# Estimating VaR and Hedging:

# A Copula-EVT Framework

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**May 2007**

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An Independent Study
Submitted in Partial Fulfillment of
the Requirement for the Degree of
Master of Science Program (Finance)

Master of Science Program in Finance (International Program)
Faculty of Commerce and Accountancy
Thammasat University, Bangkok, Thailand
May 2007

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has been approved as a partial fulfillment of the requirements for
the degree of Master of Science Program (Finance)
On May, 2007 by

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# Content

## List of Table and Charts

# Acknowledgement

I would like to thank warmly my project advisors, Asst. Prof. Dr. Arnat Leemakdej and Dr. Sutee Mokkhavesa, for giving me the opportunity to do my independent study in this interesting field and for their valuable remarks and suggestions. I would also like to express my appreciation to Asst. Prof. Dr. Suluck Pattarathammas, who was also a member of the committee and provided many valuable comments.

Moreover, I would like to thank my family for their supports and understanding during my years of study at MIF program. Finally, a special thanks goes to my brothers, sisters, and friends in the MIF program, this paper could not have been completed without their helps and supports.

# Abstract

The traditional risk measurements are usually based on the classical assumption of normal distribution of risk factors, however many literature have shown that the marginal distribution of asset log-return has fatter tail than normal. The Copula-EVT model which can handle the non-normality is suggested as an alternative. The main objective of this paper is to compare the performance of the Copula-EVT model and the traditional model in estimating the Value-at-Risk (VaR). Monte-Carlo simulation is used to simulate scenarios for log-returns of portfolio generated from multivariate distribution with Gaussian Copula and marginal distributed as normal in the center and Extreme Value Theory (EVT) in the tails. In this paper, I apply this method to estimate VaR over a one-day horizon for a portfolio containing twenty-nine Thai equities. The empirical result indicates that Copula-EVT VaR outperforms multivariate normal model. For hedging purpose, the study shows that minimum-VaR hedging provides the higher percentage of reduction in VaR by taking smaller short position in futures than the minimum variance hedge strategy.

## 1. Introduction

When the financial crisis in Asia Pacific was emerged in 1997, inefficient risk management of financial institutions was blamed as one of culprits. The capital reserves for risky assets invested by such financial institutions were insufficient to cover loss. This was mainly due to the underestimation of portfolio's risk, both the investment portfolio and the loan portfolio.

After the crisis, risk management has become more and more attractive topic for regulators, fund managers, bankers, and investors. In the market risk management, Value at Risk (VaR) has been introduced as a standard measure to quantify market risk. VaR is defined as the maximum potential loss on a given portfolio with certain confidence level within a specified investment horizon. VaR measures can be applied for both risk management and regulatory purposes. In particular, the Basel Committee on Banking Supervision (1996) at the Bank of International Settlements (BIS) has announced the Basel II standards, which forces banks and financial institutions to maintain the adequacy of capital requirements based on VaR estimates. Therefore, providing accurate VaR estimates is a crucial importance. If the risk is not properly estimated, this may lead to sub-optimal capital allocation under risk constraint or the banks may have inadequate or excess capital reserves left unused to support underlying risks.

The concept of conventional VaR is based on the classical assumption of normal distribution of the underlying risk factors. However, it is well known that most of securities' returns follow fat tail distribution (McNeil and Frey 1999); their kurtosis and skewness of the distribution are significantly different from 3 and 0 as in normality case. Since the assumption of normal distribution is violated, the traditional method of normality is not appropriate to measure VaR. As a result, researches and studies in financial field have developed various methodologies to better compute VaR.

The Extreme Value Theory (EVT) has been introduced as a branch of statistics dealing with the extreme deviations from the median of probability distributions. The EVT can be applied to risk management since the financial asset returns are usually fat-tailed, then assuming normality can lead to serious underestimates of VaR. Therefore the EVT methods which fit extreme quartiles can explain the risk for highly unusual events better than the other methodologies. Besides, there are numerous literatures that support the superior performance of the EVT based VaR to the normality VaR estimation.

In the past three years, EVT has become more popular. It has been developed to deal with the multivariate distribution and dependence structure of the securities' returns. Then the Gaussian Copula has been introduced to deal with such multivariate and joint distributions. Therefore, the Copula has been applied as a risk management tools for insurance companies, financial institutions, and the mutual funds to overcome the limits of traditional VaR model.

However, in mutual fund management and financial institution, accurate VaR measurement is not enough to avoid the financial disaster, the more important question is "how to manage such a risk?" The hedging strategy is now well established and commonly used not only by fund managers and bankers but also the corporate practitioners to offset the position in spot market by taking position in derivatives market. In practice, to hedge risk exposure by long or short derivatives, the question is what the optimal amount of such derivatives to minimize risk. In other words, the hedger need to know the Optimal Hedge Ratio. The traditional approach to estimate the hedge ratio is to find the amount of derivatives that minimize variance of the hedged portfolio, which called minimum-variance hedge ratio. This method presumes that the portfolio risk is measured by standard deviation or variance. The method inherits the normal distribution assumption of the asset returns from traditional framework. However, most asset returns are indeed characterized as fat-tailed. Adding up all assets' risks into portfolio's risk regardless of skewness and kurtosis of the hedged portfolio as suggested the minimum variance approach is prone to serious errors. Therefore, the minimum-VaR hedge ratio has been proposed as an alternative.

This paper combines the two approaches in two steps. First, I concentrate on the accuracy of estimated VaR, the proper one will lead to an optimal asset allocation and the appropriate hedge ratio. The next step is for hedging purpose; I will estimate the amount of index futures to hedge the risk of equity fund by using minimum VaR approach and compare its hedging performance with the minimum variance approach.

The contributions of this research are twofold. First, it demonstrates how to use the new method of Copula-EVT VaR estimation. Second, it provides an application of Copula-EVT VaR minimization in hedging strategies. The test of accuracy of estimated VaR and test of hedging performance will lead to the right decision on adequacy of capital reserve and better risk management.

**Research Questions**

1. Does the VaR estimated by Copula-EVT approach outperform the traditional VaR estimation?
2. Does the minimum-VaR hedge ratio outperform the minimum-variance hedge approach?

**Objectives and Benefits of the Study**

Since the financial institution and asset management firms have to monitor risks based on the VaR framework. Traditional VaR estimates mentioned above are based on the classical assumption of normal distribution and ignore the extreme situations. However, many papers prove that most financial asset returns usually distribute in fat-tails. As a result, the traditional method of normality is not appropriate to measure VaR. In addition, there are some evidences that the security returns are not distributed independently. Then the Copula should be applied to deal with multivariate and joint distribution.

This paper introduces Copula-EVT VaR, which is theoretically appropriate to deal with multivariate fat-tailed data. One of the main objectives is that to estimate VaR by using Copula-EVT approach and perfrom backtesting to test the performance of this approach comparing to the traditional one. Moreover, the study also provides the contribution of Copula-EVT VaR in hedging strategy. The purpose is to estimate the optimal hedge ratio of index futures by minimizing VaR, which is calculated from Copula-EVT estimation, to hedge the portfolio of stocks. Then the paper compare the hedging performance of the minimum-VaR hedge ratio and the minimum-variance hedge ratio.

The main benefit of this study can be divided into two parts, first, it provides a better understanding and implication about the estimation of VaR in fat-tailed environment and joint distribution of the portfolio returns. Second, this paper proposes the concept and advantage of minimum-VaR hedging compared to minimum-variance hedging. Besides, if the fat-tailed distribution can be observed in Thai Stock returns, the Copula-EVT VaR can be applied as a more appropriate approach, at least theoretically, to estimate the optimal capital requirement and to manage portfolio risk by minimum-VaR hedge ratio.

**Scope of Study**

The study will focus on how to estimate VaR by applying EVT approach. The Gaussian Copula will be applied to simulate the joint distributed multivariate risk factors. Moreover, the application of VaR in hedging strategies will be investigated in this study. Finally, the test of goodness of Copula-EVT VaR and hedging effectiveness of minimum-VaR hedge ratio are then proposed to compare with the traditional approaches.

For the data, equity data is collected from SETSMART and/or BloomBerg using daily data from January 1998 to March 2007 of stocks listed in SET50 Index before 1998.

**Limitation**

Since the index futures market has just been emerged in April 2006, the data of SET50 index futures are inefficient to analyze the result. Then, this study uses the settlement price of SET50 index futures calculated by Cost of Carry Model.

This paper proceeds as follows. Section 1 provides some background by emphasizing the significance of the problem. Section 2 reviews previous researches related to this topic. Section 3 outlines the theoretical framework. Section 4 provides the setting of formulation of the problem to be studied. Section 5 presents the empirical study, and the paper is concluded in Section 6

## 2. Literature Reviews

The Extreme Value Theory (EVT) has been introduced as a classical probability statistic focusing on the extreme event. This framework has firstly introduced in the hydrology, and then applied to the theory of insurance and finance, nowadays, there has been several number of EVT studies in the financial risk management field. Besides, the application of Copula in risk measurement has been proposed in many studies during the recent years.

Due to the conceptual simplicity, Value at Risk (VaR) has become a standard tool to quantify risk. VaR measures can be used in many applications, such as in risk management and for regulatory requirement, in particular, the Basel II has been imposed to financial institution to meet the capital requirements calculated based on VaR framework. The recent development of VaR models in finance can be found in the study by Manganelli and Engle (1999), which reviews all VaR methodologies by classified them into three main categories that are parametric, nonparametric, and semi parametric. The paper focuses on the underlying assumptions and the logical drawbacks of available methodologies; however, their empirical application is not mentioned. The EVT for risk management has been applied by Mcneil (1999), which provides an overview of EVT in risk management as a tool to measure extreme risks. The study concentrates on how the Peaks-Over-Threshold (POT) model can be a useful model in VaR estimation and expected shortfall for market risks. Moreover, he combines the stochastic volatility models, which is fitted by the ARCH/GARCH family model, and dynamic risk management where interested in the conditional return distribution to dynamic measures VaR and expected shortfall over a 1-day horizon, then compare to traditional methods namely normality and historical simulation. The backtesting and empirical study conclude that model based on normality assumption are likely to underestimate extreme risk, and model of historical simulation can only provide imprecise extreme risk estimation. EVT is the most efficient instrument to predict the size of extreme event.

Using EVT to solve the problems of normality VaR estimation, which is based on normal distribution and its result are likely underestimates the extreme risk, has become attractive in recent years. Mcneil and Frey (1999) use a Generalized Pareto Distribution (GPD) estimation based on extreme value theory to model the tail of the distribution of risk factors. They provide steps to estimate the GPD parameters by maximum likelihood estimation (MLE) and the threshold choice by applying mean squared error (MSE) technique. The result concludes that GPD approximation suggested by EVT work quite well when the returns follow asymmetric in tail. Specifically, they use backtesting to compare EVT with expected shortfall and find that the risk factors should be modeled by fat-tail distribution, as a result EVT is preferable.

Bensalah (2000) reviews some theoretical results of EVT concerning the estimation of the asymptotic distribution of the extreme observations. The paper also provides steps in applying EVT to financial risk management including data analysis, a tool to choose the high threshold, extreme VaR estimates, and the GPD approximation. The EVT techniques are applied to a series of exchange rates of Canadian/U.S. Dollars and the empirical result concludes that the EVT results apply well to the univariate case, however, the multivariate case and joint distribution of the marginal extreme distribution incorporating the market risk framework remains an open question in this study.

Similar study is proposed by Habiboellah (2005), which provides systematic steps to apply EVT in banking. The paper describes how to apply EVT in bank risk management to meet regulatory requirement. EVT provides useful tools to define the distribution function of extreme events concerning a fall in prices of financial assets held or issued by bank. The study suggests that, in banking, EVT can be used to manage all three risks regulated by Basel II, which are credit risk, market risk, and operational risk. In market risk, EVT is efficient to determine VaR, in credit and operational risk, EVT is often utilized for determining the adequate level of capital.

Wongchotiwat (2004) estimates VaR based on EVT model and apply for empirical study of risk factors in Thai market, which are SET index and Thai Baht/US Dollar exchange rate. The study uses backtesting to compare the performance of different VaR models including normality VaR and historical simulation approaches. The result indicates that, in the fat-tail environment, normal VaR fail to capture risk at high confidence level, EVT and historical simulation methods are more conservative than normal distribution.

The issue of multivariate risk management has been introduced in recent years. A Copula function is the powerful concept to manage multivariate risk factors of portfolio of asset. The first introduction of applying Copula in risk management can be found in the paper by Bouyé and Durrleman (2001), which clarifies the concept of Copula and empirical use in financial field. Khanthavit (2006) applies Copula VaR and Copula Expected Shortfall in portfolio of Thai debt instruments. The paper explains that the Thai Government Bond Yields are distributed as Logistic distribution, and estimates VaR by using Monte Carlo Simulation, in which the random numbers are generated from the Gaussian Copula function. Though the returns do not follow normal distribution, the backtesting result indicates that Normality VaR outperforms Copula VaR. However, Khanthavit suggests that the unexpected result may come from the error in parameters estimation. Applying Copula function in extreme VaR estimation is firstly presented by Clemente and Romano (2005), who describe steps to simulate the risk factors under multivariate GPD distribution and dependence structure by using Gaussian Copula and Student's t-Copula. The simulated risk factors correlate as desired based on the correlation matrix forecasted through Exponentially Weighted Moving Averages (EWMA) approach. The empirical application to estimate 99% VaR over one-day horizon of a portfolio of twenty Italian equities concludes that the Copula-EVT outperforms the traditional VaR models. The similar result can be found in the paper by Mourany and Mukherji (2005). They estimate Copula-EVT VaR of a portfolio of thirteen UK equities, the backtesting over a time window of four years shows that Copula-EVT provides more accurate VaR.

Hai He (2005) presents a Copula-EVT model to estimate portfolio VaR by simulating risk factor log-return from multivariate distribution with Gaussian and Student's t-Copula and the marginal distributions follow GPD. In this study, the correlation matrix is estimated based on both EWMA and MGARCH approach. The paper applies Copula-EVT model to estimate VaR of a portfolio containing sixty-four Chinese equities and performs the backtesting over a time window of six years and the results indicate that Copula-EVT approach outperforms the traditional VaR models.

To apply VaR in the theory of hedging, which is now well established and commonly used by the practitioners to offset the risk of spot market by taking position in derivatives market, Harris and Jian Shen (2004) propose the minimum-VaR hedge ratio instead of minimum-variance hedge ratio. The study shows that although the minimum-variance hedging can reduce the standard deviation of portfolio returns, it tends to increase the portfolio skewness and kurtosis, and consequently the utility of investors will be affected. Then the standard deviation is no longer an appropriate measure of risks since it fail to capture all of characteristics of portfolio returns that investors consider to be important. The paper introduces hedging with minimizing VaR, which consider not only standard deviation of portfolio returns but also their skewness and kurtosis. The results present that minimum-VaR hedge ratio is significantly smaller than minimum-variance hedge ratio and the minimum-VaR hedging offer a lower risk of portfolio than the risk of minimum-variance hedge portfolio.

From the literature reviews stated above, most of the studies in this area focus on either VaR estimation or optimal hedge ratio. However, this paper combines such two things by studying on how to estimate VaR accurately and contribute it to the hedging strategy. In addition, this study will provide the empirical study of the portfolio containing Thai stocks listed in SET50 index.

# 3. Theoretical Framework

This section outlines the theoretical frameworks of the key concepts used in this paper. There are, namely, four main topics, namely, Value at Risk, Extreme Value Theory, Copula, and minimum-VaR hedging.

## 3.1 Value-at-Risk

Value at Risk (VaR) has become the standard measure that financial analysts use to quantify the risk. It is defined as the maximum potential loss of a portfolio of financial instruments with a given confidential level over a certain horizon. Since VaR is intuitive and very simple to understand, its measurement is widely developed in many methodologies. The existing models for calculating VaR follow a common general structure, which are to estimate the distribution of portfolio returns and to compute the VaR of the portfolio. The main differences among VaR methods are related to the estimation of the returns distribution that is the way most of studies address the problem of how to estimate the possible changes in the value of the portfolio. The existing VaR models can be classified into three main categories, which are Parametric, Nonparametric, and Semi Parametric model.

Parametric VaR generally assumes that risk factors follow normal distribution. The assumption of independent identical distribution of standardized residual terms is a necessary device to estimate the unknown parameters of the distribution function. The drawback of this approach is that it tends to underestimate VaR, since the normality assumption seems to be inconsistent with the behavior of financial returns. However, this method is still widely used in practice because it is easy to implement and fast to calculate.

One of the most common methods for VaR estimation is the Historical Simulation, which is classified as a nonparametric model. This approach simplifies the procedure to estimate VaR, since there is no any assumption about the distribution of the portfolio returns. However, the hidden assumption behind this method is that the portfolio returns are assumed unchanged within the analytical period. The forecasted VaR under this approach are meaningful only if the historical data used in the estimation have the identical distribution. The historical simulation based VaR will be biased subject to the volatility of the historical data observed within a given period.

Many alternative methods have been introduced to estimate VaR during the recent years, such as Monte Carlo simulation, the application of Extreme Value Theory, etc. These approaches simulate normally distributes future scenarios using the distribution function of risk factor returns and uses them to reevaluate the portfolio. It estimates VaR by randomly creating many scenarios for future rates, using nonlinear pricing model to estimate the change in value for each scenario and then calculating VaR according to the worse case scenario. The biggest advantage of this method is that it is captures non-linearity and can generate an infinite number of scenarios.

Since many pioneering works proposed that most of financial data are fat-tailed, this paper will focus on the Extreme Value Theory, which has been developed to explain the characteristic of the tails.

### 3.2 Extreme Value Theory

Extreme Value Theory (EVT) has been originally introduced as the probabilistic theory for studying extreme events. EVT is a useful tool for estimating the tail of asset log-return distribution. In theory of EVT, the distinction can be made between two approached, namely, the Block Maxima approach and Peak-Over-Threshold (POT) approach. The main difference between both approaches is how the extreme data are identified. The first one is the oldest approach in EVT, which considers the maximum data in the successive period, such as ten days or three months. These selected maximum data are classified as extreme event. This study concentrates on the second approach, POT, which only considers the observations that exceed a given threshold. This model exploits data more efficiently than Block Maxima model and becomes more preferable in recent applications.

Suppose that $X = (X_1, \ldots, X_n)$ is a sequence of independent identically distributed observations with distribution function $F(x) = Pr(X_i \leq x)$. The excess over a given threshold $u$ occurs when $X_i > u$ for any i = 1, 2, ..., n and the excess over $u$ is defined by $y$. The distribution of the excess losses over the threshold u is given by;

$$F_u(y) = \Pr(X - u \leq y \mid X > u) \quad (1)$$

The distribution $F_u$ represents the probability that the value of $X$ exceeds the threshold $u$ by at most an amount of $y = x - u$ given that $x$ exceeds the threshold $u$.

From equation (1);

$$1-F_u(y) \;=\; 1-\Pr(X-u \le y \mid X>u) \;=\; \Pr(X-u>y \mid X>u) \qquad (2)$$

Given

$$1-F(u+y) \;=\; 1-\Pr(X \le u+y) \;=\; \Pr(X>u+y) \qquad (3)$$

From Bayes's Theorem, this conditional probability can be written as;

$$\Pr\left(X>u+y \mid X>U\right) = \frac{\Pr(X>u+y \cap X>u)}{\Pr(X>u)} \qquad (4)$$

$$\Pr(X>u+y \cap X>u) \;=\; \Pr\left(X>u+y \mid X>u\right)\cdot \Pr(X>u) \qquad (5)$$

Since $X - u = y$ and $X \geq u$ then $y \geq 0$;

$$\Pr(X>u+y \cap X>u) \;=\; \Pr(X>u+y) \qquad (6)$$

From equation (3), (5), and (6):

$$\Pr(X>u+y) \;=\; \Pr\left(X>u+y \mid X>u\right)\cdot \Pr(X>u) \qquad (7)$$

$$= \Pr\left(X-u>y \mid X>u\right)\cdot \Pr(X>u)$$

$$1-F(u+y) = \left[1-F_u(y)\right]\cdot\left[1-F(u)\right] \qquad (8)$$

$$F_u(y) \;=\; \frac{F(u+y)-F(u)}{1-F(u)} \qquad (9)$$

Since for $x > u$, $y = x - u$, then $x = u + y$ , I have;

$$\Pr(X \le u+y) \;=\; \Pr(X<x),\ \textit{then}\ F(u+y) = F(x) \qquad (10)$$

Therefore, from (9) and (10);

$$F_u(y) \;=\; \frac{F(x)-F(u)}{1-F(u)}$$

$$F(x) \;=\; \left[1-F(u)\right]F_u(y) \;+\; F(u) \qquad (11)$$

Pickands (1975), Balkema and de Haan (1974) show that for sufficiently high threshold $u$, the distribution function of the excess $F_u(y)$ can be approximated by the Generalized Pareto Distribution (GPD), which has the analytical form as follow;

$$G_{\xi,\beta,u}(y) = \begin{cases} 1-(1+\xi \frac{y}{\beta})^{-1/\xi} & , if \ \xi \neq 0 \\ 1-e^{-y/\beta} & , if \ \xi = 0 \end{cases} \quad (12)$$

with $$y \in \begin{cases} [0,\infty) & , if \ \xi \geq 0 \\ [0,-\beta/\xi) & , if \ \xi < 0 \end{cases}$$

Where $y = X - u$, which is the excess of $X$ over a threshold $u$. The scale parameter β and the shape parameter ξ are estimated from real data of excess returns.

From equation (11), for sufficiently high threshold $u$, $F_u(y)$ converges to the GPD in equation (12), and then I have;

$$F(x) = [1-F(u)]G_{\xi,\beta,u}(y) + F(u) \quad (13)$$

For a high threshold $u$, the last term on the right hand side can be determined by the empirical estimator $(N-N_u)/N$ where $N$ is the total number of observation and $N_u$ is the number of observation exceed the threshold u. The result, therefore, is given by:

$$F(x) = \left(\frac{N_u}{N}\right)G_{\xi,\beta,u}(y) + \frac{N-N_u}{N} \quad (14)$$

Substitute equation (12) into the equation (14), simplified $F(x)$ is;

$$F(x) = 1-\frac{N_u}{N}\left(1+\xi\frac{y}{\beta}\right)^{-1/\xi} \quad (15)$$

For a given confidence level $p$, and $y = x_p - u$, VaR is defined as

$$X_p = \text{VaR}_{\text{extremeGPD (p)}} = u + \frac{\beta}{\xi}\left[\frac{N}{N_u}(1-p)^{-\xi} - 1\right] \quad (16)$$

### 3.3 Copula Functions

Copulas are functions that describe dependencies among variables, and provide a way to create distributions to model correlated multivariate data. Using a copula, one can construct a multivariate distribution by specifying marginal distributions, and then choose a particular copula to provide a correlation structure between random variables. The distributions in higher dimensions are possible.

**Definition**[1]**:** An n-dimensional copula is a multivariate cumulative distribution function (c.d.f.) with uniform distributed margins in [0,1] (U(0,1)) and the following properties:

1. C: $[0,1]^n \rightarrow [0,1]$;
2. C is grounded and n-increasing;
3. C has margins $C_i$ which satisfy $C_i(u) = C(1, ..., 1, u, 1, ..., 1) = u$ for all $u \in [0,1]$.

It is obvious, from the above definition, that if $F_1, ..., F_n$ are univariate distribution functions, then $C(F_1(x_1), ..., F_n(x_n))$ is a multivariate c.d.f. with marginal distribution $F_1, ..., F_n$ because $u_i = F_i(x_i)$, i = 1, ..., n, is a uniform random variable. Copula functions are a useful tool to construct and simulate multivariate distributions.

The following theorem is known as **Sklar's Theorem**. It is the most important theorem about copula functions because it is used in many practical applications.

**Theorem**[2]**:** Let F be an n-dimensional c.d.f. with continuous marginal distribution $F_1, ..., F_n$. Then it has the following unique copula representation:

$$F(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n)) \tag{17}$$

Sklar's Theorem shows that, for continuous multivariate distribution functions, the univariate marginal distribution and the multivariate dependence structure can be separated. The dependence structure can be represented by a proper copula function. Moreover, the following corollary is attained from (17).

**Corollary:** Let F be an n-dimensional c.d.f. with continuous marginal distribution $F_1, ..., F_n$ and copula C (satisfying (17)). Then, for any $u = (u_1,\dots,u_n)$ in $[0,1]^n$:

$$C(u_1,...,u_n) = F(F_1^{-1}(u_1),...,F_n^{-1}(x_n)) \tag{18}$$

where $F_i^{-1}$ is the generalized inverse of $F_i$.

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[1] The original definition is given by Sklar (1959).

[2] For the proof see, Sklar (1996).

**Normal copula:** The Gaussian (or normal) copula is the copula of the multivariate normal distribution. In fact, the random vector X = ($X_1$, …,$X_n$) is multivariate normal if and only if:

1. the univariate marginal distribution $F_1$, …, $F_n$ are Gaussians;
2. the dependence structure among the marginal distribution is described by a unique copula function C (the normal copula) such that:

$$C_R^{Ga}(u_1,...,u_n) = \Phi_R(\phi^{-1}(u_1),...,\phi^{-1}(x_n)) \tag{19}$$

where $\Phi_R$ is the standard multivariate normal c.d.f. with linear correlation matrix **R** and $\phi^{-1}$ is the inverse of the standard univariate Gaussian c.d.f.

Multivariate normal is commonly applied in risk management to simulate the distribution of the *n* risk factors affecting the value of the trading portfolio (market risk).

### 3.4 Minimum-Value at Risk Hedging

Suppose that hedging portfolio is the combination of long position in spot market and taking short position in derivatives market, which is used to offset the risk exposure of an unhedging portfolio. Consequently, the return of hedging portfolio is composed of the return of unhedging portfolio and the return of derivatives, which can be written as;

$$R_H = R_U - hR_F \tag{20}$$

where $R_H$ = return of hedging portfolio

$R_U$ = return of unhedging portfolio (portfolio of securities traded in spot market)

$R_F$ = return of derivatives

h = hedge ratio, the optimal amount of derivatives that minimize risk of hedging portfolio.

When the VaR of hedging portfolio is known, to minimize VaR of the hedging portfolio by finding the optimal hedge ratio, the objective function is:

$$\min_h VaR\{R_U - hR_F\} \tag{21}$$

In this study, the numerical solution will be shown in section five.

The methodologies for estimating VaR and hedging with minimizing VaR are thoroughly described in the following section.

# 4. Methodology

This section shows the methodology in estimating GPD parameters and simulating multivariate risk factors by applying Gaussian copula. The forecasted correlation matrix of the assets contained in the portfolio is presented in this chapter. The chapter begins with the preliminary data analysis, then the method of estimating GPD parameters and threshold selection are provided. Finally, estimating correlation matrix and generating random variables from the n-dimensional Gaussian copula are described systematically.

## 4.1 Return calculation

The first step is to calculate the log-returns of stock and futures by using daily close prices, which can be presented mathematically as;

$$x_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \tag{22}$$

## 4.2 Preliminary data analysis

The preliminary statistics are proceeded to test whether the log-returns follow normal distribution as assumed by traditional VaR method and if fat-tailness of log-return distribution is observed then it is satisfied to estimate VaR using Generalized Pareto Distribution.

***Jarque-Bera test***

Initially, I test whether the distribution of log-returns follows normal distribution by using the Jarque-bera test (JB test). JB test is a goodness-of-fit measure of departure from normality, based on the sample kurtosis and skewness. The test statistic JB is defined as

$$JB = \frac{N}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \tag{23}$$

where S is the skewness, K is the kurtosis, and N is the number of observations. The statistic has an asymptotic chi-squared distribution with two degrees of freedom and can be used to test the null hypothesis that the data are from a normal distribution; since normal distribution have an expected skewness of 0 and an expected kurtosis of

3. If the probability value of the computed Chi-square statistic is sufficiently low and the JB statistic is higher than the Chi-square critical value, the null hypothesis that the series is normally distributed can be rejected.

***QQ Plots***

Quantile-Quantile (QQ) plot is used to testify whether the tail of the empirical distribution of the portfolio's log-returns follow the normal distribution. QQ plots display the sample quantiles of empirical data versus theoretical quantiles from a normal distribution. When the distributions of log-returns are normal, the plot will be close to linear. If the empirical data are fat-tailed, the graph will show a curve to the top at the left end or the bottom at the right end.

Let $X_1, \ldots, X_n$ be the succession of random variables i.i.d., and $X_{n,n} < \ldots < X_{1,n}$ be the sequence of random variable namely the order statistics. $F_n(x)$ be the empirical distribution function. Note that a particular quantile of the empirical distribution is defined by $F_n(X_{k,n}) = (n-k+1)/n$ and F is the estimated parametric distribution or normal distribution of the data.

The graph of QQ plots can be defined by the following set of the points,

$$QQ\ Plots = \left\{ X_{k.\,n},\, F^{-1}\left(\frac{n-k+1}{n}\right)\right\} \qquad (24)$$

If the normal distribution model fits the data well, this graph will have a linear form. Thus, the graph makes it possible to compare various estimated models and choose the best. The more linear the QQ plot, the more appropriate the model in terms of goodness of fit. In this paper, fat-tail distribution is considered, the QQ plots, which QQ Plots of upward quantile are higher than the normal or the QQ Plots of downward quantile are lower than the normal distribution.

### 4.3 Forecasted variance-covariance matrix

To standardize historical return series and simulate log-returns $X$ of the $n$ risk factors during the time [t, t+1], the mean return vector $\mu_{t+1}$ is usually assumed to be equal to zero. The elements of the nxn variance-covariance matrix $\Sigma_{t+1}$ are the forecasted variances and covariances among the $n$ risk factor log-returns in time step [t, t+1].

Analytically: $\Sigma_{t+1} = [\sigma_{i,j,t+1}]$, i, j = 1,…,n, where $\sigma_{i,i,t+1} = \sigma^2_{i,i,t+1}$.

In the traditional risk management applications the variance-covariance matrix is forecasted by using the Exponentially Weighted Moving Averages (EWMA). Following this technique, the variance of risk factor i in time step (t,t+1) is:

$$\sigma^2_{i,t+1} = (1-\lambda)\sum_{k=1}^{N}\lambda^{k-1}x^2_{i,t-k+1}, \qquad i = 1, ..., n \qquad (25)$$

The covariance between risk factors i and j is the following:

$$\sigma_{i,j,t+1} = (1-\lambda)\sum_{k=1}^{N}\lambda^{k-1}x_{i,t-k+1}\,x_{j,t-k+1}, \qquad i, j = 1, ..., n \qquad (26)$$

where $\lambda$ = decay factor (in RiskMetrics, it is assumed $\lambda = 0.94$)

$x_{i,t-k+1} = \ln(P_{i,t-k+1}/P_{i,t-k})$, k = 1, ..., n, log-return of asset i

N = total number of historical log-returns used in the estimation.

Using the EWMA method the earlier data have a higher weight in the estimation of variances and covariance depending on the decay factor. The smaller the decay factor, the greater the weight given to recent events. If the decay factor is equal to one, the model reduces to an equally weighted.

### 4.4 Estimating parameters

To find the appropriate Generalized Pareto Distribution (GPD) of the log-returns distribution, the tail parameters ($\xi$), as well as the scaling parameter ($\beta$) have to be determined by fitting the GPD to the actual data. The Maximum Likelihood Estimation (MLE) can be applied with the following log-likelihood function;

$$L(\xi,\beta \mid x) = \begin{cases} -N_u \ln(\beta) - \left(\frac{1}{\xi}+1\right)\sum_{i=1}^{N_u}\left(1+\frac{\xi}{\beta}(x_i-u)\right) & \text{if } \xi \neq 0 \\ -N_u \ln(\beta) - \left(\frac{1}{\beta}\right)\sum_{i=1}^{N_u}(x_i-u) & \text{if } \xi = 0 \end{cases} \qquad (26)$$

where $N_u$ is the number of exceeding log-returns over threshold u, and $x_i$ is the log-returns, which is exceed the threshold u. the parameters, $\xi$ and $\beta$, can be estimated by using MATLAB 7.3.0 R2006b software[3].

[3] MATLAB software is developed and patented by MathWorks. (www.mathworks.com)

From the c.d.f. of GPD mentioned in section 3, the following marginal distributions used to simulate the risk factor standardized log-returns.

$$F_i(x) = \begin{cases} \frac{N_{u_i}^L}{N}\left(1+\xi_i^L \frac{\left|x-u_i^L\right|}{\beta_i^L}\right)^{-1/\xi_i^L} & (x < u_i^L) \\ \Phi(x) & (u_i^L \le x \le u_i^R), \quad \text{i = 1, ..., N} \\ 1-\frac{N_{u_i}^R}{N}\left(1+\xi_i^R \frac{x-u_i^R}{\beta_i^R}\right)^{-1/\xi_i^R} & (x \ge u_i^R) \end{cases} \tag{27}$$

where Φ is the standardized normal c.d.f. $N_u^L$ is the number of negative log-returns exceeding threshold $-u^L$ and $N_u^R$ is the number of log-returns exceeding $u^R$.

In order to apply EVT correctly, the historical data have to be independent determinations sampled from a common c.d.f.. So, the variances estimated with EWMA are used for filtering data. Analytically:

$$z_{i,t-j+1} = \frac{x_{i,t-j+1}}{\sigma_{i,t-j+1}}, \text{ i = 1, ..., n; j = 1, 2, ..., N-74} \tag{28}$$

The number of standardized log-returns is N-74 since the 74 older observations are lost in the variance and covariance estimation[4].

***Choice of threshold***

The EVT approach considers extreme observations exceeding a threshold that is high enough. The threshold selection is subject to the trade-off between variance and bias. When using a big amount of data (low threshold), the estimated tail index is more precise with less variance but biased because some observations from the center of the distribution, which is irrelevant to the tail analysis, are taken into account. On the other hand, using less data (high threshold), the volatility of the estimator is higher but is less biased.

In order to estimate the threshold u, Clemente and Romano (2005) suggest the following steps;

[4] In RiskMetrics, it is assumed N = 74, that means 74 historical data are used in the EWMA estimation procedure.

- Calculate the standardized normal c.d.f. of historical standardized log-returns.
- Select the upper threshold $u_i^R$ as the highest x that the standardized normal c.d.f. $\Phi(x) < 1 - \frac{N_x^R}{N}$, where $N_x^R$ is the number of historical standardized log-returns exceeding x.
- The lower threshold $u_i^L$ has been selected as the lowest x that the standardized normal c.d.f. $\Phi(x) > \frac{N_x^L}{N}$, where $N_x^L$ is the number of historical standardized negative log-returns exceeding -x.

### 4.5 Simulation

From the equation (19);

$$C_R^{Ga}(u_1,...,u_n) = \Phi_R(\phi^{-1}(u_1),...,\phi^{-1}(x_n)) \quad (19)$$

where $\Phi_R$ is the standard multivariate normal c.d.f. with linear correlation matrix **R**, its elements $R_{ij} = \Sigma_{ij} / \Sigma_{ii}\, \Sigma_{jj}$, and $\phi^{-1}$ is the inverse of the standard univariate Gaussian c.d.f.

To estimate the correlation matrix R of the Gaussian copula, this paper applies the following steps provided by Clemente and Romano (2005);

- Transforming dataset of the standardized log-returns log-return of asset i, i = 1,…,n, from equation (28) $(z_1^t,...,z_n^t)$ t = 1,…,N into the univariate on [0,1], $(\hat{x}_1^t,...,\hat{x}_n^t)$, by using the marginal distribution in equation (27).
- Use the inverse of c.d.f. for the standardized normal distribution to obtain the $\varsigma_t = \{\Phi^{-1}(\hat{x}_1^t),...,\Phi^{-1}(\hat{x}_n^t)\}$, t = 1,…,N.
- Calculate $\hat{\mathbf{R}}$ based on the transformed dataset $\{\varsigma_t\}$, t = 1, 2, ..., N-74 using EWMA mentioned in (25) and (26).

If the matrix **R** is positive definite, then there are some nxn matrix **A** such that **R** = $\mathbf{AA}^T$ assuming that the random variables $Z_1, ..., Z_n$ are independent standard normal. The random vector $\mu + AZ$ (where $Z = (Z_1, ..., Z_n)^T$ and the vector $\boldsymbol{\mu} \in R^n$) is multivariate normally distributed with mean vector **μ** and correlation matrix **R**.

The matrix **A** can be determined by the Cholesky decomposition of **R**. Then, random variable with with mean vector **μ** and correlation matrix **R** can be generated from n-dimensional Gaussian copula by using the following algorithm:

- Find the Cholesky matrix decomposition **A** of the matrix **R**, where **R** = **AA**$^T$.
- Simulate k independent standard normal random variables **v** = $(v_1,\ldots,v_k)^T$, v is a vector kx1.
- Generate the random numbers **w**, that have the correlation related to estimated correlation matrix **R**, by setting **w** = **Av**.
- Calculate the c.d.f. for the standardized normal distribution of $w_i$, $p_i = \Phi(w_i)$, i = 1,…,n.
- For Monte-Carlo scenario, transform $(p_1,\ldots,p_n)^T \sim C_R^{Ga}$ to daily log-return of n assets **X**, by using the inverse of the GPD distribution function.

  Then, obtain $z = (z_1,\ldots,z_n)^T = \left(F_1^{-1}(p_1),\ldots,F_n^{-1}(p_n)\right)^T$;

$$z_i = F_i^{-1}(p_i) = \begin{cases} u_i^L - \dfrac{\beta_{i_i}^L}{\xi_i^L}\left[\left(\dfrac{N}{N_{u_i}^L}(p_i)\right)^{-\xi_i^L} - 1\right] & (p_i < \Phi(u_i^L)) \\ \Phi^{-1}(p_i) & (\Phi(u_i^L) \le p_i \le \Phi(u_i^R)), \\ u_i^R + \dfrac{\beta_{i_i}^R}{\xi_i^R}\left[\left(\dfrac{N}{N_{u_i}^R}(1-p_i)\right)^{-\xi_i^R} - 1\right] & (p_i > \Phi(u_i^R)) \end{cases} \quad (29)$$

  where i =1, ..., N

- Then, rescale the standardized log-returns using square roots of the EWMA variances estimated by equation (25), $x = (x_1,\ldots,x_n)^T = (z_1\sigma_{1,t+1},\ldots,z_n\sigma_{n,t+1})^T$.
- Then, the log-returns follow Gaussian Copula are already obtained.

### 4.6 VaR calculation

To describe the procedure of estimating portfolio 99% VaR over one-day horizon, assume the portfolio contains one position for each of the n assets. The portfolio at time t is;

$$P_t = \sum_{i=1}^{n} P_{i,t}$$ , where $P_{i,t}$ = market price of asset i, i = 1,…,n, at time t. (30)

From simulated Gaussian Copula log-returns, I simulate s = 10,000 Monte-Carlo scenario for each asset log-returns, $R_{ij}$, i = 1,…,n; j = 1,…,10,000, over the time horizon [t,t+1]. On time t+1, the portfolio will be revalued as;

$$P_{j,t+1} = \sum_{i=1}^{n} P_{i,t} \exp\{R_{i,j}\}, \qquad j = 1,\ldots,10{,}000 \tag{31}$$

The portfolio losses in each scenario j can be explained as;

$$L_j = P_t - P_{j,t+1} = \sum_{i=1}^{n} \left[P_{i,t} - P_{i,t} \exp\{R_{i,j}\}\right] = \sum_{i=1}^{n} P_{i,t}\left(1 - \exp\{R_{i,j}\}\right) \quad , j = 1,\ldots,10{,}000 \tag{32}$$

Therefore, to determine the 99% VaR from this distribution, ordering the 10,000 value of $L_j$ in increasing order, the 99% VaR is the 9,900$^{th}$ ordered scenario.

Finally, use Loss Function to compare Copula-EVT based VaR with the traditional approaches.

### 4.7 Evaluating VaR performance

To compare the predictability among VaR models and select the most accurate one, Lopez (1998) proposed a measure of relative performance that can be used to monitor the performance of VaR estimates. The general form of a loss function is

$$C_{m,t+1} = \begin{cases} f(r_{t+1}, VaR_{m,t}) & \text{if} \quad r_{t+1} < VaR_{m,t} \\ g(r_{t+1}, VaR_{m,t}) & \text{otherwise} \end{cases} \tag{33}$$

where $C_{m,t+1}$ represents the numerical scores generated for individual VaR model m. The score for the complete regulatory sample is

$$C_m = \frac{1}{T}\sum_{t=1}^{T} C_{m,t+1} \tag{34}$$

The scores are constructed with a negative orientation, which low values of loss functions are preferred because it indicates the lower loss in risk management. The best VaR model is selected by comparing the expected score of complete regulatory. A model, which minimizes the expected loss, is preferred over the other models.

In this paper, I apply following three criteria of loss function to evaluate the relative performance of various VaR forecasts.

*(1) Binary Loss Function*

The loss function implied by the binomial method is

$$C_{m,t+1} = \begin{cases} 1 & \text{if } \; r_{t+1} < VaR_{m,t} \\ 0 & \text{otherwise} \end{cases} \tag{35}$$

If a loss exceeding the VaR is observed, it is called "exception". This approach is frequency known as "Backtesting". The model simply considers with the number of exceptions rather than the magnitude of these exceptions. If a VaR model provides an accurate estimate, the summation of $C_{m,\ t+1}$ will equal to 0.01 multiply by no. of time windows ($T$) for the 99$^{th}$ percentile VaR.

*(2) Regulatory loss function (RLF)*

The regulatory loss function or magnitude loss function takes account of the magnitude of the exceptions when the failures of model occur. The loss function is defined by:

$$C_{m,t+1} = \begin{cases} (r_{t+1} - VaR_{m,t})^2 & \text{if } \; r_{t+1} < VaR_{m,t} \\ 0 & \text{otherwise} \end{cases} \tag{36}$$

This model reflects the penalty due to the failure of a model. The score increases with the magnitude of exception and can provide the information on how the underlying VaR model underestimates the risk. However, it is impossible to perform the hypothesis testing because the distribution of loss function is unknown.

*(3) Firm's loss function (FLF)*

For the firm, there is a conflict between the loss protection and the profit maximization. A VaR estimate, which produces "too high" values of VaR will lead to "too much" reserve capital, imposing the high opportunity cost to the firm. The FLF is defined as follows:

$$C_{m,t+1} = \begin{cases} (r_{t+1} - VaR_{m,t})^2 & \text{if } \; r_{t+1} < VaR_{m,t} \\ -\alpha VaR_{m,t} & \text{otherwise} \end{cases} \tag{37}$$

Where α represents the opportunity cost of capital.

*Hypothesis Testing*

For significance testing, P. Kupiec (1995) proposes the analysis of exceptions (*N*) based on the observation (*T*). This approach presents the method to do the hypothesis testing, whether the no. of exceptions (*N*) is “too small” or “too large” under the null hypothesis:

$$H_0: \quad p = 0.01$$

The log-likelihood ratio test statistic is given by:

$$LR_{uc} = -2\ln\left[(1-p)^{T-N} p^N\right] + 2\ln\left\{[1-(N/T)]^{T-N}(N/T)^N\right\} \qquad (38)$$

which is asymptotically distributed chi-square with one degree of freedom under the null hypothesis that *p* is true probability. Therefore, the null hypothesis can be rejected at the 95% confidence level if LR > 3.8415. In short, the VaR model would be accept, if LR < 3.8415.

## 4.8 Hedging

At this step, the VaR of portfolio of stocks traded in spot market or called unhedging portfolio is already known. The next step is to estimate VaR of hedged portfolio, which minimize risk exposure of unhedging portfolio by taking short position in futures, the return of hedge portfolio can be written by; $R_H = R_U - hR_F$ as described in section 3.

To estimate VaR of hedging portfolio, I repeat the steps to find the Copula-EVT based VaR as described above. Moreover, the futures are considered as the securities added in portfolio.

Then use the numerical method to minimize the VaR of hedging portfolio by estimating the optimal amount of hedged futures or called optimal hedge ratio;

$$\min_h VaR\{R_U - hR_F\}$$

Finally, compute the amount of risk reduction and return of hedging portfolio, and then compare with the minimum-variance hedge ratio.

## 5. Empirical Results

In this section, the models described in section 4 are applied to estimate the 99% VaR over one-day investment horizon for a portfolio of stocks. The paper also compares the accuracy of Copula-EVT model and the Monte-Carlo Simulation based on multivariate normal distribution. The effectiveness of models is evaluated by performing loss functions and Kupiec's testing. Moreover, the paper evaluates the effectiveness in risk reduction and return of portfolio hedged by Minimum-VaR Hedge Ratio comparing to the Minimum-Variance Hedge Ratio.

The data set consists of daily closing prices of 29 stocks, which are listed in SET50 before 1998, from January 1, 1998 to March 15, 2007 obtaining approximately 2,256 observations for each series. The observed historical data is divided into 2 parts; time series of 1,521 observations from January 1, 2001 to March 15, 2007 are used to estimate relevant parameters and predicting VaR, and the remaining of 735 observations from January 1, 1998 to December 31, 2000 are preserved for back-testing procedure.

For futures prices, since the futures market has been introduced in Thailand for only 11 months from April 28, 2006, the observed historical prices are insufficient to satisfy the basic assumption of asymptotic properties of extreme value theory. Therefore, this paper applies the method of Cost of Carry model to estimate futures prices by using the closing prices of SET50 index from January 1, 2001 to March 15, 2007.

### 5.1 Preliminary data analysis

The preliminary statistics are proceeded to test the normality of log-returns as assumed by traditional VaR method and if fat-tailness of log-return distribution is observed then it is satisfied to estimate VaR using Generalized Pareto Distribution.

Initially, the Jarque-bera (JB) test is applied to test the normality of log-return series. If the probability value of the computed chi-square statistic is sufficiently low and the JB statistic is higher than the chi-square critical value, one can reject the null hypothesis that the return series is normally distributed. The following table presents the summary of Jarque-Bera (JB) test of stocks' log-return series;

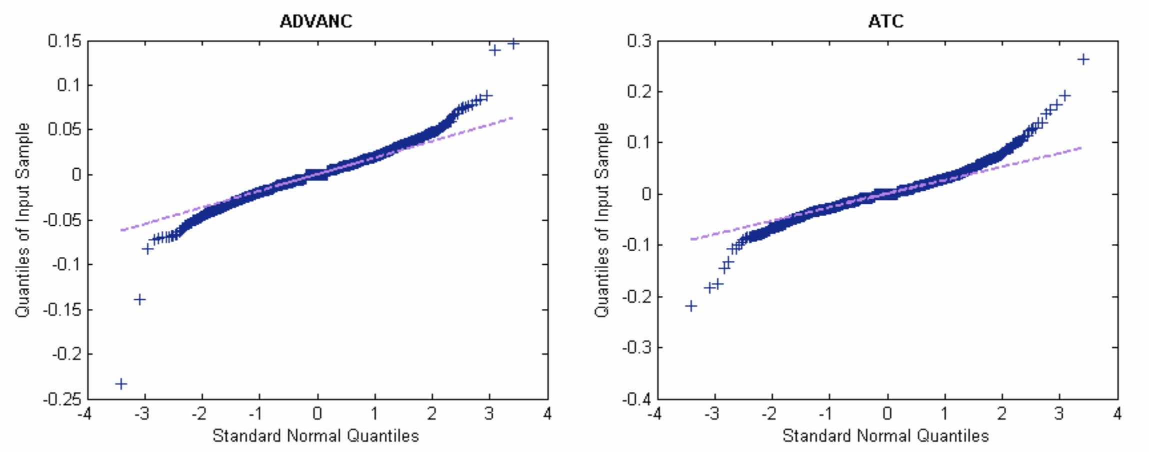
Table 1: Summary of Jarque-Bera (JB) test of normality

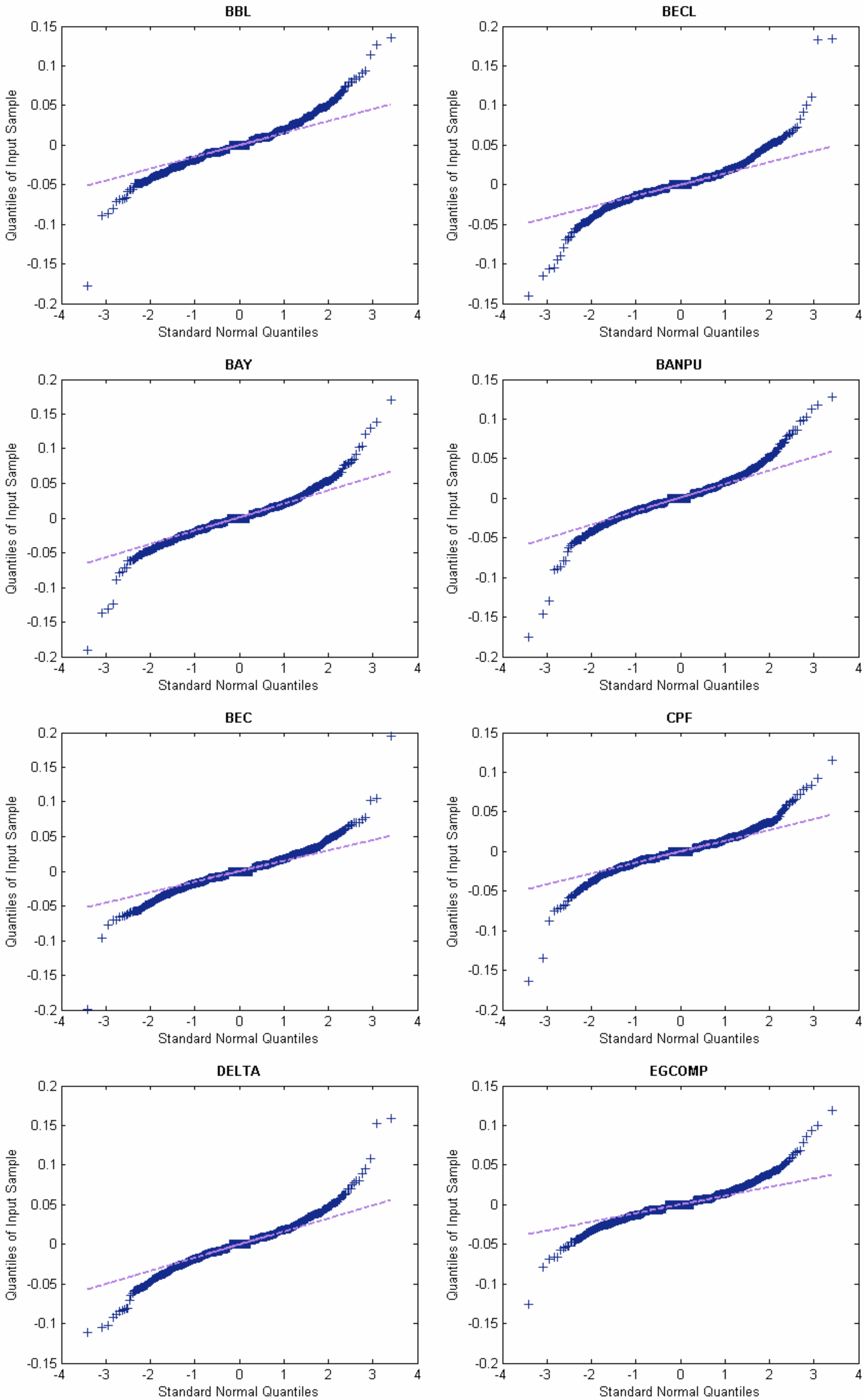
| | JB-stat | P-Value | | JB-stat | P-Value | | JB-stat | P-Value |
|---|---|---|---|---|---|---|---|---|
| ADVANC | 4,949.89 | 0.0000 | HANA | 1,075.61 | 0.0000 | SCCC | 1,785.65 | 0.0000 |
| ATC | 2,089.82 | 0.0000 | IRPC | 5,139.75 | 0.0000 | SCB | 4,480.87 | 0.0000 |
| BBL | 1,647.35 | 0.0000 | ITD | 4,666.70 | 0.0000 | THAI | 5,211.80 | 0.0000 |
| BECL | 7,142.85 | 0.0000 | KBANK | 2,100.52 | 0.0000 | TUF | 626.36 | 0.0000 |
| BAY | 2,931.40 | 0.0000 | KTB | 4,562.70 | 0.0000 | TCAP | 4,457.76 | 0.0000 |
| BANPU | 2,967.67 | 0.0000 | LH | 397.33 | 0.0000 | TISCO | 1,252.44 | 0.0000 |
| BEC | 5,956.38 | 0.0000 | PTTEP | 3,269.65 | 0.0000 | TMB | 7,591.46 | 0.0000 |
| CPF | 4,932.65 | 0.0000 | RCL | 1,607.33 | 0.0000 | TPIPL | 3,549.47 | 0.0000 |
| DELTA | 1,680.28 | 0.0000 | SSI | 2,274.88 | 0.0000 | TRUE | 3,704.55 | 0.0000 |
| EGCOMP | 2,313.93 | 0.0000 | SCC | 912.42 | 0.0000 | | | |

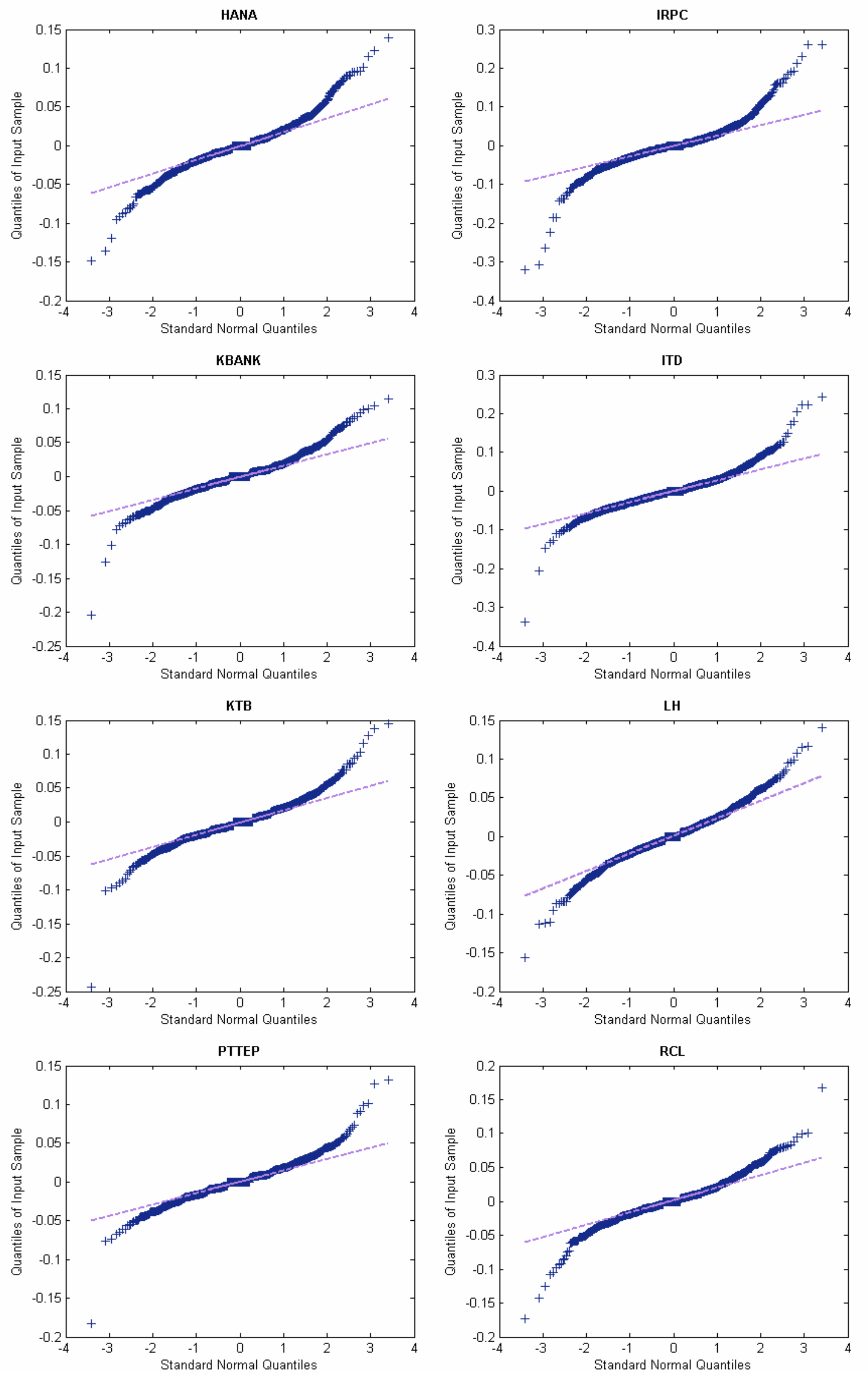
According to the Table 1, it can be seen that the values of JB statistics are sufficiently high and the probability value are sufficiently low to reject the null hypothesis of normal distribution for all data series. As a result, the paper can conclude that the historical price of each asset deviate from the normal distribution.

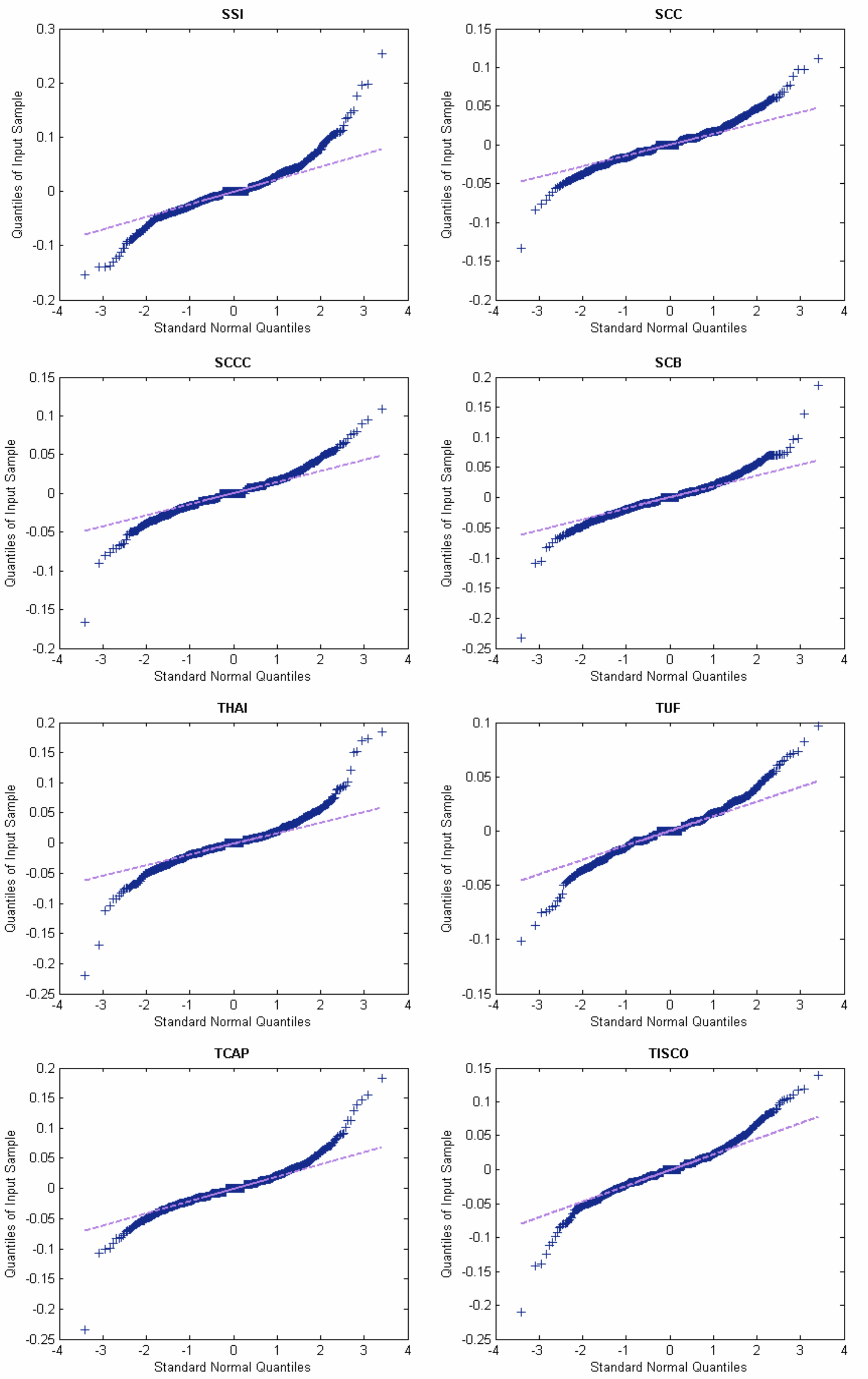
To estimate the tail distribution, Quantile-Quantile (QQ) plot is applied to test the normality of tail distribution. When the distributions of log-returns are normal, the plot will be close to linear. If the empirical data are fat-tailed, the QQ Plots of upward quantile are higher than the normal or the QQ Plots of downward quantile are lower than the normal distribution. Figure 1 presents the QQ Plot of each asset;

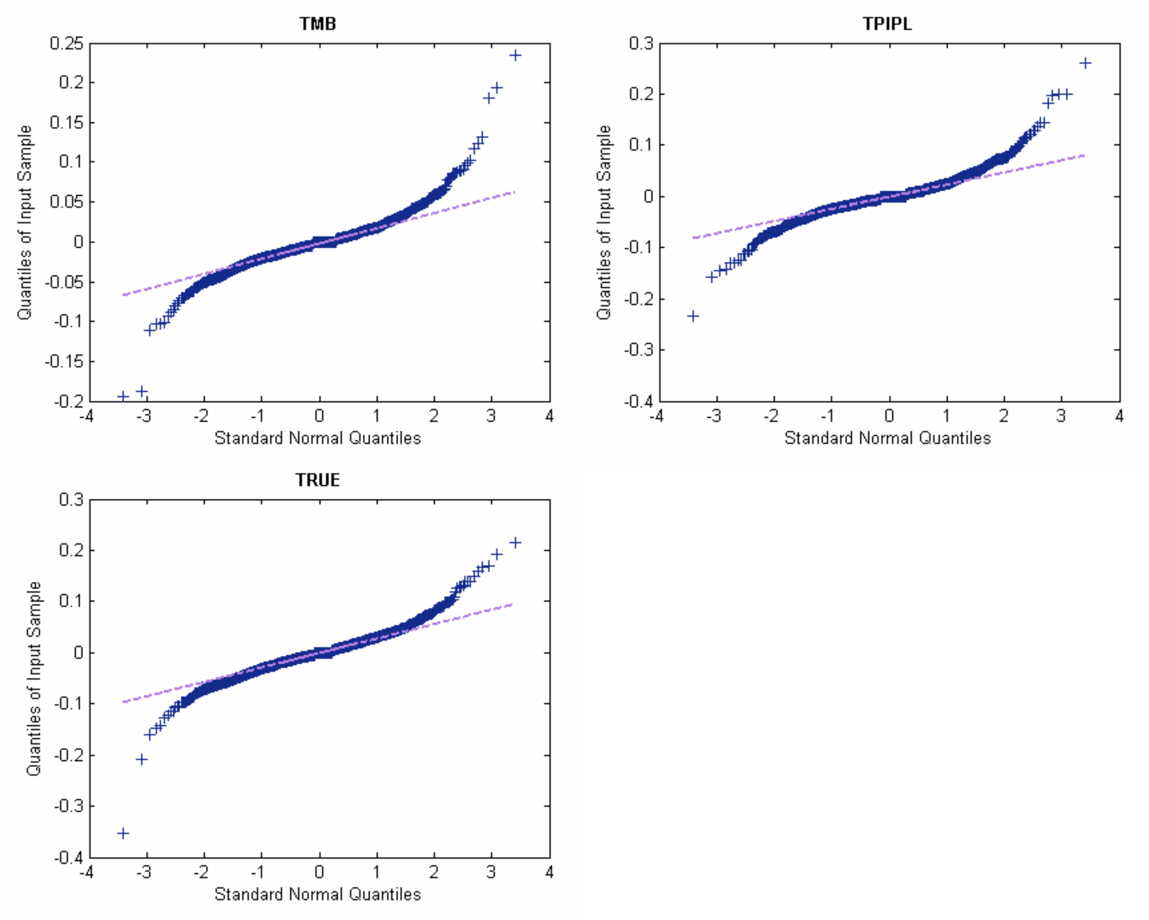
Figure 1: QQ Plot of each asset vs. Standard Normal




BBL
Quantiles of Input Sample
Standard Normal Quantiles
BECL
Quantiles of Input Sample
Standard Normal Quantiles
BAY
Quantiles of Input Sample
Standard Normal Quantiles
BANPU
Quantiles of Input Sample
Standard Normal Quantiles
BEC
Quantiles of Input Sample
Standard Normal Quantiles
CPF
Quantiles of Input Sample
Standard Normal Quantiles
DELTA
Quantiles of Input Sample
Standard Normal Quantiles
EGCOMP
Quantiles of Input Sample
Standard Normal Quantiles


HANA
IRPC
KBANK
ITD
KTB
LH
PTTEP
RCL
Quantiles of Input Sample
Standard Normal Quantiles


SSI
Quantiles of Input Sample
Standard Normal Quantiles
SCC
Quantiles of Input Sample
Standard Normal Quantiles
SCCC
Quantiles of Input Sample
Standard Normal Quantiles
SCB
Quantiles of Input Sample
Standard Normal Quantiles
THAI
Quantiles of Input Sample
Standard Normal Quantiles
TUF
Quantiles of Input Sample
Standard Normal Quantiles
TCAP
Quantiles of Input Sample
Standard Normal Quantiles
TISCO
Quantiles of Input Sample
Standard Normal Quantiles




The Figure 1 shows the deviation from the linear line, which indicates the non-normality of all data series. The outward curve in the tails indicates the increased deviation from normality. At both upper tail and lower tail, the QQ Plots show the greater density in the sample data relative to the normal. This indicates that the quantiles of return series or the cumulative probabilities are concentrated at the tails of the distribution or called "fat-tail". Therefore, it is justified to use these return series for estimating VaR by using Generalized Pareto Distribution.

## 5.2 Estimating Parameters

As described in section 4.4, in order to estimate the GPD parameters, the parameters of the marginal distribution in equation (27) are calibrated from the 1,446 standardized filtered data[5] in equation (28), which are filtered by using the volatility estimated by EWMA approach. The tail parameters $\xi$ and the scale parameters $\beta$ have been estimated using only tail data by maximum likelihood method. In Table 2 and Table 3, the estimated parameters for the tails of the marginal distributions are shown.

[5] 1,446 = 1,520 – 74, where 74 historical data are used in the EWMA estimation procedure.

Table 2: Estimated GPD parameters and threshold return for the left tail of the marginal distribution

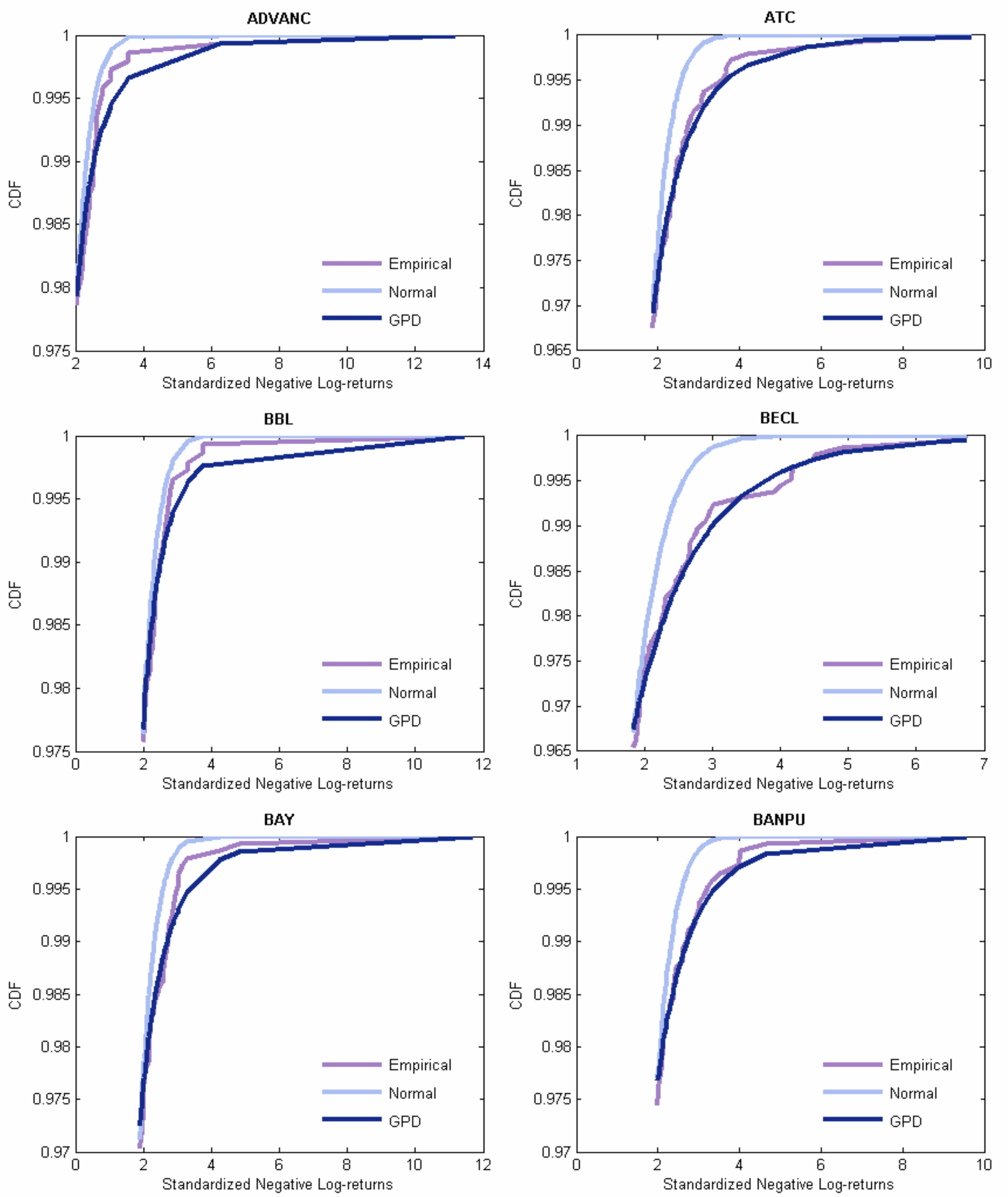
| | Threshold u | Tail ξ | Scale β | NL/N | | Threshold u | Tail ξ | Scale β | NL/N |
|---|---|---|---|---|---|---|---|---|---|
| ADVANC | -2.006 | 0.368 | 0.579 | 0.022 | LH | -1.774 | 0.016 | 0.816 | 0.038 |
| ATC | -1.830 | 0.244 | 0.759 | 0.033 | PTTEP | -2.135 | 0.478 | 0.623 | 0.015 |
| BBL | -1.948 | 0.287 | 0.538 | 0.025 | RCL | -1.891 | 0.241 | 0.774 | 0.028 |
| BECL | -1.763 | 0.099 | 0.927 | 0.035 | SSI | -1.943 | 0.396 | 0.652 | 0.024 |
| BAY | -1.827 | 0.217 | 0.692 | 0.030 | SCC | -1.886 | 0.241 | 0.592 | 0.029 |
| BANPU | -1.901 | 0.189 | 0.767 | 0.026 | SCCC | -1.807 | 0.342 | 0.572 | 0.035 |
| BEC | -1.575 | 0.298 | 0.520 | 0.057 | SCB | -2.046 | 0.388 | 0.548 | 0.019 |
| CPF | -1.894 | 0.382 | 0.703 | 0.028 | THAI | -1.761 | 0.112 | 0.870 | 0.037 |
| DELTA | -1.796 | 0.226 | 0.885 | 0.035 | TUF | -1.731 | 0.031 | 0.768 | 0.042 |
| EGCOMP | -2.142 | 0.163 | 0.899 | 0.016 | TCAP | -1.919 | 0.277 | 0.708 | 0.026 |
| HANA | -1.835 | 0.233 | 0.708 | 0.033 | TISCO | -1.975 | 0.301 | 0.760 | 0.023 |
| IRPC | -2.090 | 0.614 | 0.545 | 0.018 | TMB | -1.983 | 0.416 | 0.641 | 0.023 |
| ITD | -2.054 | 0.253 | 0.955 | 0.019 | TPIPL | -2.198 | 0.000 | 1.293 | 0.014 |
| KBANK | -1.779 | 0.283 | 0.479 | 0.037 | TRUE | -1.733 | 0.300 | 0.582 | 0.042 |
| KTB | -2.187 | 0.309 | 1.053 | 0.014 | | | | | |

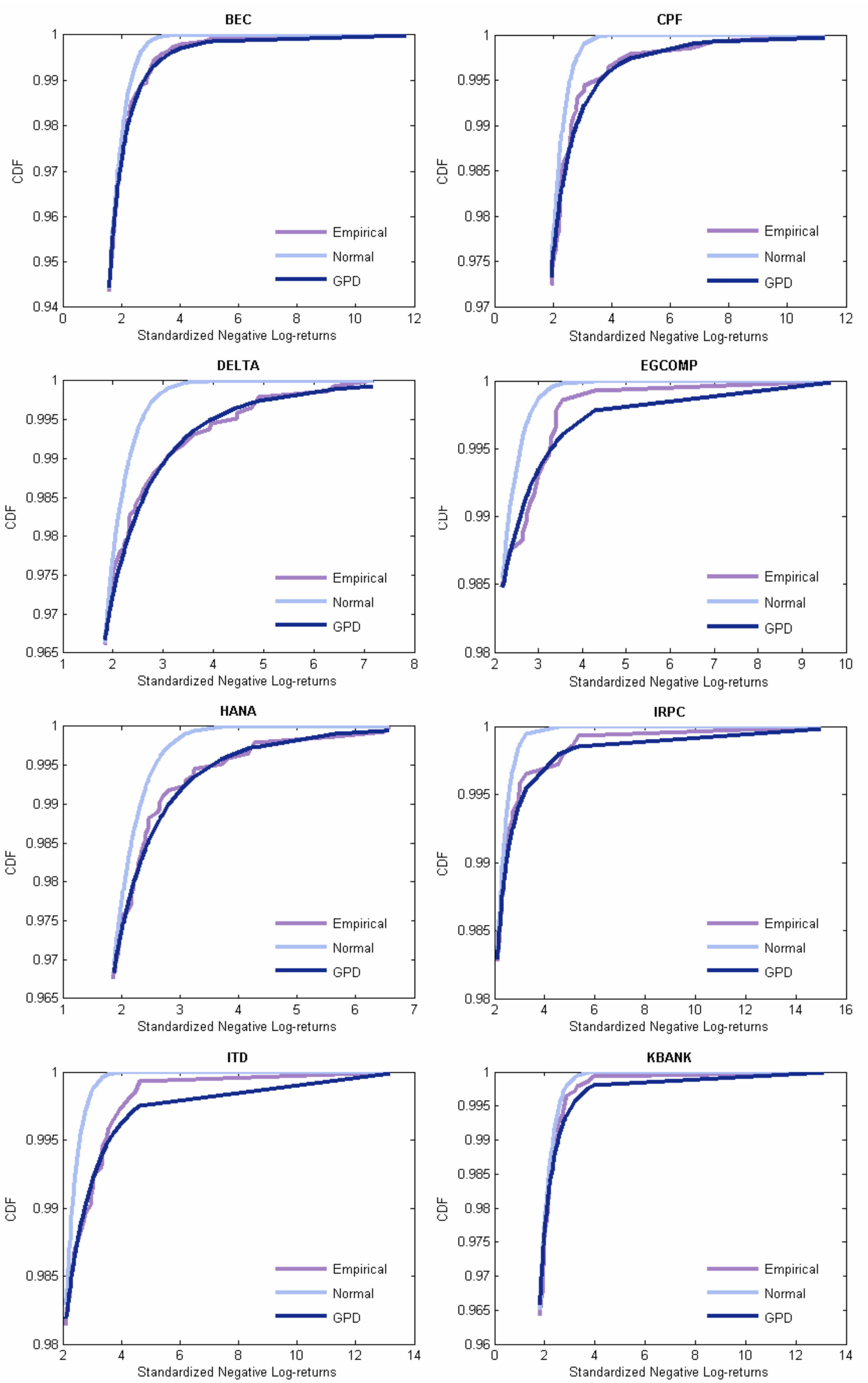
Table 3: Estimated GPD parameters and threshold return for the right tail of the marginal distribution

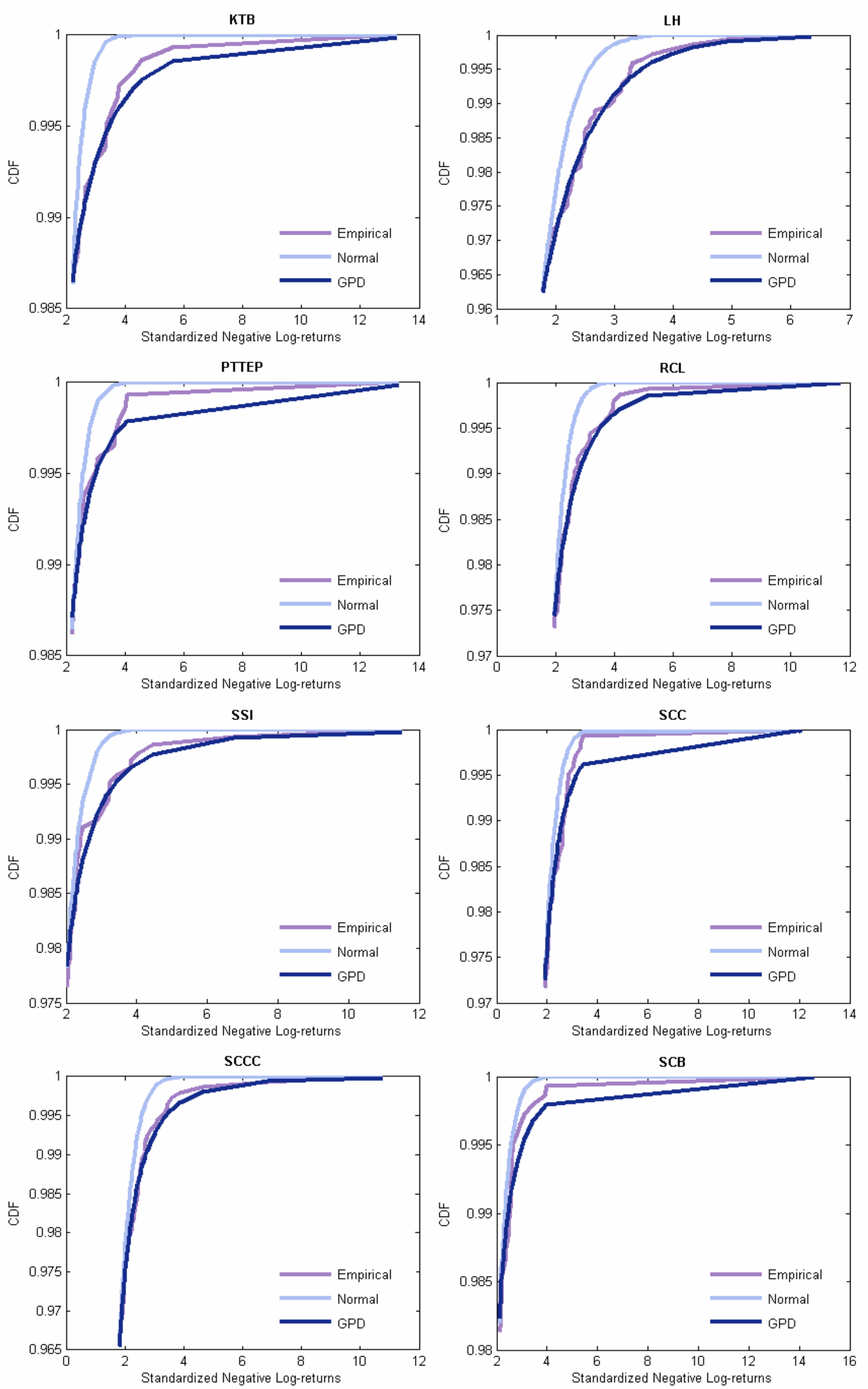
| | Threshold u | Tail ξ | Scale β | Nu/N | | Threshold u | Tail ξ | Scale β | Nu/N |
|---|---|---|---|---|---|---|---|---|---|
| ADVANC | 1.400 | -0.075 | 0.796 | 0.080 | LH | 1.307 | -0.072 | 0.692 | 0.095 |
| ATC | 1.531 | 0.073 | 0.763 | 0.063 | PTTEP | 1.262 | 0.045 | 0.735 | 0.103 |
| BBL | 1.240 | -0.064 | 0.722 | 0.107 | RCL | 1.163 | -0.015 | 0.750 | 0.120 |
| BECL | 1.375 | 0.033 | 0.698 | 0.084 | SSI | 1.609 | 0.060 | 0.945 | 0.053 |
| BAY | 1.209 | -0.134 | 0.797 | 0.113 | SCC | 1.099 | 0.017 | 0.712 | 0.132 |
| BANPU | 1.201 | 0.091 | 0.670 | 0.114 | SCCC | 1.402 | -0.064 | 0.809 | 0.080 |
| BEC | 1.558 | -0.208 | 0.999 | 0.060 | SCB | 1.094 | -0.022 | 0.706 | 0.136 |
| CPF | 1.570 | 0.059 | 0.903 | 0.056 | THAI | 1.633 | 0.417 | 0.583 | 0.051 |
| DELTA | 1.717 | 0.109 | 0.770 | 0.041 | TUF | 1.193 | -0.012 | 0.690 | 0.115 |
| EGCOMP | 1.384 | -0.177 | 1.041 | 0.083 | TCAP | 1.448 | -0.066 | 0.832 | 0.073 |
| HANA | 1.638 | -0.124 | 1.099 | 0.051 | TISCO | 1.474 | -0.216 | 1.011 | 0.069 |
| IRPC | 1.448 | 0.123 | 0.870 | 0.073 | TMB | 1.520 | 0.219 | 0.713 | 0.062 |
| ITD | 1.380 | 0.118 | 0.728 | 0.083 | TPIPL | 1.421 | 0.131 | 0.843 | 0.075 |
| KBANK | 1.361 | -0.072 | 0.859 | 0.085 | TRUE | 1.666 | 0.089 | 0.885 | 0.046 |
| KTB | 1.364 | 0.144 | 0.668 | 0.086 | | | | | |

In Figure 2, the tail distributions of the standardized filtered negative log-returns of 29 equities are plotted. These graphs show how the GPD fits the empirical distribution better than the standardized normal distribution.

Figure 2: The tail distributions of the standardized filtered negative log-returns of 29 equities




BEC
CPF
DELTA
EGCOMP
HANA
IRPC
ITD
KBANK
CDF
Standardized Negative Log-returns
Empirical
Normal
GPD


KTB
CDF
Standardized Negative Log-returns
Empirical
Normal
GPD
1
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CDF
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Empirical
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GPD
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PTTEP
CDF
Standardized Negative Log-returns
Empirical
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Standardized Negative Log-returns
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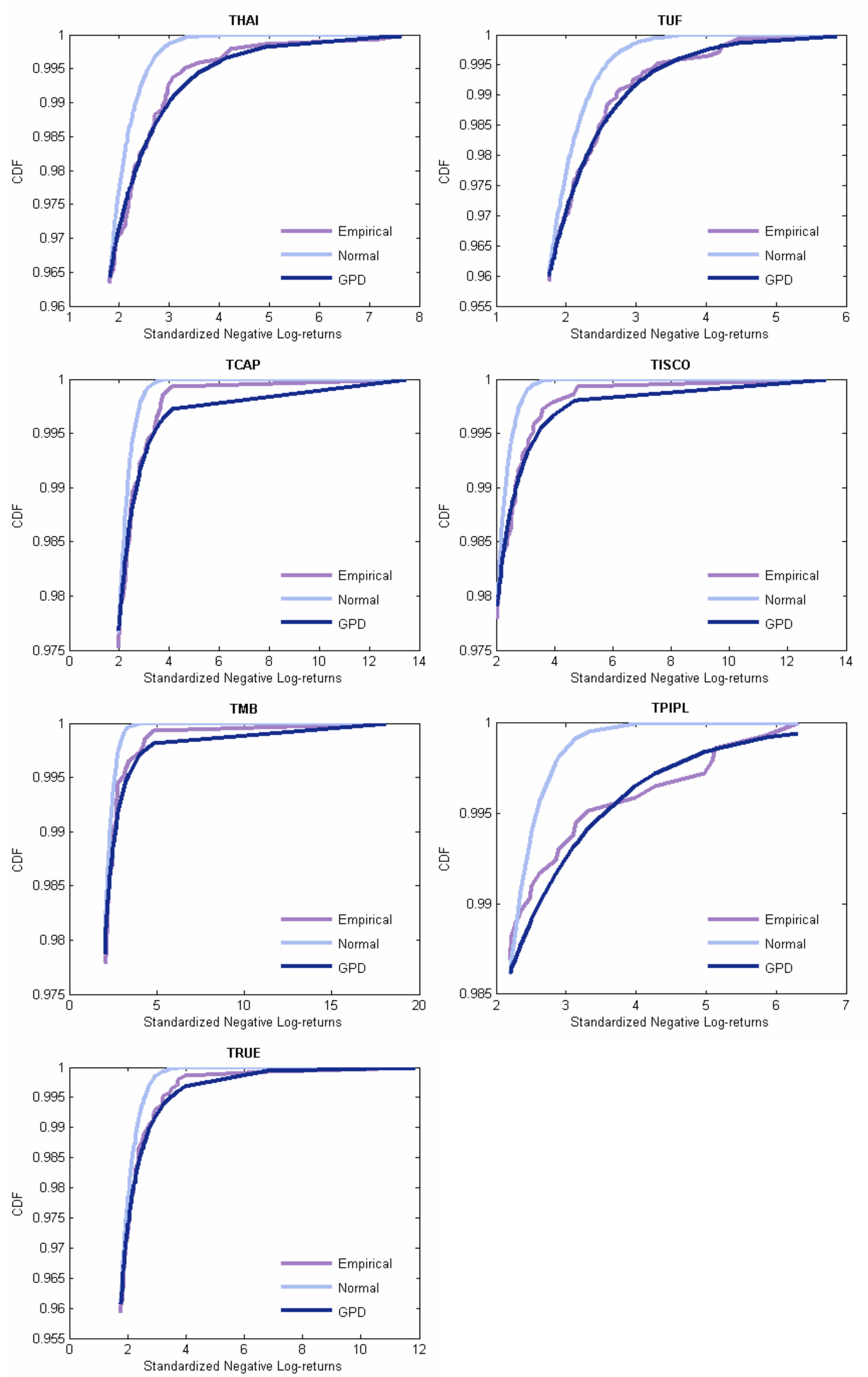
16


THAI
TUF
TCAP
TISCO
TMB
TPIPL
TRUE
CDF
Standardized Negative Log-returns
Empirical
Normal
GPD

### 5.3 VaR calculation

To estimate 99% VaR over a one-day horizon, this paper assumes the portfolio, which contains one position for each of 29 equities. The portfolio value as of time t is:

$$P_t = \sum_{i=1}^{29} P_{i,t} \; = 1{,}515.25 \text{ Baht}$$

where $P_{i,t}$ is the market price of stock i at time t .

Assume standing on March 15, 2007, I have simulated 10,000 Monte-Carlo scenarios for each asset log-returns over one-day time horizon based on the following distribution;

1) multivariate normal distribution
2) multivariate distribution with Gaussian Copula and EVT marginal distribution

Then compute the portfolio value at time t+1 and express the 10,000 portfolio losses scenarios. Finally, I order the 10,000 values of portfolio losses from lowest to highest (the possible profits are considered as negative losses). The 99% VaR is the 9,900$^{th}$ ordered scenario. The portfolio 99% VaR estimated from two different log-return distributions is shown in Table 4.

Table 4: Portfolio 99% VaR estimated assuming two different log-return distributions in value and percentage of maximum loss

| Confidence level | Copula-EVT | | Multivariate Normal | |
|---|---|---|---|---|
| | Baht | % | Baht | % |
| 99% | -31.3868 | -2.09 | -33.701 | -2.25 |

VaR estimate for March 16, 2007

Figure 3 : Portfolio profit and loss distribution based on GPD and normal distribution

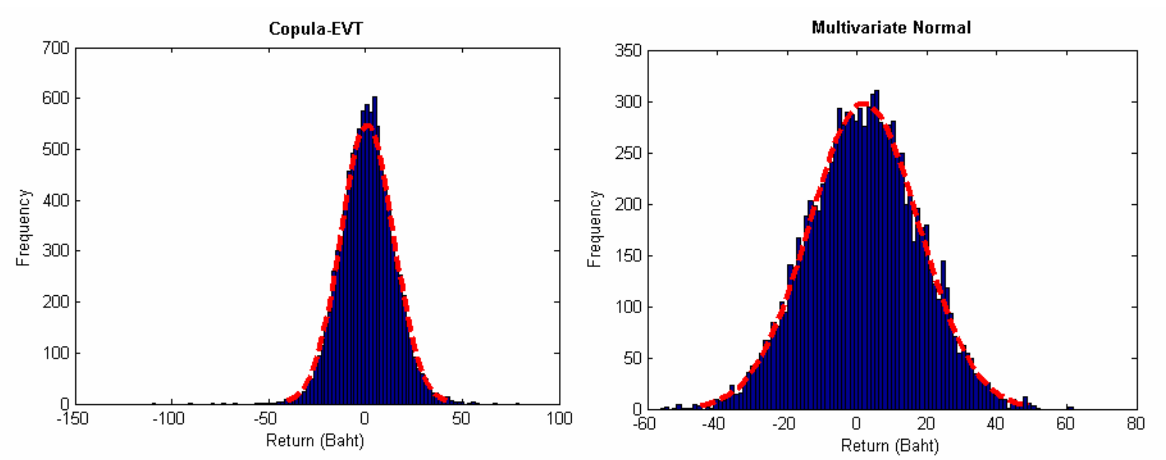


Table 4 summarizes the VaR estimates from Copula-EVT and normal distribution approaches. The Copula-EVT estimation has produced the lower VaR forecast comparing to the normal distribution model. This result differs from other literatures which indicate that the Copula-EVT usually provides the higher absolute VaR estimates than the normality approach. However, this result can be explained by backtesting process.

### 5.4 Evaluating VaR performance

Table 4 shows that the Copula-EVT model has computed the lower VaR than the normal distribution model. However, the study performs backtesting over 735-days time window and the result shows that the Copula-EVT VaR (in absolute value) is usually higher than the multivariate normal VaR.

Figure 4: 99% VaR estimation and effective portfolio return over the time window of 735 days

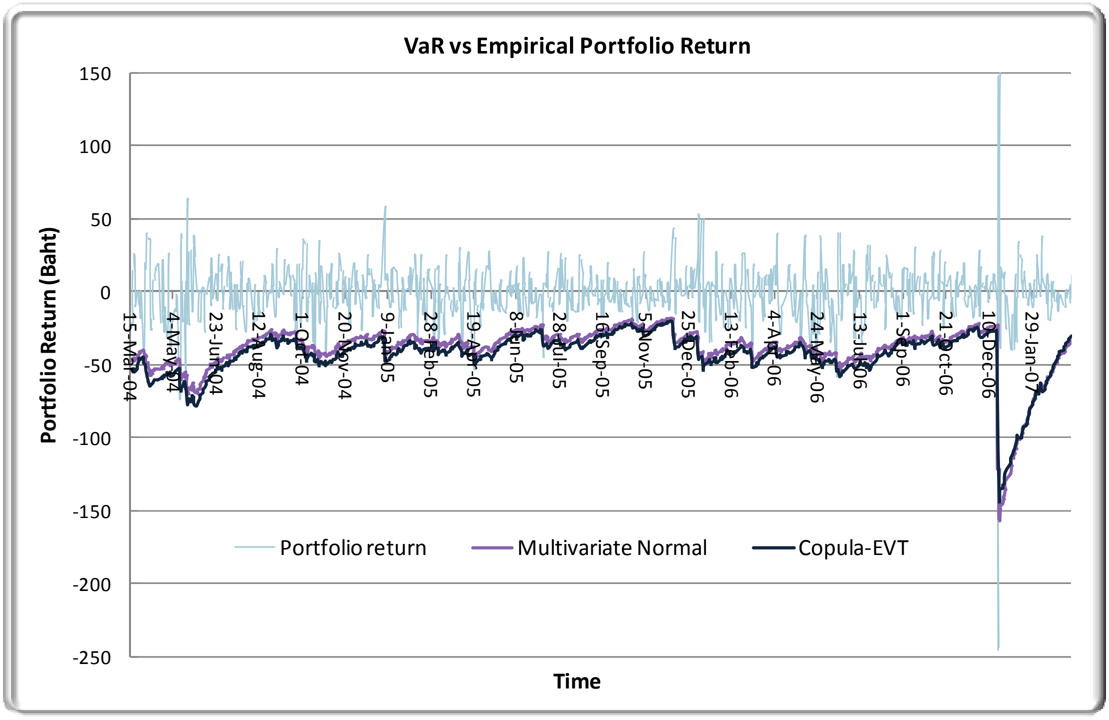


Table 5: Evaluating VaR performance

| Methods | Copula-EVT | Multivariate Normal |
|---|---|---|
| Binary loss function (Backtesting) | | |
| - No. of exceptions | 11 | 14 |
| - % of exceptions* | 1.497% | 1.905% |
| Regulatory loss function (RLF) | 64.878 | 68.253 |
| Firm's loss function (FLF)** | 66.366 | 69.598 |
| Kupiec's LR | 1.589*** | 4.803 |

Note: * For 99% confidence level, the good estimation should provide the percentage of exception close to 1%.

** To estimate FLF, assume $\alpha = 3.49\%$ per annum, which is an average of 1-year government bond yield over the time window of 735 days.

***Accept null hypothesis that $p = 0.01$ at 95% confidence level ($LR < 3.8415$).

Table 5 summarizes the value of three loss functions and Kupiec's LR that estimated for 99% VaR. The backtesting result shows that the percentage of loss over the VaR based on Copula-EVT models are closer to the expected confidence level (1%) than the multivariate normal model. The Copula-EVT method provides the lower value for both RLF and FLF than the multivariate normal method. These lower values indicate that Copula-EVT produces lower economic losses, both for a regulator and for a risk manager. Moreover, the Copula-EVT provides a sufficient low value of Kupiec's LR to accept the null hypothesis that p = 0.01. As a result, the Copula-EVT model is acceptable to estimate 99% while the multivariate normal method underestimates the probability of large losses.

The interesting point is that after the "Black Tuesday", December 19, 2006, the normal approach provides the higher VaR estimates than the Copula-EVT approach. However, due to the insufficient no. of observations, the study cannot conclude whether normal distribution provides overestimated VaR, or the Copula-EVT provides underestimated VaR after the extreme situation.

### 5.5 Hedging Effectiveness

The study provides the testing of hedging effectiveness comparing between Minimum Variance Hedge Ratio (MVHR) and Minimum VaR Hedge Ratio (minVaR) by applying the Copula-EVT VaR estimation.

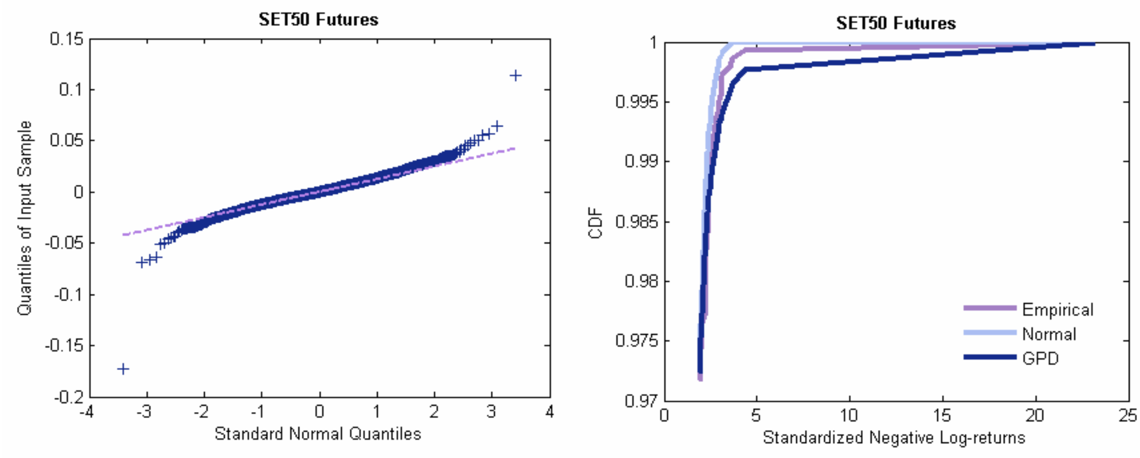
For SET50 index futures, since the SET50 index futures have been traded in the market for only 11 months, then the no. of observations are not sufficient to apply the EVT approach. Therefore, this paper uses the theoretical price of futures calculated by

Cost of Carry Model as the proxy of futures prices. The Table 6 shows the GPD parameters of tail distribution of SET50 index futures.

Table 6: Estimated GPD parameters and threshold returns for the tails of the marginal distribution

| Left-tail Parameters | | | | Right-tail Parameters | | | |
|---|---|---|---|---|---|---|---|
| **Threshold u** | **Tail ξ** | **Scale β** | **$N_L$/N** | **Threshold u** | **Tail ξ** | **Scale β** | **$N_U$/N** |
| -2.0060 | 0.3677 | 0.5790 | 0.0221 | -1.7737 | 0.0157 | 0.8163 | 0.0380 |

Figure 5: QQ Plots of SET50 index futures vs. Standard Normal (left figure) and the tail distribution of negative log-returns of SET50 index futures (right figure)




Applying the numerical technique described in section 4.6, I simulated 50 portfolios of 29 stocks by randomize the weight of each assets in the portfolio (assume the total investment of Baht 1 million) to test the hedging effectiveness of two approaches.

Figure 6: The effect of hedging with Minimum-Variance and Minimum-VaR hedging

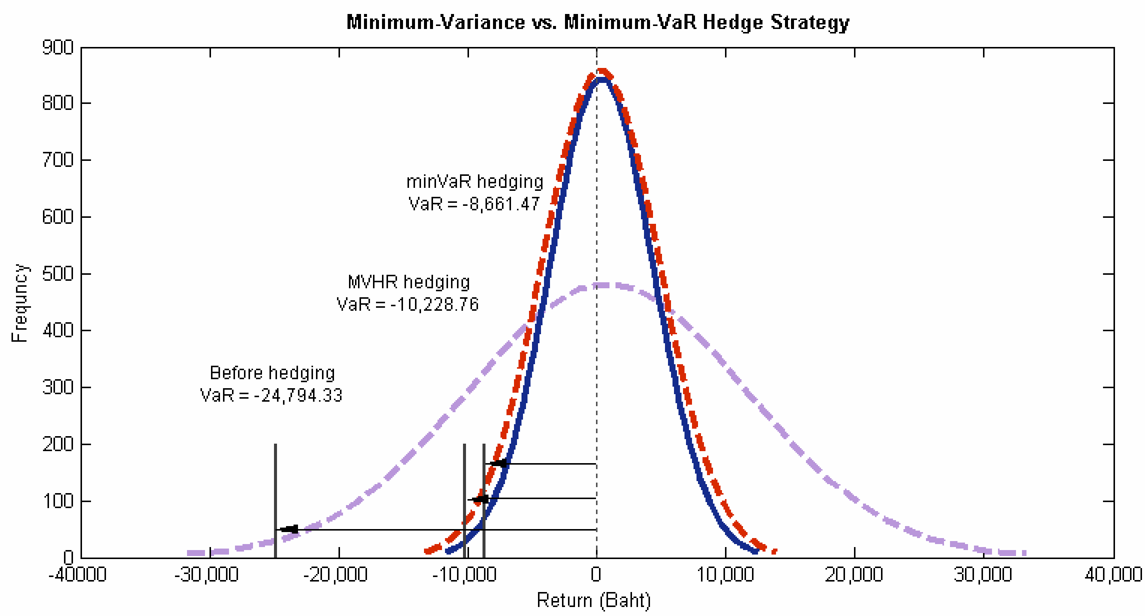


Figure 6 shows the effect of hedging with minimize VaR of portfolio 11th, which has the highest reduction in VaR. Without hedging, the 99% portfolio VaR is Baht 24,794.31. If portfolio risk manager decides to minimize VaR by taking short position in SET50 index futures, after hedging, the VaR is reduced by 65.07% to Baht 8,661.47. As shown in the figure, both minimum-VaR and minimum-variance hedging narrow the distribution of portfolio returns and hedging does not change the mean of the distribution. However, the figure also shows that assessing the amount of the futures hedging strategy in terms of standard deviation is misleading, since the standard deviation does not fully describe the risk in the case of asymmetric return distributions and its kurtosis and skewness deviate from normal distribution. In short, when the returns are not normally distributed, to minimize variance will lead to a suboptimal risk of hedging portfolio.

Table 7: Hedging effectiveness of Minimum-VaR hedge ratio vs. Minimum-Variance hedge ratio

| Portfolio | Hedge ratio | | Standard deviation | | | VaR | | | Reduction in SD | | Reduction in VaR | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | minVaR | MVHR | Unhedged | minVaR | MVHR | Unhedged | minVaR | MVHR | minVaR | MVHR | minVaR | MVHR |
| 1 | 0.9860 | 1.0929 | 1.75% | 0.55% | 0.53% | 30,170.62 | 11,796.67 | 12,632.84 | 68.57% | 69.71% | 60.90% | 58.13% |
| 2 | 0.9826 | 1.0888 | 1.77% | 0.62% | 0.60% | 30,278.18 | 12,054.76 | 12,495.63 | 64.97% | 66.10% | 60.19% | 58.73% |
| 3 | 0.9285 | 1.0463 | 1.71% | 0.61% | 0.59% | 29,500.70 | 12,936.59 | 13,333.52 | 64.33% | 65.50% | 56.15% | 54.80% |
| 4 | 0.9050 | 1.0516 | 1.71% | 0.62% | 0.58% | 28,774.87 | 11,766.15 | 12,164.71 | 63.74% | 66.08% | 59.11% | 57.72% |
| 5 | 0.8463 | 0.9824 | 1.58% | 0.52% | 0.48% | 26,558.08 | 10,558.52 | 11,693.65 | 67.09% | 69.62% | 60.24% | 55.97% |
| 6 | 0.9038 | 1.0558 | 1.73% | 0.66% | 0.61% | 29,898.35 | 13,616.88 | 13,881.87 | 61.85% | 64.74% | 54.46% | 53.57% |
| 7 | 0.7858 | 0.9477 | 1.54% | 0.58% | 0.53% | 25,237.71 | 10,609.04 | 11,792.93 | 62.34% | 65.58% | 57.96% | 53.27% |
| 8 | 0.8539 | 0.9567 | 1.55% | 0.53% | 0.50% | 24,281.50 | 9,415.76 | 10,793.30 | 65.81% | 67.74% | 61.22% | 55.55% |
| 9 | 0.8488 | 0.9666 | 1.56% | 0.54% | 0.51% | 26,477.05 | 12,215.47 | 13,065.03 | 65.38% | 67.31% | 53.86% | 50.66% |
| 10 | 0.8269 | 0.9458 | 1.54% | 0.55% | 0.52% | 25,510.66 | 11,346.10 | 12,436.86 | 64.29% | 66.23% | 55.52% | 51.25% |
| 11 | 0.8410 | 0.9760 | 1.56% | 0.49% | 0.45% | 24,794.31 | 8,661.47 | 10,228.76 | 68.59% | 71.15% | 65.07% | 58.75% |
| 12 | 0.8786 | 1.0339 | 1.69% | 0.63% | 0.58% | 28,170.50 | 12,078.45 | 12,810.49 | 62.72% | 65.68% | 57.12% | 54.53% |
| 13 | 0.8650 | 1.0102 | 1.63% | 0.55% | 0.51% | 26,474.96 | 10,033.79 | 11,115.66 | 66.26% | 68.71% | 62.10% | 58.01% |
| 14 | 0.7772 | 0.9968 | 1.62% | 0.65% | 0.56% | 24,381.08 | 10,521.72 | 12,613.73 | 59.88% | 65.43% | 56.84% | 48.26% |
| 15 | 0.8315 | 0.9646 | 1.55% | 0.51% | 0.47% | 25,259.16 | 10,427.17 | 11,787.23 | 67.10% | 69.68% | 58.72% | 53.33% |
| 16 | 0.8830 | 1.0261 | 1.66% | 0.59% | 0.54% | 27,768.26 | 12,278.77 | 13,102.74 | 64.46% | 67.47% | 55.78% | 52.81% |
| 17 | 0.8907 | 1.0129 | 1.63% | 0.55% | 0.51% | 27,293.04 | 10,832.50 | 11,397.56 | 66.26% | 68.71% | 60.31% | 58.24% |
| 18 | 0.8528 | 1.0207 | 1.65% | 0.58% | 0.52% | 27,971.40 | 11,956.00 | 12,827.34 | 64.85% | 68.48% | 57.26% | 54.14% |
| 19 | 0.8576 | 0.9998 | 1.61% | 0.55% | 0.50% | 26,866.94 | 10,172.38 | 10,979.07 | 65.84% | 68.94% | 62.14% | 59.14% |
| 20 | 0.8572 | 0.9823 | 1.59% | 0.55% | 0.51% | 28,457.06 | 12,616.32 | 12,990.08 | 65.41% | 67.92% | 55.67% | 54.35% |
| 21 | 0.8148 | 0.9444 | 1.55% | 0.60% | 0.57% | 24,928.15 | 10,286.89 | 11,258.60 | 61.29% | 63.23% | 58.73% | 54.84% |
| 22 | 0.8298 | 0.9607 | 1.53% | 0.46% | 0.41% | 24,459.63 | 8,966.39 | 10,482.98 | 69.93% | 73.20% | 63.34% | 57.14% |
| 23 | 0.8602 | 0.9590 | 1.55% | 0.53% | 0.51% | 25,868.53 | 10,939.11 | 11,604.67 | 65.81% | 67.10% | 57.71% | 55.14% |
| 24 | 0.8244 | 0.9458 | 1.52% | 0.49% | 0.46% | 26,735.98 | 11,138.70 | 11,903.02 | 67.76% | 69.74% | 58.34% | 55.48% |
| 25 | 0.8913 | 1.0090 | 1.62% | 0.54% | 0.50% | 27,826.43 | 11,410.39 | 11,885.78 | 66.67% | 69.14% | 58.99% | 57.29% |
| 26 | 0.8559 | 1.0130 | 1.64% | 0.58% | 0.53% | 25,507.45 | 9,819.12 | 11,450.00 | 64.63% | 67.68% | 61.50% | 55.11% |
| 27 | 0.8780 | 1.0081 | 1.63% | 0.56% | 0.52% | 27,689.47 | 12,264.85 | 13,175.91 | 65.64% | 68.10% | 55.71% | 52.42% |
| 28 | 0.7643 | 0.9660 | 1.57% | 0.62% | 0.54% | 23,433.44 | 8,906.03 | 10,857.03 | 60.51% | 65.61% | 61.99% | 53.67% |
| 29 | 0.8169 | 0.9392 | 1.51% | 0.48% | 0.45% | 24,502.67 | 8,587.58 | 9,707.96 | 68.21% | 70.20% | 64.95% | 60.38% |
| 30 | 0.8060 | 0.9930 | 1.62% | 0.63% | 0.56% | 27,358.98 | 11,700.10 | 12,813.70 | 61.11% | 65.43% | 57.23% | 53.16% |

| Portfolio | Hedge ratio | | Standard deviation | | | VaR | | | Reduction in SD | | Reduction in VaR | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | minVaR | MVHR | Unhedged | minVaR | MVHR | Unhedged | minVaR | MVHR | minVaR | MVHR | minVaR | MVHR |
| 31 | 0.8327 | 0.9876 | 1.58% | 0.51% | 0.45% | 26,444.99 | 9,863.22 | 10,799.84 | 67.72% | 71.52% | 62.70% | 59.16% |
| 32 | 0.8835 | 1.0378 | 1.67% | 0.58% | 0.53% | 28,625.43 | 11,492.08 | 12,210.87 | 65.27% | 68.26% | 59.85% | 57.34% |
| 33 | 0.8568 | 0.9973 | 1.59% | 0.51% | 0.46% | 28,010.53 | 10,875.57 | 11,615.72 | 67.92% | 71.07% | 61.17% | 58.53% |
| 34 | 0.9150 | 1.0124 | 1.64% | 0.56% | 0.54% | 28,210.91 | 11,957.71 | 12,576.64 | 65.85% | 67.07% | 57.61% | 55.42% |
| 35 | 0.8752 | 0.9742 | 1.57% | 0.53% | 0.51% | 28,243.95 | 10,527.89 | 10,759.23 | 66.24% | 67.52% | 62.73% | 61.91% |
| 36 | 0.8913 | 1.0367 | 1.68% | 0.58% | 0.54% | 27,159.37 | 10,530.13 | 11,496.98 | 65.48% | 67.86% | 61.23% | 57.67% |
| 37 | 0.9489 | 1.0128 | 1.62% | 0.48% | 0.47% | 28,049.22 | 9,929.71 | 10,518.35 | 70.37% | 70.99% | 64.60% | 62.50% |
| 38 | 0.8988 | 1.0054 | 1.62% | 0.53% | 0.51% | 27,211.13 | 9,559.12 | 10,450.00 | 67.28% | 68.52% | 64.87% | 61.60% |
| 39 | 0.8544 | 1.0395 | 1.68% | 0.60% | 0.53% | 27,811.49 | 11,287.88 | 12,271.85 | 64.29% | 68.45% | 59.41% | 55.87% |
| 40 | 0.8698 | 0.9825 | 1.58% | 0.51% | 0.48% | 26,811.24 | 10,772.32 | 11,906.71 | 67.72% | 69.62% | 59.82% | 55.59% |
| 41 | 0.8431 | 1.0082 | 1.65% | 0.63% | 0.58% | 26,645.67 | 11,618.71 | 12,562.69 | 61.82% | 64.85% | 56.40% | 52.85% |
| 42 | 0.7928 | 0.9642 | 1.57% | 0.59% | 0.53% | 25,424.18 | 10,445.98 | 11,419.57 | 62.42% | 66.24% | 58.91% | 55.08% |
| 43 | 0.7983 | 0.9664 | 1.57% | 0.57% | 0.51% | 25,383.57 | 10,814.90 | 12,053.40 | 63.69% | 67.52% | 57.39% | 52.51% |
| 44 | 0.8138 | 0.9562 | 1.54% | 0.53% | 0.48% | 24,508.34 | 9,485.47 | 10,712.68 | 65.58% | 68.83% | 61.30% | 56.29% |
| 45 | 0.8024 | 0.9563 | 1.55% | 0.57% | 0.52% | 24,907.87 | 10,656.25 | 11,948.92 | 63.23% | 66.45% | 57.22% | 52.03% |
| 46 | 0.8979 | 0.9579 | 1.56% | 0.53% | 0.52% | 27,432.44 | 11,985.69 | 12,071.16 | 66.03% | 66.67% | 56.31% | 56.00% |
| 47 | 0.9743 | 1.0259 | 1.66% | 0.54% | 0.53% | 28,814.89 | 10,976.25 | 11,401.47 | 67.47% | 68.07% | 61.91% | 60.43% |
| 48 | 0.9411 | 1.0403 | 1.67% | 0.52% | 0.50% | 30,311.27 | 12,587.57 | 13,094.41 | 68.86% | 70.06% | 58.47% | 56.80% |
| 49 | 0.8223 | 0.9504 | 1.54% | 0.54% | 0.51% | 26,317.90 | 9,845.78 | 10,676.02 | 64.94% | 66.88% | 62.59% | 59.43% |
| 50 | 0.8545 | 0.9834 | 1.57% | 0.50% | 0.46% | 26,821.70 | 10,492.13 | 11,208.33 | 68.15% | 70.70% | 60.88% | 58.21% |
| **Average** | **0.8622** | **0.9958** | **1.61%** | **0.56%** | **0.52%** | **26,911.63** | **10,912.36** | **11,820.75** | **65.43%** | **67.95%** | **59.49%** | **56.02%** |

Table 7 shows that the minVaR hedge ratios are considerably smaller than MVHR, suggesting that the smaller short positions in futures are required to minimize VaR than to minimize variance of portfolio. Although the minVaR portfolios have the standard deviation that is a bit higher than the MVHR portfolios, the minVaR hedging provides the larger reduction in VaR compared with MVHR hedging. However, since the returns of assets do not follow the normal distribution, the standard deviation is theoretically not sufficient to explain the risks of the assets.

Table 8: Actual return of hedging portfolios by Minimum-VaR hedge strategy and Minimum-Variance hedge strategy

| No. | 1-day | | 15-day | | 30-day | | 45-day | | 60-day | |
|---|---|---|---|---|---|---|---|---|---|---|
| | MinVaR | MVHR | MinVaR | MVHR | MinVaR | MVHR | MinVaR | MVHR | MinVaR | MVHR |
| 1 | 0.79% | 0.87% | 0.23% | 0.25% | 0.90% | 0.54% | 0.32% | (0.10%) | 1.74% | 1.08% |
| 2 | 0.44% | 0.51% | 0.34% | 0.35% | 1.30% | 0.95% | 0.81% | 0.40% | 0.79% | 0.13% |
| 3 | 0.65% | 0.73% | 0.15% | 0.16% | 0.49% | 0.10% | (0.39%) | (0.85%) | 1.61% | 0.89% |
| 4 | 0.41% | 0.51% | 0.89% | 0.91% | 1.87% | 1.39% | 1.19% | 0.62% | 1.64% | 0.74% |
| 5 | 0.27% | 0.36% | 1.18% | 1.19% | 1.46% | 1.01% | 0.99% | 0.46% | 1.52% | 0.68% |
| 6 | 0.54% | 0.64% | 0.28% | 0.30% | 0.82% | 0.32% | 0.29% | (0.31%) | 0.96% | 0.02% |
| 7 | 0.65% | 0.76% | 1.13% | 1.15% | 1.44% | 0.90% | 1.20% | 0.57% | 2.38% | 1.38% |
| 8 | 0.27% | 0.34% | 0.46% | 0.48% | 0.87% | 0.53% | 0.45% | 0.04% | 0.42% | (0.21%) |
| 9 | 0.36% | 0.44% | 0.84% | 0.86% | 1.18% | 0.79% | (0.60%) | (1.06%) | (0.91%) | (1.64%) |
| 10 | 0.09% | 0.17% | 0.85% | 0.87% | 1.04% | 0.65% | (0.07%) | (0.54%) | (1.00%) | (1.73%) |
| 11 | 0.28% | 0.38% | 0.59% | 0.61% | 1.23% | 0.78% | 0.26% | (0.26%) | (0.08%) | (0.91%) |
| 12 | 0.42% | 0.53% | 1.28% | 1.30% | 1.82% | 1.31% | 0.45% | (0.15%) | 0.49% | (0.47%) |
| 13 | 0.06% | 0.16% | (0.20%) | (0.18%) | 0.78% | 0.30% | 0.63% | 0.07% | 1.26% | 0.36% |
| 14 | 0.32% | 0.47% | 1.00% | 1.03% | 1.68% | 0.95% | 1.39% | 0.53% | 1.05% | (0.30%) |
| 15 | 0.10% | 0.19% | 1.13% | 1.15% | 1.95% | 1.51% | 1.40% | 0.88% | 0.90% | 0.08% |
| 16 | 0.57% | 0.66% | 0.19% | 0.21% | 1.18% | 0.71% | 0.43% | (0.13%) | 1.00% | 0.12% |
| 17 | 0.25% | 0.34% | 0.16% | 0.18% | 0.92% | 0.51% | 1.12% | 0.65% | 2.45% | 1.70% |
| 18 | 0.54% | 0.65% | 0.54% | 0.56% | 1.39% | 0.84% | (0.04%) | (0.70%) | 0.66% | (0.38%) |
| 19 | 0.27% | 0.37% | 0.53% | 0.55% | 1.36% | 0.89% | 0.96% | 0.40% | 1.91% | 1.03% |
| 20 | 0.12% | 0.21% | 1.01% | 1.02% | 1.18% | 0.77% | 0.40% | (0.09%) | 0.45% | (0.32%) |
| 21 | 0.63% | 0.72% | (0.06%) | (0.04%) | 0.86% | 0.43% | 0.69% | 0.18% | 2.36% | 1.56% |
| 22 | 0.38% | 0.47% | 0.92% | 0.93% | 1.41% | 0.98% | 0.50% | (0.01%) | (0.66%) | (1.46%) |
| 23 | 0.32% | 0.38% | (0.05%) | (0.04%) | 0.68% | 0.35% | 0.40% | 0.02% | 1.16% | 0.56% |
| 24 | 0.23% | 0.31% | 0.98% | 0.99% | 1.11% | 0.70% | (0.25%) | (0.72%) | (0.55%) | (1.30%) |
| 25 | 0.23% | 0.31% | 1.40% | 1.42% | 2.21% | 1.82% | 1.29% | 0.84% | 1.73% | 1.00% |
| 26 | 0.22% | 0.33% | (0.37%) | (0.35%) | 0.33% | (0.19%) | 0.40% | (0.21%) | 0.67% | (0.30%) |
| 27 | 0.28% | 0.37% | 0.10% | 0.12% | 1.18% | 0.74% | (0.01%) | (0.52%) | 0.33% | (0.47%) |
| 28 | 0.25% | 0.39% | (0.31%) | (0.28%) | 1.09% | 0.43% | 0.98% | 0.19% | 1.58% | 0.34% |
| 29 | 0.00% | 0.08% | 0.45% | 0.47% | 1.01% | 0.60% | 0.42% | (0.05%) | 0.43% | (0.32%) |
| 30 | (0.03%) | 0.10% | 1.17% | 1.19% | 2.22% | 1.60% | 1.15% | 0.42% | 0.17% | (0.98%) |
| 31 | 0.49% | 0.59% | 0.25% | 0.27% | 0.44% | (0.08%) | 0.33% | (0.28%) | 2.06% | 1.10% |
| 32 | 0.40% | 0.51% | 0.76% | 0.78% | 1.56% | 1.05% | 1.53% | 0.93% | 2.62% | 1.67% |
| 33 | 0.39% | 0.48% | 1.20% | 1.22% | 1.62% | 1.15% | 1.10% | 0.56% | 1.92% | 1.05% |
| 34 | 0.23% | 0.30% | 0.63% | 0.64% | 1.07% | 0.75% | 0.56% | 0.18% | 1.65% | 1.05% |
| 35 | 0.43% | 0.49% | 1.25% | 1.26% | 1.32% | 0.99% | 1.07% | 0.68% | 2.56% | 1.95% |
| 36 | 0.57% | 0.67% | 0.49% | 0.50% | 1.19% | 0.70% | 0.06% | (0.50%) | 0.10% | (0.79%) |
| 37 | 0.34% | 0.38% | 0.30% | 0.31% | 0.75% | 0.54% | (0.32%) | (0.57%) | 0.37% | (0.03%) |
| 38 | 0.32% | 0.39% | 1.38% | 1.39% | 1.94% | 1.59% | 1.46% | 1.04% | 1.52% | 0.86% |
| 39 | 0.12% | 0.24% | 0.86% | 0.88% | 1.73% | 1.12% | 1.27% | 0.55% | 1.19% | 0.05% |
| 40 | 0.17% | 0.24% | 0.84% | 0.86% | 1.08% | 0.71% | 0.71% | 0.27% | 0.72% | 0.02% |
| 41 | 0.05% | 0.16% | 0.44% | 0.46% | 0.70% | 0.15% | 0.79% | 0.15% | 0.66% | (0.35%) |
| 42 | 0.38% | 0.49% | 0.60% | 0.63% | 1.11% | 0.54% | 0.71% | 0.05% | 1.12% | 0.06% |
| 43 | 0.08% | 0.19% | 1.43% | 1.45% | 1.78% | 1.22% | 1.75% | 1.09% | 0.86% | (0.18%) |
| 44 | 0.09% | 0.18% | 0.33% | 0.35% | 0.59% | 0.11% | 0.19% | (0.36%) | (0.44%) | (1.32%) |
| 45 | (0.01%) | 0.10% | 1.49% | 1.51% | 2.00% | 1.49% | 0.98% | 0.38% | 0.65% | (0.30%) |
| 46 | 0.36% | 0.40% | 0.60% | 0.61% | 0.99% | 0.79% | 0.15% | (0.08%) | 1.51% | 1.14% |
| 47 | 0.21% | 0.25% | 0.98% | 0.98% | 1.16% | 0.98% | 1.12% | 0.92% | 1.62% | 1.30% |
| 48 | 0.35% | 0.42% | 0.94% | 0.95% | 1.33% | 1.00% | 0.16% | (0.22%) | 0.57% | (0.04%) |
| 49 | 0.35% | 0.44% | 1.28% | 1.29% | 1.53% | 1.10% | 1.91% | 1.41% | 2.38% | 1.59% |
| 50 | 0.21% | 0.29% | 1.21% | 1.23% | 1.54% | 1.11% | 0.79% | 0.29% | 1.08% | 0.28% |
| **Avg.** | **0.31%** | **0.40%** | **0.68%** | **0.70%** | **1.25%** | **0.80%** | **0.66%** | **0.14%** | **1.02%** | **0.20%** |

Note: Actual returns for 1-day, 15-day, 30-day, 45-day, and 60-day are calculated from that actual price of assets on March 16, March 30, April 17, April 30, and May 15, 2007 respectively.

Table 8 presents the actual returns of hedging portfolio for 1-day, 15-day, 30-day, 45-day, and 60-day investment horizons comparing between minimum-VaR and minimum-variance hedge ratio. For 1-day and 15-day holding period, returns of minimum-variance hedging portfolio are a bit higher than the minimum-VaR hedging portfolio, however, the risk of portfolios are also higher. For longer investment horizons, which are 30-day, 45-day, and 60-day, at lower level of risk, minimum-VaR hedging portfolios provide the higher return than the minimum-variance hedging portfolios. As a result, by applying the minimum-VaR hedging strategy, the hedging performance can be improved both for risk reduction and return maximization purposes.

## 6. Conclusion

The empirical evidence shows that the log-returns of stocks do not follow multivariate normal distribution as assumed in the traditional risk measurement models. The log-returns of stocks has ticker tail than normal or called “fat-tail”. This paper applies the Copula-EVT to simulate 10,000 scenarios of portfolio log-returns based on multivariate distribution with Gaussian Copula and Extreme Value Theory marginal distribution. Moreover, the paper proposes the method of Minimum-VaR hedging strategy to estimate no. of short position in derivatives required to minimize risk of portfolio.

The study has estimated 99% VaR for a portfolio of 29 Thai equities by assuming asset log-returns follow EVT distribution with Gaussian Copula and multivariate normal distribution. I estimates 99% portfolio VaR over one-day horizon by using daily data of stock price series over 6 years time window. To test the accuracy of VaR estimates, the paper performs backtest over 735 days time window, and performs various forms of loss functions including binary loss function, regulatory loss function (RLF), and firm’s loss function (FLF) to compare the performance of VaR estimations. In addition, I do the Kupiec’s hypothesis testing, whether the no. of exceptions are acceptable under desired probability. The empirical study shows that the Copula-EVT outperforms the multivariate normal model. The backtesting result indicates the percentage of exceptions of Copula-EVT is closer to the confidence level 1% than the one of traditional model. The Copula-EVT model provides lower values of loss function for both RLF and FLF, which indicate that this model produces lower economic losses comparing to the normal one. The Kupiec’s test shows that the Copula-EVT model is acceptable to estimate 99% while the multivariate normal method underestimates the probability of large losses.

For hedging purpose, I construct random 50 portfolios of 29 Thai equities, and then estimate hedge ratio from both minimum variance hedge and minimum VaR hedge strategies. To compare hedging effectiveness, I compute the percentage of reduction in standard deviation and 99% VaR of hedging portfolio. The result shows that the minimum-VaR hedging provides the higher percentage of reduction in VaR by taking smaller short position in futures than the minimum variance hedge strategy.

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## Appendix A1: Sample Equities

| Ticker | Name |
|---|---|
| ADVANC | Advanced Info Service Public Company Limited |
| ATC | Aromatics Thailand Public Company Limited |
| BBL | Bangkok Bank Public Company Limited |
| BECL | Bangkok Expressway Public Company Limited |
| BAY | Bank of Ayudhya Public Company Limited |
| BANPU | Banpu Public Company Limited |
| BEC | BEC World Public Company Limited |
| CPF | Charoen Pokphand Foods Public Company Limited |
| DELTA | Delta Electronics Thai Public Company Limited |
| EGCOMP | Electricity Generating Public Company Limited |
| HANA | Hana Microelectronics Public Company Limited |
| IRPC | IRPC Public Company Limited |
| ITD | Italian-Thai Development Public Company Limited |
| KBANK | Kasikornbank Public Company Limited |
| KTB | Krung Thai Bank Public Company Limited |
| LH | Land and Houses Public Company Limited |
| PTTEP | PTT Exploration & Production Public Company Limited |
| RCL | Regional Container Lines Public Company Limited |
| SSI | Sahaviriya Steel Industries Public Company Limited |
| SCC | Siam Cement Public Company Limited |
| SCCC | Siam City Cement Public Company Limited |
| SCB | Siam Commercial Bank Public Company Limited |
| THAI | Thai Airways International Public Company Limited |
| TUF | Thai Union Frozen Products Public Company Limited |
| TCAP | Thanachart Capital Public Company Limited |
| TISCO | Tisco Bank Public Company Limited |
| TMB | TMB Bank Public Company Limited |
| TPIPL | TPI Polene Public Company Limited |
| TRUE | True Corp Public Company Limited |