whereas drying time increase similar with drying temperature increase and rate of energy consumption reduces as shown in Figure 5.14.

### 3.3 Total rate of energy consumption



**Figure 5.15** The graph shows the total energy consumption rate of drying from the experimental with simulation

Figure 5.15 found that the total energy consumption is decreased when the drying temperature increased and the temperature of drying up to the time it takes to dry will be less. Therefore, the total energy consumption is reduced. But considering the drying velocity of hot air up found that rate of total energy consumption will increase as the time of drying. Such as, the drying air temperature hot air velocity and drying time--cause changes in the total energy consumption.