Forecasting Term Structure of Government Bond Yields in Thailand : Nelson-Siegel Approach

Pongpit Pinsai

MASTER OF SCIENCE PROGRAM IN FINANCE

(INTERNATIONAL PROGRAM)

FACULTY OF COMMERCE AND ACCOUNTANCY

THAMMASAT UNIVERSITY, BANGKOK, THAILAND

May 2007

Forecasting Term Structure of Government Bond Yields in Thailand : Nelson-Siegel Approach

by

Pongpit Pinsai

An Independent Study
Submitted in Partial Fulfillment of
the Requirement for the Degree of
Master of Science Program (Finance)

Master of Science Program in Finance (International Program)
Faculty of Commerce and Accountancy
Thammasat University, Bangkok, Thailand
May 2007

Thammasat University

Faculty of Commerce and Accountancy

An Independent Study

by

Pongpit Pinsai

“ Forecasting Term Structure of Government Bond Yields in Thailand : Nelson-Siegel Approach ”

has been approved as a partial fulfillment of the requirements for
the degree of Master of Science Program (Finance)
On May, 2007 by

Main Advisor (……. ……. ……………………)
Asst. Prof. Dr. Suluck Pattarathammas

Co-advisor (……. ……………… …………..)
Assoc. Prof. Dr. Tatre Jantarakolica

Acknowledgements

My appreciation is expresses to Asst. Dr. Suluck Pattarathammas, my independent study advisor, for guiding me financial and analytical skills. His sincere guidance gave me an invaluable experience. I am also grateful to Assoc. Dr. Tatre Jantrarakolica, my independent study co-advisor, for his generous advises and his research methodology course. In addition, I also got useful comments from Dr. Sutee Mokkavesa, my independent study committee, which benefit me so much.

My special thanks are expressed to MIF program and Thammasat University. My two and a half years in MIF had paved me a prosperous working life. Totally, my six and a half years in this university had fulfilled me a living life.

Finally, I owe a deep gratitude to my beloved family for their supporting and encouragement.

Abstract

In investment area, market participants are always curious about what a shape of the yield curve is and how the yield curve will move in the future horizon. In Thailand, the studies of yield curve have been done in narrow areas, modeling and data extracting. Also, the researches of forecasting yield and interest rate were in the form of interest rate model. In this paper, the Nelson-Siegel with dynamic factors model proposed by Diebold & Li (2005) was studied. The original Nelson-Siegel yield curve was reinterpreted as a three factors dynamic model. The forecasting results are convincing, for 1-month horizon, the Nelson-Siegel with AR (1) (λ = 0.05978) outperforms all competitors models for all maturities and the Nelson-Siegel with VAR (1) (λ = 0.05978 and 0.03446) work well for the 6 and 12-month horizon. This study used spot rate data from the Thai Bond Market Association (ThaiBMA) from July 2001 to March 2007.

1. Introduction

In the world of economics and finance, there are a lot of indicators that play important roles. Among these indicators, yield curve provides a valuable resource for users. In the view of policy makers, yield curve can be used for tracking information about inflation (Fama, 1975), output growth (Estrella & Hardouvelis, 1991) which are important macroeconomic variables. Furthermore, the curve also contains information about the market expectation of interest rate in the future. For investors, most of analysis and pricing activities of bonds and bond related assets require information from yield curves.

Yield curve is the plotted graph that describes the relationship between a particular yield and a time to maturity of bond with the same credit rating. Anyway, ordinary yield curves have an important drawback because they assume the same level of reinvestment rate so zero coupon yield curve or spot yield curve is required (In this study the term “spot” will be used).

Spot yield curve, similar to ordinary yield curve, is the relationship between spot yield, which is a yield of zero coupon bond, and a time to maturity of bond with the same credit rating. Because zero coupon bond pays interest and principal at maturity date so reinvestment problem are solved. If there is a liquid zero coupon bond market spot yield curve can find directly. But in fact, in many bond market ,such as Thailand, there are few traded zero coupon bonds or no traded zero coupon bond exist, the spot yield curve can be derived from the yield curve which are called theoretical spot yield curve. Sometimes the spot yield curve is used in the term of the term structure of interest rates.

The common way to derive the term structure of interest rates is a bootstrapping technique. However this approach has problems in practice therefore the correct presentation of the term structure of interest rates is an active research in this area.

The studies of the term structure of interest rates have been the active area of research for long time. The pioneer of this kind of knowledge is David Durand (1942). This study attempted to fit the French yield curve from the scatter points of data.

Up to present, This field of study can be roughly categorized into 2 families : non-parametric and parametric. The non-parametric family is sometimes called piecewise polynomial or splines family. The well known and acceptable approach was presented by McCulloch (1971). This method, polynomial spline, attempted to produce a continuous discount function by restricting assumption that the bond will pay coupon continuously. However, polynomial spline method has some problems so McCulloch modified the model and proposed cubic spline as presented by McCulloch (1975). This model is the spline with 3 orders which allow more flexibility compared with the polynomial spline model. The most advanced and widely used spline-family model is smoothing spline. This kind of model is selected as a benchmark by central bank of United States and central bank of Japan. Broadly, this technique is used to derive the complete set of discount factors from the market bond prices or yields. The main objective of smoothing spline model is to optimize a criterion that incorporates a smoothing condition. This model will reduce amount of curvature at the expense of worse fitting (James & Webber 2000). There are several forms of smoothing spline such as natural smoothing spline, jump smoothing spline, maximal smoothing spline, B-spline and etc.

Another family that mentioned is parametric family. It is called parametric because the model estimates the forward rate by using parametric function. According to the survey conducted by BIS (Table1), the result is that most central banks rely on parametric models for interest rate and term structure modeling used for macroeconomic policy decision. Nelson & Siegel (1985) is the predecessor of this family. The authors imposed 4 parameters for fitting the curve. With its 4 parameters, Nelson & Siegel model is doubted about its flexibility, says, it can generate only 1 hump and cannot match arbitrary term structure with satisfied precision. The main advantage of Nelson & Siegel model is its simplicity for estimation. To make Nelson & Siegel method more flexible number of academic people extended the model by number of methods. The well known and widely used one is Svensson (1994). Svensson added 2 more parameters into the model that make the curve more flexible. This adjustment allowed for spoon-shaped curve with one more hump. Recently, Diebold & Li (2005) tried to empower the Nelson & Siegel model by employing dynamic concept. This work applied Nelson & Siegel model in an explicitly out of sample forecasting perspective. The authors found that the three-time varying parameters will be interpreted as factors that may be estimated with high efficiency. To prove this argument, the authors compared the model with other 12 models and measured by root mean square error method. For enhancing this kind of model Diebold, Li together with Yue attempted to model the inter-country yield curve. Diebold & Li & Yue (2006) tried to answer the question about the existence of the global yield curve. The authors extracted the global factors from bond yields of United States, Germany, Japan, and the United Kingdom. They found that the global yield curve did exist and it was economically important.

In sum, the area of yield curve and term structure modeling and forecasting are the active area of research in finance and economics. A number of models are developed and used in the markets. However, to model the curves, there is a trade-off between accuracy and smoothness. The curves built for serving the former objective will be appropriate for pricing fixed-income or fixed-income related products. Alternatively, the curves built for serving the latter objective can be used as economic indicators.

In Thailand, Thamchamrassri (2006) employed B-spline method to model government bond zero coupon yield curve. This study estimated 4 models : non-restricted discount fitting, restricted discount fitting, spot fitting and forward fitting. The results were compared with those of the Thai bond market association (ThaiBMA) and the author found that the ThaiBMA curve is in appropriate confident interval of the selected criteria. Jornjaroen (1999) studied the relationship between the term structure of interests of Thailand and major economic indicators such as inflation and GDP (Use manufacturing product index : MPI as a proxy). This study found that, surprisingly, term structure of interest rates and expected inflation have no significant relationship. The explanation for the result was that during the period of study the real interest of Thailand was very fluctuate so nominal interest rate did not move closely with inflation. For GDP, the study found that GDP and term structure of interest rates have a reasonable relationship especially at 3 years time to maturity. For forecasting objective, Promchan (2004) studied interest rate models using data of Thai government bonds from January 1999 to January 2004. This study reveals that CIR model outperforms Vasicek model. Recently, Khanthavit & Jaroenjitrkam (2006) studied Vasicek and CIR models by using Kalman filtering technique which can solve all previous defects. Moreover, this study used the longest period of data that have ever done in Thailand, 1,266 days, July 2 2001 to September 8 2006. The authors found that both models are not reasonable for describing interest rate behavior in Thailand. However, if focusing only on forecasting ability measured by root mean square error (RMSE) and mean absolute error (MAE), Vasicek model outperforms CIR model significantly.

Up to present, the studies of interest rate and yield curve forecasting in Thailand have been done several times but few techniques have been used, broadly, spline family model and equilibrium model. This study uses the technique of Diebold & Li (2005), Nelson-Siegel with dynamic factors model, which reinterprets original Nelson-Siegel model as a three time-varying dynamic factors which estimates the parameters for each month. This model focuses on out-of-sample perspective and Diebold & Li (2005) got a high efficiency result on the U.S. treasury evidence

In this paper, the technique called Nelson-Siegel curve will be studied. This technique and its family are used widely in central bank of many counties as shown in table below. Furthermore, such technique was reinterpreted by Diebold & Li (2005) as a dynamic model which forecast well out-of-sample. Diebold & Li (2005) interpreted Nelson-Siege model as a three time-varying parameters, level, slope and curvature, which evolve dynamically. Such study proposed and estimated autoregressive model for the parameters and got an encouraging result.

Table 1 Estimation methods surveyed by BIS

Central Bank	Method	Minimized Error
Belgium	Svensson or Nelson-Siegel	Weighted prices
Canada	Merill Lynch Exponential Spline	Weighted prices
Finland	Nelson-Siegel	Weighted prices
France	Svensson or Nelson-Siegel	Weighted prices
Germany	Svensson	Yields
Italy	Nelson-Siegel	Weighted prices
Japan	Smoothing Splines	Prices
Norway	Svensson	Yields
Spain	Svensson	Weighted prices
Sweden	Smoothing Spline and Svensson	Yields
Switzerland	Svensson	Yields
United States	Smoothing spline	Bills : Weighted prices Bonds : Prices

Source : BIS

The benefit of Nelson-Siegel modeling technique compared with other techniques is that it performs smoothing curve and it is a quick estimation technique. So, individual fixed-income analysts, fixed-income traders, portfolio managers or even individual investor can model their own the term structure of interest rates in order make their own decisions. By applying dynamic concept, this technique is more attractive because the parameter will evolve dynamically, period by period, and forecasting power of the model is enhanced.

In this paper, government bonds are used because most of active bonds in Thailand are government bonds.

2. Some fundamental concepts

2.1 Yield

Yield on any investment is a return that will make the present of the cash flows equal the initial cost of the investment. There are many types of yield measurements used in the market for different purposes. Some selected measurements will be described below :

Yield to maturity (YTM) is the most frequently used for measuring the return from holding any bonds until maturity. It is the discount rate that equates the present of all cash flows to the initial cost of the investment. YTM is used as quoted yield in the bond market. Anyway, YTM has some drawbacks because it assumes the constant reinvestment rate in all cash flows received.

The alternative and more accurate measurement might be to calculate the present values of cash flows by using the discount rate that represents the market's view on the interest rate, known as forward rate. However, forward rate is an implied interest rate because the actual interest rate in the market might be different from the implied interest rate. Therefore, a measurement by using forward rate could be done in practice either.

2.2 Zero-coupon bond

A zero-coupon bond is bond that pays bullet of cash at obligated date in the future. Since there is no coupon payment for the bond holders, a zero coupon bond is sold at discount value.

Yield from zero-coupon bond is called spot yield or zero-coupon yield. Because it pays only one cash flow at the maturity date so reinvestment problem is eliminated.

2.3 Parametric model of term structure estimation

The term structure of interest rates is a continuous function of spot rates and maturities. This means the matching of spot rate to any maturities in order to price the cash flows in any period in the future. Because of the lacking of zero coupon bond in the market hence the parametric function is employed. This function relates spot rates and time to maturity. Nelson-Siegel function presents a flexible function for the forward rates that can be used to obtain spot rates.

3. Nelson-Siegel model

The purpose of the ordinary Nelson-Siegel (1987) model is to estimate the shape of yield curve with few parameters, sometimes called parsimonious model. This model is able to capture various forms of yield curves.

Nelson-Siegel assumed the instantaneous forward rate as

$$f_t(\tau) = b_{1t} + b_{2t} e^{-\lambda_t \tau} + b_{3t} \lambda_t e^{-\lambda_t \tau} \quad \textbf{(1)}$$

f_t = Forward rate

b_{it} = Parameter that determines shape of forward curve which must be estimated

λ_t = Decay rate

τ = Time to maturity

The corresponding function for spot rate can be derived by integrating the forward rate function and divide by the amount of time to maturity

$$Y_t(\tau) = b_{1t} + b_{2t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + b_{3t}\left(-e^{-\lambda_t \tau}\right) \qquad \textbf{(2)}$$

This is original Nelson-Siegel (1987) spot rate model which imply the spot rate is an equally-weighted average forward rate.

From the above functions, Diebold & Li (2005) used factorization where $b_{1t} = \beta_{1t}$, $b_{2t} = \beta_{2t} + \beta_{3t}$ and $b_{3t} = \beta_{3t}$ and interpret them as the three-factor model

$$Y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right) \qquad \textbf{(3)}$$

Where

Y_t = Spot rate

β = Parameter that determines shape of forward curve which must be estimated

β_{1t} implies long term interest rate

β_{2t} implies spread between short term and long term interest rate

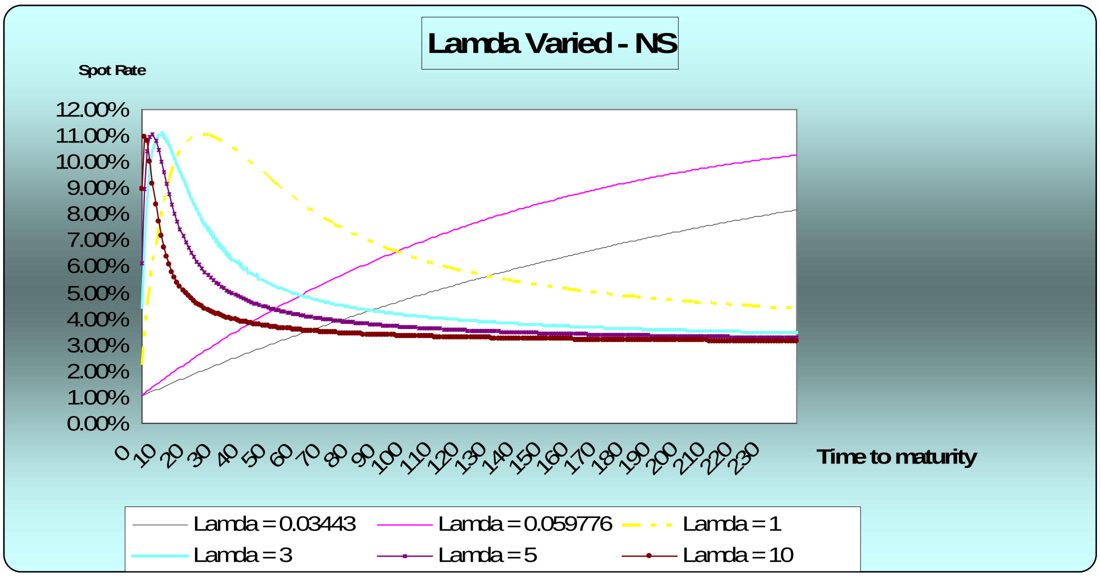
β_{3t} implies shape of the curve

λ_t = Decay rate

τ = Time to maturity

The parameter λ_t governs the exponential decay rate; the small values of λ_t produce slow decay and can fit the curve at long maturities, while large values of λ_t produce fast decay and better fit the curve at short maturities. The figure1 depicted below show the sensitivity analysis of λ_t in Nelson-Siegel model.

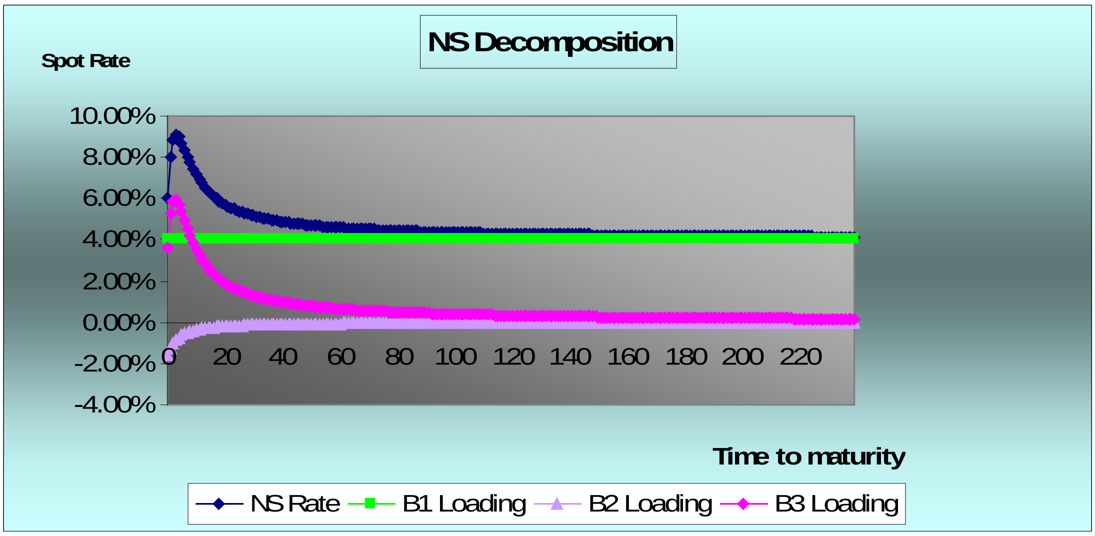
Figure 1 : Sensitivity analysis of Nelson-Siegel model by varying λ value



Note : $\beta_1 = 0.03$, $\beta_2 = -0.02$, $\beta_3 = 0.3$

The loading on β_{1t} is 1 which is a constant that does not decay to zero in the limit; hence it may be view as a long term factor. The loading on β_{2t} is $\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}\right)$ which is a function that starts 1 but decays monotonically and quickly to 0; hence it may be viewed as short term factor. The loading on β_{3t} is $\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right)$ which starts at 0, increases, and then decay to 0; hence it may be viewed as medium term factor. All three factor loadings can be plotted as below

Figure 2 : Decomposition of factor loadings of Nelson-Siegel model



Note : $\beta_2 = -0.02$, $\beta_3 = -0.2$, $\lambda = 2$

The three factors can also be interpreted in term of level, slope and curvature.

The long term factor, β_{1t}, governs yield curve level. Increasing in β_{1t} will increase all yields equally, as the loading is identical at all maturities, thereby changing the level of yield curve. The short term factor, β_{2t}, is closely related to the yield curve slope. The increasing in β_{2t} increases short yields more than long yields because the short rate loaded on β_{2t} is more heavily, thereby changing the level of yield curve. The medium term factor, β_{3t}, is closely related to curvature. The increasing in β_{3t} will has little effect on very short or very long yields, which loaded minimally on it, but increase medium term yields, thereby increasing yield curve curvature. The graphs depicted below show the sensitivity analysis of each parameter in Nelson-Siegel model.

Figure 3 : Sensitivity analysis of Nelson-Siegel model by varying β_1 value

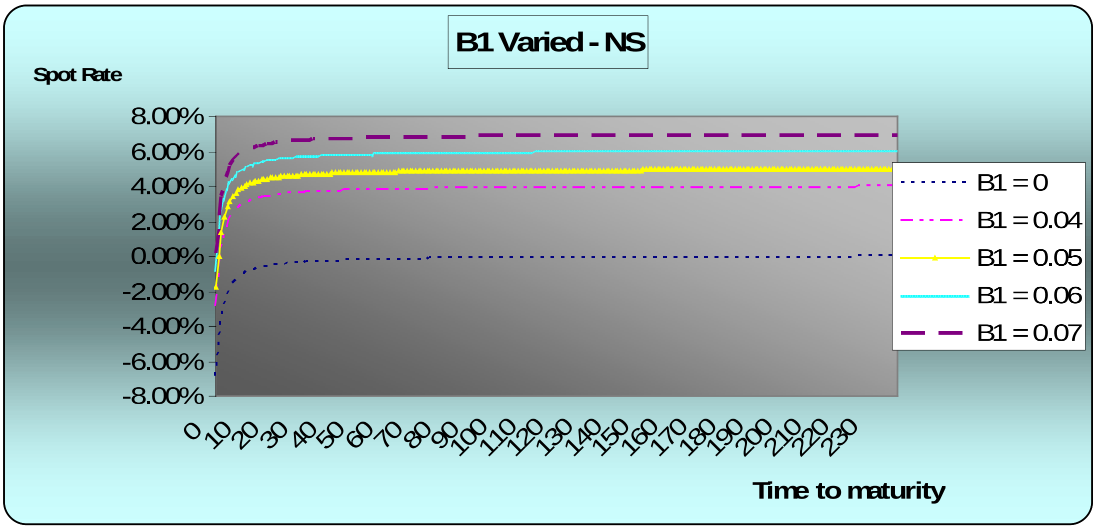
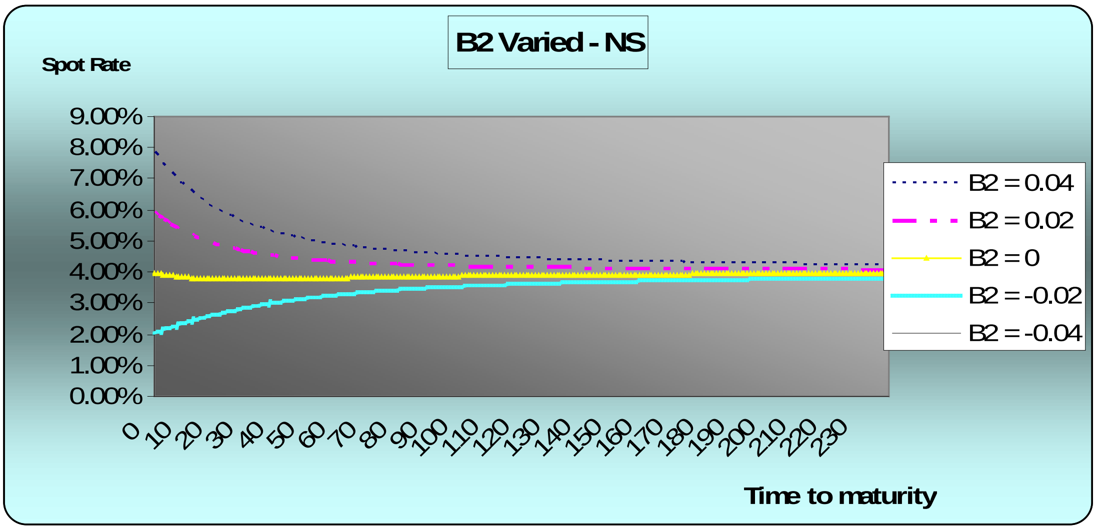
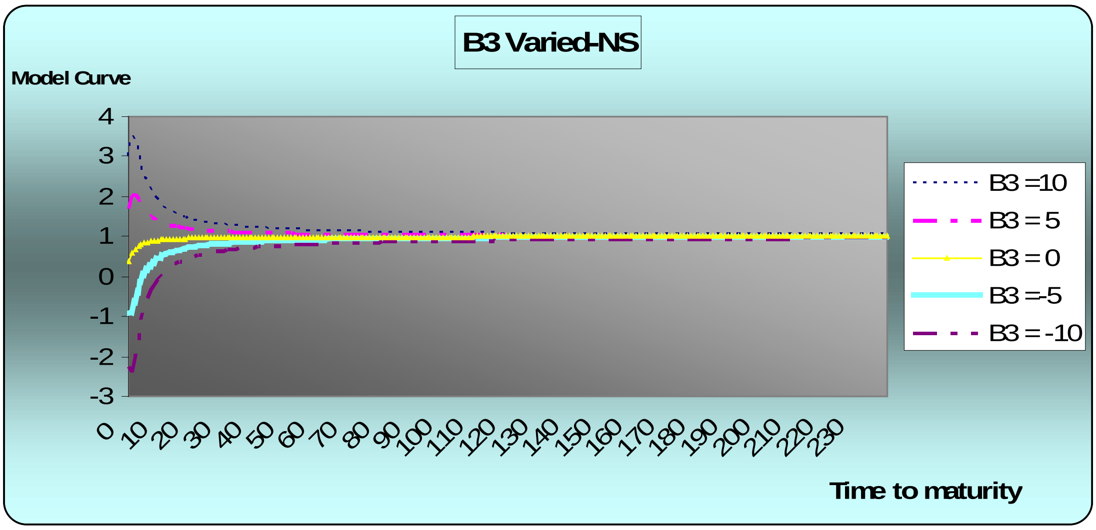


Figure 4 : Sensitivity analysis of Nelson-Siegel model by varying β_2 value



Note : β_1 = 0.04, β_3 = -0.08, λ = 0.05978

Figure 5 : Sensitivity analysis of Nelson-Siegel model by varying β_3 value



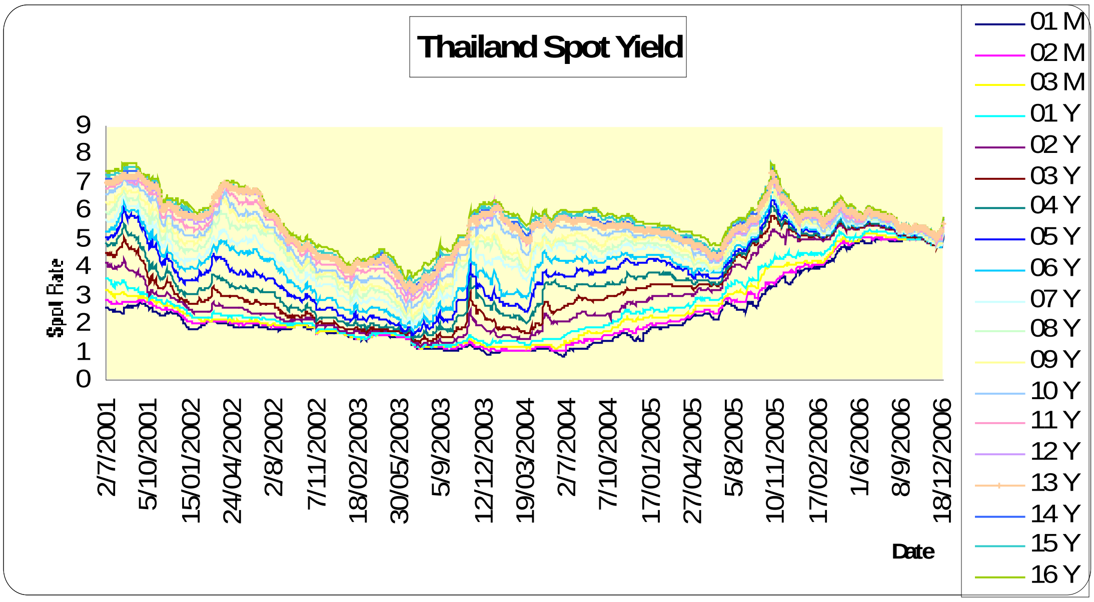
Note : $\beta_1 = 1$, $\beta_2 = -1$, $\lambda = 1$

4. Modeling and forecasting term structure

4.1 The data

Since the term structure requires short-term, medium-term and long-term zero coupon bond so the end-of-month spot yields from government bonds with maturity ranging from 1 to 16 years are required. This study obtains information from the Thai bond market association (ThaiBMA) from July 2001 to March 2007.

The required information excludes bond with embedded option, callable and convertible bonds, because the yield of such bonds reflect the price of options so they cannot be compared with standard bonds. Moreover, tax effect and transaction costs will be ignored for simplicity.



Source : ThaiBMA, 2001:07 to 2007:03

The Thai bond market has grown rapidly after 1997, financial crisis year. In 1998 Thai government issued government bonds fist time in the decade to support financial institutions and has continuously issued bonds to finance budget deficit. The Thai bond market association has also continuously developed and provided us yields on synthetic zero-coupon bonds (Spot Yields) from generalized bootstrapping method originating in July 2001.

In the present, the Thai bond market is dominated by the short-term bonds as shown in the table 2.

Table 2 Numbers of Thai government bonds categorized by time to maturities

TTM	**No. of T-bills and Government bonds**
Less than 1 year	45
1 – 5 years	12
5 – 10 years	10
10 – 15 years	7
More than 15 years	3

Source : ThaiBMA at 31 March 2007

Table 3 Descriptive statistics, yield curve from 2001:01 to 2007:03

Maturity		Mean	Standard Deviation	Max	Min
Years	Months				
0.08	1	2.42811	1.31868	5.03331	0.84882
0.25	3	2.53103	1.30552	5.05359	1.00502
0.50	6	2.65554	1.29935	5.21356	1.12315
1	12	2.79788	1.29947	5.31538	1.15525
2	24	3.09840	1.33302	5.54320	1.17814
3	36	3.35782	1.29773	5.83724	1.34905
4	48	3.62948	1.21864	6.16369	1.45146
5	60	3.94359	1.15164	6.39796	1.56799
6	72	4.22481	1.06546	6.67100	1.73832
7	84	4.50147	0.97216	6.78530	1.89784

8	96	4.71190	0.94860	6.86142	2.06042
9	108	4.86674	0.96202	6.95462	2.23643
10	120	5.07240	0.94704	7.18844	2.50550
11	132	5.27291	0.92231	7.23057	2.76328
12	144	5.35221	0.90352	7.26566	2.97388
13	156	5.42284	0.89237	7.35337	3.13010
14	168	5.50015	0.89391	7.54263	3.24704
15	180	5.58660	0.89649	7.65101	3.34991
16	192	5.69277	0.87320	7.68463	3.55823

Source : ThaiBMA

4.2 Nelson-Siegel curve fitting

Estimation procedure of the parameters can be done by step by step estimation.

Firstly, evaluate equation **(3)** with all λ.

$$Y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right)$$

Where

Y_t = Spot rate

β = Parameter that determines shape of forward curve which must be estimated

β_{1t} implies long term interest rate

β_{2t} implies spread between short term and long term interest rate

β_{3t} implies shape of the curve

λ_t = Decay rate

τ = Time to maturity

From Equation (3), The non-linear least square technique could be employed for estimating parameters $\{\beta_{1t}, \beta_{2t}, \beta_{3t}, \lambda_t\}$. This technique follows ordinary Nelson & Siegel (1987) standard, however, for forecasting purpose such technique is inappropriate because the value of λ affects the value of β_{it}. Hence forecasting error of

λ value will affect $\beta_{it.}$ This paper uses 2 methods to fix the value of λ. The first one is proposed by Dieblod & Li (2005) which instead λ_t at a specified value which allows us using ordinary least square to estimate betas for each t. The value of λ_t is determined by maximizing the loading on medium-term factor. Medium-term is commonly defined as 2 or 3 year maturities so the average number, 30 months, were selected. Another one is propose by Cavin & Chen (2005) which use the median value, this study uses mean value, of λ resulting from non-linear least square. This β-fixed technique is called β stabilization. After stabilizing we expect the consistent movement of the β_i parameters and the convincing out-of-sample forecasting performances.

Next, by using optimization technique, the sum of square error will be minimized. And because most of government bonds in Thailand are short maturity bonds so the weighted technique will be employed. This study uses duration weighted technique of Bliss (1997). The technique weights the short region of yield curve more heavily when fitting the model.

$$\min \sum_{i=1}^{Nt} (w_i \varepsilon_i)^2 \quad \textbf{(4)}$$

Where

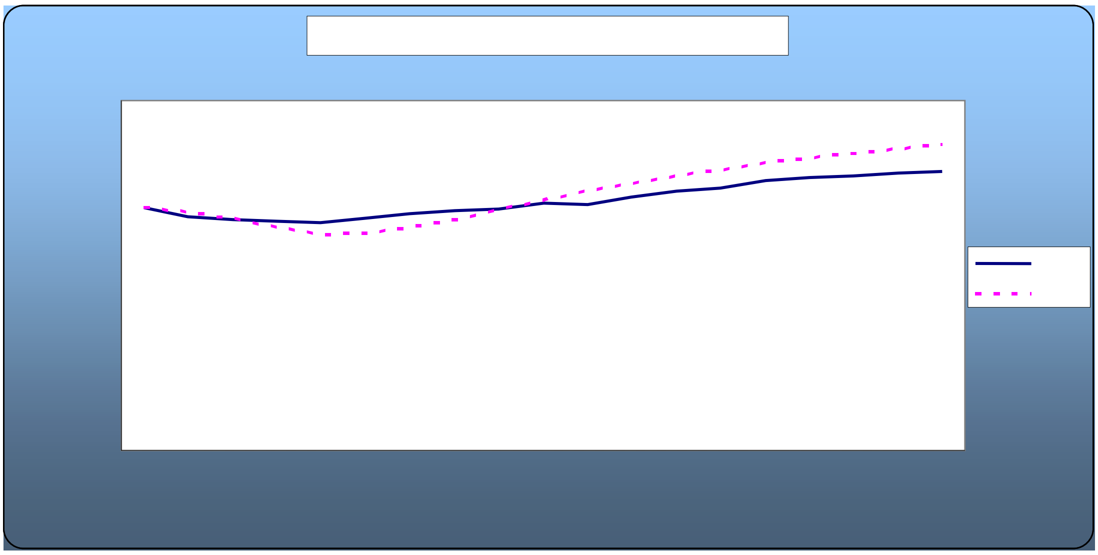
$$w_i = \frac{1/d_i}{\sum_{j=1}^{Nt} 1/d_j}$$

Where

w_i = Weight

d_i = Macaulay Duration

ε_i = Error Term



Note : Data at 30 March 2007, λ = 0.03443

4.3 Forecasting Nelson-Siegel yield curve

Because of this paper models and forecasts the Nelson-Siegel with dynamic factor concept so the model follows univariate AR(1) process :

$$\hat{Y}_{t+h/t}(\tau) = \hat{\beta}_{1,t+h/t} + \hat{\beta}_{2,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \hat{\beta}_{3,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \quad \textbf{(5)}$$

Where

Y_t= Spot rate

β = Parameter that determines shape of forward curve which must be estimated

$$\hat{\beta}_{i,t+h/t} = \hat{c}_i + \hat{\gamma}_i \hat{\beta}_{it} \quad \text{, I = 1, 2, 3 …}$$

λ_t = Decay rate (Fixed at 0.05996 and 0.03443)

τ = Time to maturity

4.4 Model comparison

A good approximation to yield curve dynamics should not only fit well in-sample forecasting but also well out-of-sample forecasting. Diebold & Li (2005) compared out-of-sample forecasting result from Nelson-Siegel model to 12 competitors, in this study the 6 of those 12 models are selected.

Nelson-Siegel with VAR(1) dynamic factors model

The Nelson-Siegel with multivariate VAR(1) process is

$$\hat{Y}_{t+h/t}(\tau) = \hat{\beta}_{1,t+h/t} + \hat{\beta}_{2,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \hat{\beta}_{3,t+h/t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \qquad \textbf{(6)}$$

Where

$$\hat{\beta}_{i,t+h/t} = \hat{c} + \hat{\Gamma}\hat{\beta}_{it}$$

Random walk model

The random walk model function is

$$\hat{Y}_{t+h/t}(\tau) = Y_t(\tau) \qquad \textbf{(7)}$$

Where

Y_t= Spot rate

τ = Time to maturity

Slope regression model

The slope regression function is

$$\hat{Y}_{t+h/t}(\tau) - Y_t(\tau) = \hat{c}(\tau) + \hat{\gamma}(\tau)(Y_t(\tau) - Y_t(3)) \quad \textbf{(8)}$$

Where

Y_t= Spot rate at time t

τ = Time to maturity

AR (1) on yield curve levels model

The AR (1) on yield curve levels function is

$$\hat{Y}_{t+h/t}(\tau) = \hat{c}(\tau) + \hat{\gamma} Y_t(\tau) \quad \textbf{(9)}$$

Where

Y_t= Spot rate at time t

τ = Time to maturity

VAR (1) on yield levels model

The VAR (1) on yield levels function is

$$\hat{Y}_{t+h/t} = \hat{c} + \hat{\Gamma} Y_t \quad \textbf{(10)}$$

Where

Y_t = Spot rate at time t

= [Y_t (1), Y_t (3), Y_t (6), Y_t (12), Y_t (24), Y_t (36), Y_t (48) . . . Y_t (192)]

Note that, because all Y_t have unit root so by doing first difference, Y_t - $Y_{t-1,}$ will result the yield changes.

The selected models can give the forecasted yield from historical yield input. To compare these models, out-of-sample forecasting results will be examined. This paper employs two criteria to measure the accuracy of the selected models.

Root mean square error (RMSE)

$$\text{RMSE} = \sqrt{\left[\frac{1}{n}\Sigma\{Y_t - \Sigma(Y_t)\}^2\right]}$$

Mean absolute error (MAE)

$$\text{MAE} = \frac{1}{n}\sum\left|Y_t - \text{E}(Y_t)\right|$$

The comparison will be done by using data of spot yields reported by the Thai bond market association.

5. Out-of-sample forecasting performance

5.1 β stabilization result

By following Diebold & Li (2005) the value of fixed-λ is 0.05978[1]. From this figure the estimated β_i series was done. We found that β_2 is a unit root while β_1 and β_3

[1] This result is a little bit different from Diebold & Li (2005) which the result is 0.0609 but the difference is not affect the maximized medium term value so much. The maximized medium term value is 0.29843 and 0.29838 for this paper and Diebold & Li (2005), respectively.

are not. Next, by using mean value from the non-linear least square estimation we got the value of fixed-λ equals 0.03443 and we found that the series of β_1, β_2 and β_3 are unit root.

5.2 out-of-sample forecasting results

A good forecasting should not fit well only in-sample but should also be able to fit out-of-sample data. With Nelson-Siegel model, this study has already decomposed it into 3 series of parameters, β_1, $\beta_{2,}$ $\beta_{3,}$ thus forecasting term structure of government bond yields is, in fact, forecasting β_1, β_2 and β_3 (λ was fixed at 0.05978 and 0.03443 from the previous discuss).

The estimation uses data beginning from July, 2001 to March, 2007. In tables below compare h-month-ahead out-of-sample forecasting results from Nelson-Siegel model with those of competitors for maturities of 1M, 3M, 6M, 1Y, 2Y, 3Y . . . 16Y for the forecasted horizons of h = 1M, 6M, 12M.

Insert Table 4-9

The 1-month-ahead forecasting results shown in table 4 and table 7 reveal the result from Nelson-Siegel with AR(1) factor dynamics (λ = 0.05978) outperforms all models for all maturities. The 6-month-ahead forecasting results appeared in table 5 and table 8, we can see that for a short maturities which are 1 month to 3 years the Nelson-Siegel with VAR(1) factor dynamics (λ = 0.05978) is the optimal method while for the longer maturities the Nelson-Siegel with VAR(1) factor dynamics (λ = 0.03443) beats all competitors. Notice that, the value 0.05978 stemmed from maximizing the loading on medium term which set equals 2.5 years. In the same way, when we solve back for the value of 0.03443 the matched maturity is 4.47 years. That why VAR model with lambda 0.05978 is the appropriate model for forecasting 1 month to 3 year–yield and VAR model with lambda 0.03443 is appropriate for the longer ones. Lastly, the 12-month-ahead forecasting results shown in table 6 and table 9 affirm the efficiency of the Nelson-Siegel with VAR (1) factor dynamics model.

The results differ from those of Diebold & Li (2005) which the AR (1) dynamic factor model works well especially in the long forecasting horizon. According to Ashley, Granger and Schmalensee (1980), if the forecast of y from x from a VAR is superior to the forecast from an AR model for y then x carries information about y and hence x causes y. In our case, β_1, β_2 and β_3 might contain information for each other so VAR model outperforms AR model. Notice that, the magnitude of RMSE of this study is lower than that of Diebold & Li (2005) in every forecasting horizon and selected time to maturity. This might imply that the Nelson-Siegel with dynamic factor model suits Thai data.

The forecasting results are convincing but somewhat out of the weighted optimization objective. The objective of weighted function of Bliss (1997) was to give more optimizing accuracy to bonds with short maturities but from the all forecasting results of Nelson-Siegel with dynamic factors model, bonds with long maturities were better fit out-of-sample forecasting.

6. Conclusion

In Thailand, the studies of yield curve have been done in narrow areas, modeling and data extracting. Also, the researches of forecasting yield and interest rate were executed in the form of interest rate models. In this independent study, a new method proposed by Diebold & Li (2005) was adopted. The original Nelson-Siegel yield curve (1987) was reinterpreted as a three factors dynamic model. The curve analysis was done and found that the reinterpreted Nelson-Siegel curve is a combination of level, slope and curvature. To forecast the yield curve, we forecast the parameters that control such level, slope and curvature. As Diebold & Li (2005) did, this paper attempts to fix the value of λ to stabilize the value of forecasted β_1, β_2, and β_3. Furthermore, on optimization process we apply Bliss (1997) weighted function to emphasize short-duration bonds forecasting accuracy because most of government bonds in Thailand are short-term bonds.

The simple forecasting approach, AR (1) and VAR (1), were adopted in Nelson-Siegel with dynamic factors model and other simple regression methods were done for comparison. After out-of-sample result comparison, the encouraging result appeared. The Nelson-Siegel with AR (1) factor dynamics (λ = 0.05978) beats every models in 1-month forecasting horizon and The Nelson-Siegel with VAR (1) factor dynamics worked well in 1-month and 12-month forecasting horizon.

The recommendations for further study are modeling other parametric models such as Svensson (1994) with dynamic factors concept. Also, various figures can be done during estimation and optimization process which are, for instance, relaxing or developing β stabilization and duration weighted function methods. Another important recommendation is finding the way to vary the lambda value consistently with the time to maturity being estimated.

Bibliography

Ashley, R., C.W.j. Granger, & R. Schmalensee (1980) “Advertising and Aggregate Consumption : An Analysis of Causality” , *Econometrica*, 22, 1149-1167.

BIS Paper NO. 25 (2005) “Zero-Coupon Yield Curves : Technical Documentation” , Monetary and Economic Department, Bank for International Settlements.

Bliss, R., (1997) “Testing Term Structure Estimation Methods” , *Advance in Futures and Options Research*, 9, 97-231.

Choudhry, M. (2004) *Analysing and Interpreting Yield Curve*. Singapore: Wiley

Chris, B. (2002) *Introductory Econometrics for Finance*, United Kingdom: Cambridge

Diebold, F. X., & Li, C. (2005) “Forecasting Term Structure of Government Bond Yields” ,*Journal of Econometrics*, 130, 337-364.

Diebold, F. X., & Li, C., & Yue, V. Z. (2006) “Global Yield Curve Dynamics and Interaction : A Generalized Nelson-Siegel Approach” , NBER Working Paper.

James, J., & Webber, N. (2000). *Interest Rate Modelling*. Chichester: Wiley

Khanthavit, A. (2001) *Fixed-income Investment Analysis*. Bangkok: Thammasat University Press (inThai).

Khanthavit, A., & Jaroenjitrkam, A. (2006) “A Statistical Examination of Short-termed One-Factor Interest Models for Thailand’s Bond Market”, 14th Annual Professor Sangvien Indaravijaya Conference on Thailand’s Financial Markets Working Paper, Department of Finance and Accountancy, Thammasat University.

McCulloch, J. H. (1971) “Measuring the Term Structure of Interest Rates” , *Journal of Business*, 44, 19-31.

Nelson, C.R., & Siegel, A. F. (1985) ”Parsimonious Modeling of Yield Curves for US. Treasury Bills” , *Journal of Business*, 60, 473-490.

Jornjaroen, N. (1999) “*Term Structure of Interest Rates and Economic Indicators of Thailand*” , Master Thesis, Thammasat University, Bangkok.

Philippe, C., & Xuelin C. (2005) “Forecasting Term Structure of Swiss Government Bond Yields” , Working Paper, UBS Business Banking S.A.

Promchan, C. (2004) “*An Estimation of Term Structure of Interest Rates in the Thai Bond Market*” , Master Thesis, Chulalongkorn University, Bangkok.

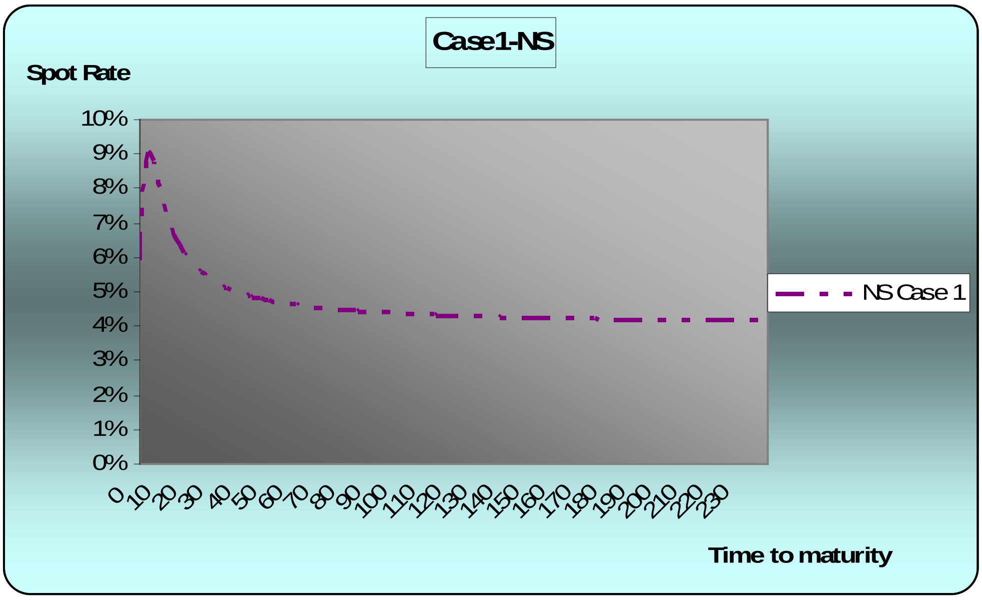
Sangmanee, A. (2001) “Information Content of Financial Markets” , Bangkok of Thailand Symposium Working Paper.

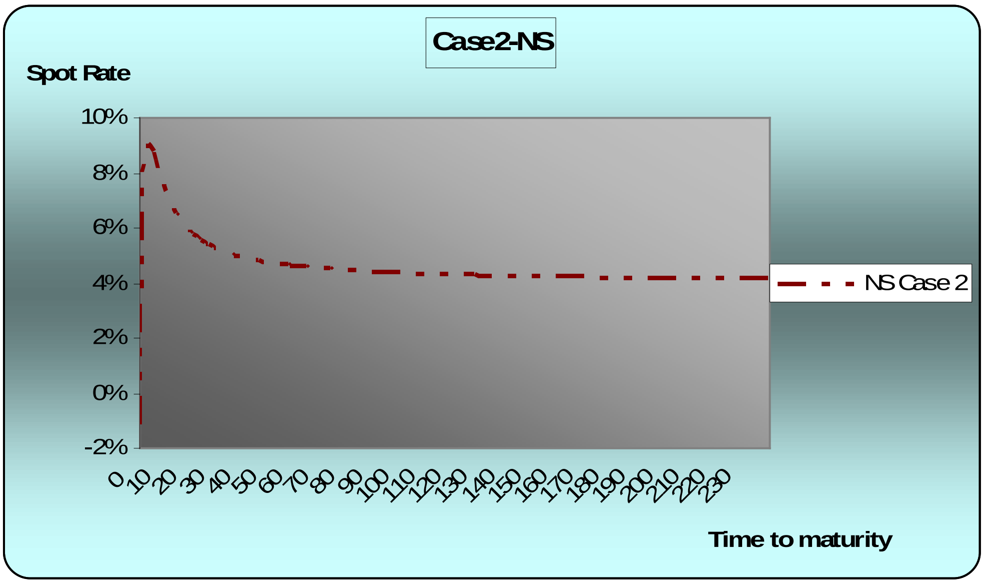
Siegel, A. F., & Nelson, C.R. (1987) “Long Term Behavior of Yield Curves” , *Journal of Financial and Quantitative Analysis*, 23, 105-110.

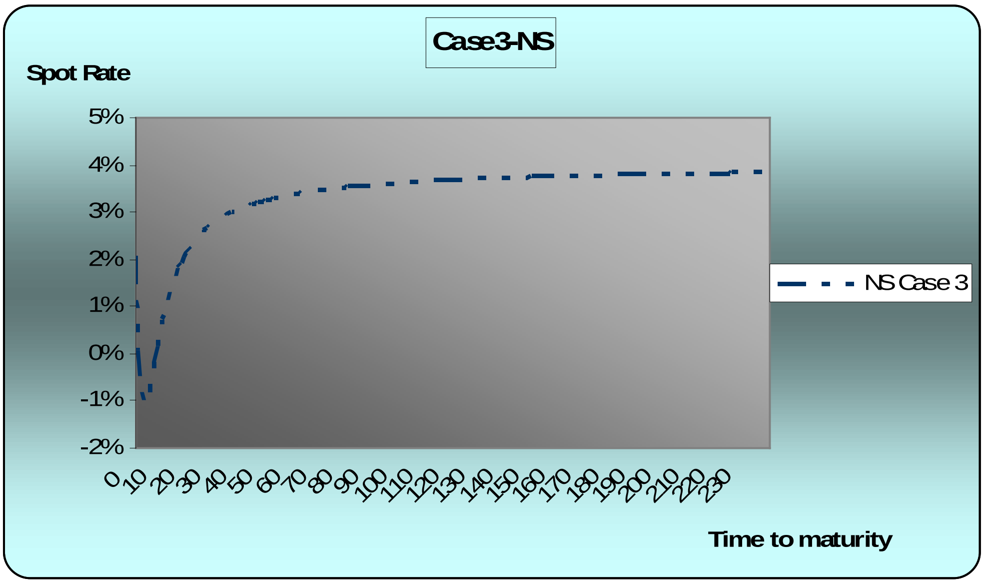
Thamchamrassi, K. (2006) “*Estimating Term Structure of Interest Rates for Thai Government Bonds : A B-Spline Approach*” , Independent Study, Thammasat University, Bangkok.

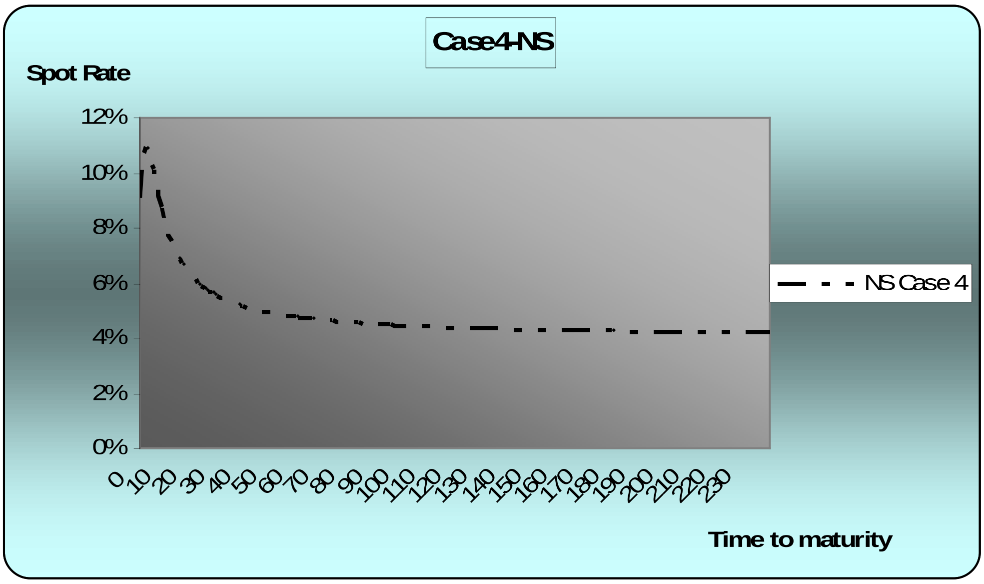
Appendix

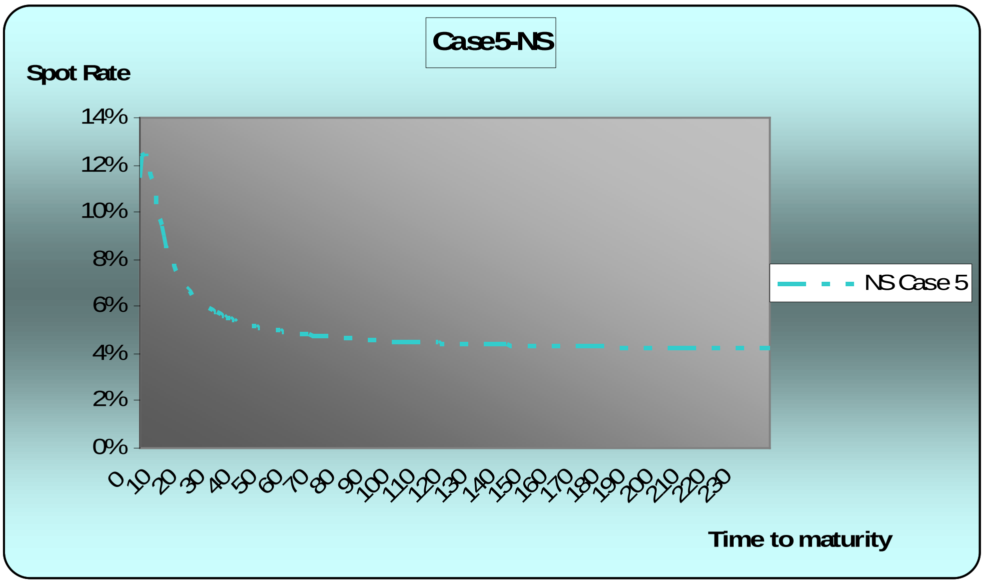
Nelson-Siegel model can generate various forms of yield curves and term structure of interest rates. The graphs below depict the possible shapes.

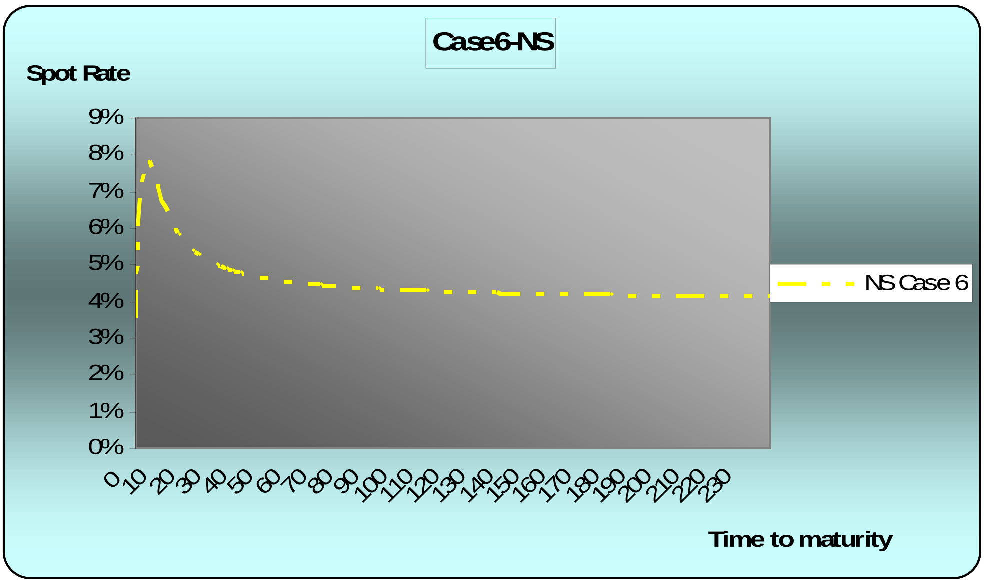


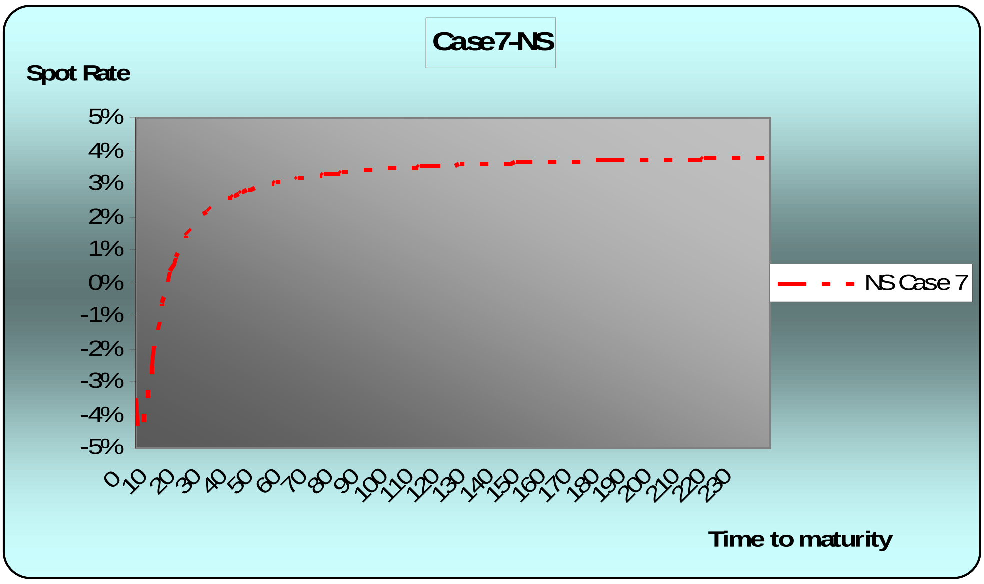

Case2-NS
Spot Rate
10%
8%
6%
4%
2%
0%
-2%
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230
Time to maturity
NS Case 2

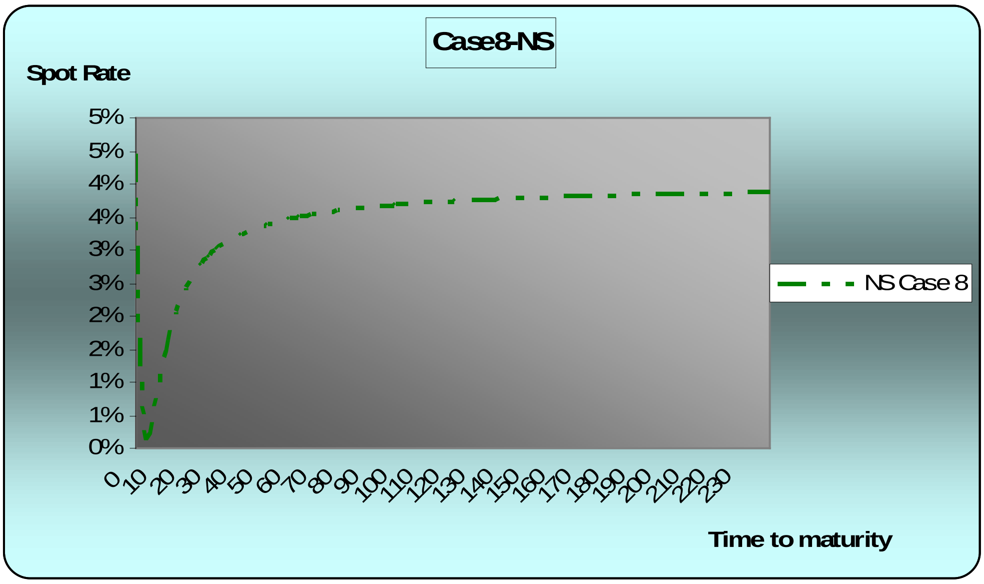

Case3-NS
Spot Rate
5%
4%
3%
2%
1%
0%
-1%
-2%
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230
Time to maturity
NS Case 3

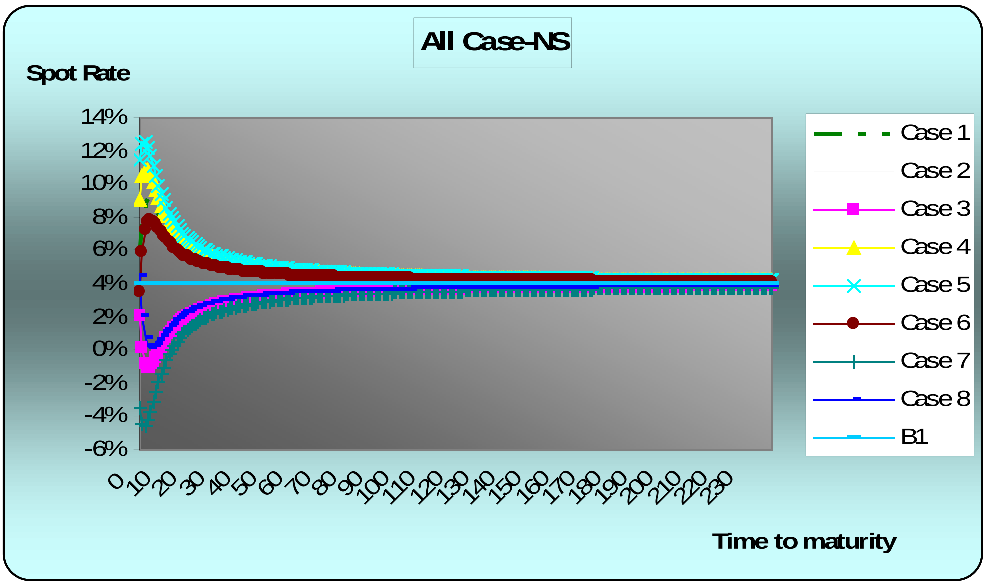

Case4-NS
Spot Rate
12%
10%
8%
6%
4%
2%
0%
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
NS Case 4
Time to maturity


Case5-NS
Spot Rate
14%
12%
10%
8%
6%
4%
2%
0%
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
NS Case 5
Time to maturity


Case6-NS
Spot Rate
9%
8%
7%
6%
5%
4%
3%
2%
1%
0%
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230
NS Case 6
Time to maturity


Case7-NS
Spot Rate
5%
4%
3%
2%
1%
0%
-1%
-2%
-3%
-4%
-5%
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230
NS Case 7
Time to maturity


Case8-NS
Spot Rate
5%
5%
4%
4%
3%
3%
2%
2%
1%
1%
0%
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
NS Case 8
Time to maturity


All Case-NS
Spot Rate
14%
12%
10%
8%
6%
4%
2%
0%
-2%
-4%
-6%
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
Case 1
Case 2
Case 3
Case 4
Case 5
Case 6
Case 7
Case 8
B1
Time to maturity

<u>Remark</u>

Shape	β_1	β_2	β_3	λ_1	**Condition**
1	+	-	+	+	$\lvert\beta_1\rvert >= \lvert\beta_2\rvert$
2	+	-	-	+	$\lvert\beta_1\rvert >= \lvert\beta_2\rvert$
3	+	+	-	+	$\lvert\beta_1\rvert >= \lvert\beta_2\rvert$
4	+	+	+	+	$\lvert\beta_1\rvert >= \lvert\beta_2\rvert$
5	+	+	+	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$
6	+	-	+	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$
7	+	-	-	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$
8	+	+	-	+	$\lvert\beta_1\rvert < \lvert\beta_2\rvert$