# EXECUTIVE SUMMARY

## Executive Summary

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ซึ่งมีรายละเอียดงานวิจัยโดยสรุป ดังนี้

Let $\mathcal{H}$ be a real Hilbert space equipped with norm $\|\cdot\|$ and inner product $\langle\cdot,\cdot\rangle$ and let $2^{\mathcal{H}}$ and $CB(\mathcal{H})$ denote for the family of all the nonempty subsets of $\mathcal{H}$ and the family of all the nonempty closed bounded subsets of $\mathcal{H}$, respectively. As usual, we will define $D : CB(\mathcal{H}) \times CB(\mathcal{H}) \to [0, \infty)$, the Hausdorff metric on $CB(\mathcal{H})$, by

$$D(A,B) = \max\left\{\sup_{x\in A}\inf_{y\in B}\|x-y\|, \sup_{y\in B}\inf_{x\in A}\|x-y\|\right\}, \quad \text{for all} \quad A,B \in CB(\mathcal{H}).$$

Let $(\Omega, \Sigma, \mu)$ be a complete $\sigma$- finite measure space and $\mathcal{B}(\mathcal{H})$ be the class of Borel $\sigma$-fields in $\mathcal{H}$. A mapping $x : \Omega \to \mathcal{H}$ is said to be measurable if $\{t \in \Omega : x(t) \in B\} \in \Sigma$, for all $B \in \mathcal{B}(\mathcal{H})$. We will denote by $\mathcal{M}_{\mathcal{H}}$ for a set of all measurable mappings on $\mathcal{H}$, that is, $\mathcal{M}_{\mathcal{H}} = \{x : \Omega \to \mathcal{H} | x \text{ is a measurable mapping}\}$.

Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two real Hilbert spaces. Let $F : \Omega \times \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_1$ and $G : \Omega \times \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_2$ be single-valued mappings. Let $U : \Omega \times \mathcal{H}_1 \to CB(\mathcal{H}_1), V : \Omega \times \mathcal{H}_2 \to CB(\mathcal{H}_2)$ and $M_i : \Omega \times \mathcal{H}_i \to 2^{\mathcal{H}_i}$ be set-valued mappings, for $i = 1, 2$. In this paper, we will consider the following problem: find measurable mappings $a, u : \Omega \to \mathcal{H}_1$ and $b, v : \Omega \to \mathcal{H}_2$ such that $u(t) \in U(t, a(t)), v(t) \in V(t, b(t))$ and

$$\begin{cases} 0 \in F(t, a(t), v(t)) + M_1(t, a(t)), \\ 0 \in G(t, u(t), b(t)) + M_2(t, b(t)), \quad \forall t \in \Omega. \end{cases} \tag{1}$$

The problem of type (1) is called the system of random set-valued variational inclusion problem. If $a, u : \Omega \to \mathcal{H}_1$ and $b, v : \Omega \to \mathcal{H}_2$ are solutions of problem (1), we will denote by $(a, u, b, v) \in SRSVI_{(M_1,M_2)}(F, G, U, V)$. Notice that, if $U : \Omega \times \mathcal{H}_1 \to \mathcal{H}_1$ and $V : \Omega \times \mathcal{H}_2 \to \mathcal{H}_2$ are two single-valued mappings then the problem (1) reduces to the following problem: find $a : \Omega \to \mathcal{H}_1$ and $b : \Omega \to \mathcal{H}_2$ such that

$$\begin{cases} 0 \in F(t, a(t), V(t, b(t))) + M_1(t, a(t)), \\ 0 \in G(t, U(t, a(t)), b(t)) + M_2(t, b(t)), \quad \forall t \in \Omega. \end{cases} \tag{2}$$

In this case, we will denote by $(a, b) \in SRSI_{(M_1,M_2)}(F, G, U, V)$.

We provide the following lemma, and use it for proving our main result.

**Lemma A:** Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two real Hilbert spaces. Let $F : \Omega \times \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_1$ and $G : \Omega \times \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_2$ be single-valued mappings. Let $U : \Omega \times \mathcal{H}_1 \to CB(\mathcal{H}_1), V : \Omega \times \mathcal{H}_2 \to CB(\mathcal{H}_2)$ and $M_i : \Omega \times \mathcal{H}_i \to 2^{\mathcal{H}_i}$ be a set-valued mappings for $i = 1, 2$. Assume that $M_i$ are random $(A_i, m_i, \eta_i)$ - monotone mappings, and $A_i : \Omega \times \mathcal{H}_i \to \mathcal{H}_i$ be random $(r_i, \eta_i)$- strongly monotone mappings, for $i = 1, 2$. Then we have the following statements:

(i) if $(a, u, b, v) \in SRSVI_{(M_1,M_2)}(F, G, U, V)$ then for any measurable functions $\rho_1, \rho_2 : \Omega \to (0, \infty)$ we have

$$\begin{cases} a(t) = J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \left[A_1(t, a(t)) - \rho_1(t)F(t, a(t), v(t))\right], \\ b(t) = J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}} \left[A_2(t, b(t)) - \rho_2(t)G(t, u(t), b(t))\right], \quad \text{for all } t \in \Omega. \end{cases}$$

(ii) if there exist two measurable functions $\rho_1, \rho_2 : \Omega \to (0, \infty)$ such that

$$\begin{cases} a(t) = J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \left[A_1(t, a(t)) - \rho_1(t)F(t, a(t), v(t))\right], \\ b(t) = J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}} \left[A_2(t, b(t)) - \rho_2(t)G(t, u(t), b(t))\right], \end{cases}$$

for all $t \in \Omega$, then $(a, u, b, v) \in SRSVI_{(M_1,M_2)}(F, G, U, V)$.

However, due to Lemma A, we see that the following assumptions should be needed.

**Assumption** $(\mathcal{A})$ :

$\mathcal{A}(a)$ $\mathcal{H}_1$ and $\mathcal{H}_2$ are separable real Hilbert spaces.

$\mathcal{A}(b)$ $\eta_i : \Omega \times \mathcal{H}_i \times \mathcal{H}_i \to \mathcal{H}_i$ are random $\tau_i$-Lipschitz continuous single-valued mappings, for $i = 1, 2$.

$\mathcal{A}(c)$ $A_i : \Omega \times \mathcal{H}_i \to \mathcal{H}_i$ are random $(r_i, \eta_i)$-strongly monotone and random $\beta_i$-Lipschitz continuous single-valued mappings, for $i = 1, 2$.

$\mathcal{A}(d)$ $M_i : \Omega \times \mathcal{H}_i \to 2^{\mathcal{H}_i}$ are random $(A_i, m_i, \eta_i)$-monotone set-valued mappings, for $i = 1, 2$.

$\mathcal{A}(e)$ $U : \Omega \times \mathcal{H}_1 \to CB(\mathcal{H}_1)$ is a random $\phi_1$-$D$-Lipschitz continuous set-valued mapping and $V : \Omega \times \mathcal{H}_2 \to CB(\mathcal{H}_2)$ is a random $\phi_2$-$D$-Lipschitz continuous set-valued mapping.

$\mathcal{A}(f)$ $F : \Omega \times \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_1$ is a random single-valued mapping which has the following conditions:

(i) $F$ is a random $(c_1, \mu_1)$- relaxed cocoercive with respect to $A_1$ in the third argument and a random $\alpha_1$- Lipschitz continuous in the third argument,

(ii) $F$ is a random $\zeta_1$- Lipschitz continuous in the second argument.

$\mathcal{A}(g)$ $G : \Omega \times \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_2$ is a random single-valued mapping which has the following conditions:

(i) $G$ is a random $(c_2, \mu_2)$-relaxed cocoercive with respect to $A_2$ in the second argument and a random $\alpha_2$-Lipschitz continuous in the second argument,

(ii) $G$ is a random $\zeta_2$-Lipschitz continuous in the third argument.

Now, we are in position to present our main results.

**Main Theorem (I)** Assume that Assumption $(\mathcal{A})$ holds and there exist two measurable functions $\rho_1, \rho_2 : \Omega \to (0, \infty)$ such that $\rho_i(t) \in \left(0, \frac{r_i(t)}{m_i(t)}\right)$, for each $i = 1, 2$ and

$$\begin{aligned} \frac{\tau_1(t)}{r_1(t) - \rho_1(t)m_1(t)} \sqrt{\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)\alpha_1^2(t)c_1(t) + \rho_1^2(t)\alpha_1^2(t)} < 1 - \frac{\tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)}{r_2(t) - \rho_2(t)m_2(t)}; \\ \frac{\tau_2(t)}{r_2(t) - \rho_2(t)m_2(t)} \sqrt{\beta_2^2(t) - 2\rho_2(t)\mu_2(t) + 2\rho_2(t)\alpha_2^2(t)c_2(t) + \rho_2^2(t)\alpha_2^2(t)} < 1 - \frac{\tau_1(t)\rho_1(t)\zeta_1(t)\phi_2(t)}{r_1(t) - \rho_1(t)m_1(t)}, \end{aligned} \tag{3}$$

for all $t \in \Omega$. Then the problem (1) has a solution.

In particular, we have the following result.

**Main Theorem (II)** Let $U : \Omega \times \mathcal{H}_1 \to \mathcal{H}_1$ and $V : \Omega \times \mathcal{H}_2 \to \mathcal{H}_2$ be two random single-valued mappings. Assume that Assumption $(\mathcal{A})$ holds and there exist measurable functions $\rho_1, \rho_2$ satisfy (3). Then problem (2) has a unique solution.

In the proof of Main Theorem (II), in fact, we have constructed a sequence of measurable mappings $\{(a_n, b_n)\}$ and show that its limit point is nothing but the unique element of $SRSI_{(M_1,M_2)}(F, G, U, V)$. In this section, we will consider the stability of such a constructed sequence.

Let $F, G, M_i, \eta_i, A_i$ and $\rho_i$, for $i = 1, 2$, be random mappings defined as in Main Theorem (II). Now, for each $t \in \Omega$, if $\{(x_n(t), y_n(t))\}$ is any sequence in $\mathcal{H}_1 \times \mathcal{H}_2$. We will consider the sequence $\{(S_n(t), T_n(t))\}$ which is defined by

$$\begin{aligned} S_n(t) &= J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}}[A_1(t, x_n(t)) - \rho_1(t)F(t, x_n(t), V(t, y_n(t)))], \\ T_n(t) &= J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}}[A_2(t, y_n(t)) - \rho_2(t)G(t, U(t, x_n(t)), y_n(t))], \end{aligned} \tag{4}$$

where $U : \Omega \times \mathcal{H}_1 \to \mathcal{H}_1$ and $V : \Omega \times \mathcal{H}_2 \to \mathcal{H}_2$ and $t \in \Omega$. Consequently, we put

$$\delta_n(t) = \|(x_{n+1}(t), y_{n+1}(t)) - (S_n(t), T_n(t))\|^+. \tag{5}$$

Meanwhile, let $Q : \Omega \times \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_1 \times \mathcal{H}_2$ be defined by

$$Q(t, a(t), b(t)) \left(J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}}[A_1(t, a(t)) - \rho_1(t)F(t, a(t), b(t))], J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}}[A_2(t, b(t)) - \rho_2(t)G(t, a(t), b(t))]\right) \tag{6}$$

for all $a \in \mathcal{M}_{\mathcal{H}1}, b \in \mathcal{M}_{\mathcal{H}2}, t \in \Omega$. In view of Lemma A, we see that $(a, b) \in SRSI_{(M_1,M_2)}(F, G, U, V)$ if and only if $(a, b) \in F(Q)$.

Now, we prove the stability of the sequence $\{(a_n, b_n)\}$ with respect to mapping $Q$, defined by (6).

**Main Theorem (III)** Assume that Assumption $(\mathcal{A})$ holds and there exist $\rho_1, \rho_2$ satisfy (3). Then for each $t \in \Omega$, we have $\lim_{n\to\infty} \delta_n(t) = 0$ if and only if $\lim_{n\to\infty}(x_n(t), y_n(t)) = (a(t), b(t))$, where $\delta_n(t)$ are defined by (5) and $(a(t), b(t)) \in F(Q)$.