Chapter 4

Conclusion

The following results are all main theorems of this thesis:

1. *Let D be a nonempty subset of a metric space (X, d) and let $I : D \to D$, $T : D \to CL(D)$, and I and T satisfy the property (W.P.) on D. If T is a generalized I-contraction on D, then $C(I, T) \neq \varnothing$. Moreover, if $IIv = Iv$ for some $v \in C(I, T)$, then $F(I, T) \neq \varnothing$.*

2. *Let D be a nonempty subset of a normed space X. Let $I : D \to D$, $T : D \to CL(D)$, I and T satisfy the property (W.P.) on D, and the pair (I,T) satisfies the CPC on D. If T is a generalized I-nonexpansive on D, then $C(I, T) \neq \varnothing$. Moreover, if IIv=Iv for some $v \in C(I, T)$, then $F(I, T) \neq \varnothing$.*

3. *Let D be a nonempty subset of a normed space X. Let $I : D \to D$, $T : D \to CL(D)$, I and T satisfy the property (W.P.) on D, and (I-T)(D) be closed. If T is a generalized I-nonexpansive on D, then $C(I, T) \neq \varnothing$. Moreover, if IIv=Iv for some $v \in C(I, T)$, then $F(I, T) \neq \varnothing$.*

4. *Let D be a q-starshaped subset of a normed space X. Let $I : D \to D$, $T : D \to CL(D)$, I and T satisfy the property (W.P.) on D, I(D)=D, D weakly compact,*

and I-T is demiclosed at 0. If T is generalized I-nonexpansive on D, then $C(I,T) \neq \varnothing$. *Moreover, if IIv=Iv for some* $v \in C(I,T)$, *then* $F(I,T) \neq \varnothing$.

5. *Let* X *be a normed space,* $I : X \to X$, $T : X \to CL(X)$, $M \subseteq X$, $I(B_M(p)) = B_M(p)$, *I and T satisfy the property (W.P.) on* $B_M(p)$, *the pair (I,T) satisfies the CPC on* $B_M(p)$, *and*

$$\sup_{y \in Tx} \|y - p\| \leq \|Ix - p\|$$

for all $x \in B_M(p)$.

If T is a a generalized I-nonexpansive on $B_M(p)$, *then* $C(I,T) \cap B_M(p) \neq \varnothing$. *Moreover, if IIv=Iv for some* $v \in C(I,T) \cap B_M(p)$, *then* $F(I,T) \cap B_M(p) \neq \varnothing$.

6. *Let* X *be a normed space,* $I : X \to X$, $T : X \to CL(X)$, $M \subseteq X$, $I(B_M(p)) = B_M(p)$, *I and T satisfy the property (W.P.) on* $B_M(p)$, $(I - T)(B_M(p))$ *be closed, and*

$$\sup_{y \in Tx} \|y - p\| \leq \|Ix - p\|$$

for all $x \in B_M(p)$.

If T is a generalized I-nonexpansive on $B_M(p)$, *then* $C(I,T) \cap B_M(p) \neq \varnothing$. *Moreover, if IIv=Iv for some* $v \in C(I,T) \cap B_M(p)$, *then* $F(I,T) \cap B_M(p) \neq \varnothing$.

7. *Let* X *be a normed space,* $I : X \to X$, $T : X \to CL(X)$, $M \subseteq X$, $I(B_M(p)) = B_M(p)$, $B_M(p)$ *be weakly compact and q-starshaped, I and T satisfy property (W.P.) on* $B_M(p)$, *I-T be demiclosed at 0, and*

$$\sup_{y \in Tx} \|y - p\| \leq \|Ix - p\|$$

for all $x \in B_M(p)$.

If T is a generalized I-nonexpansive on $B_M(p)$, *then* $C(I,T) \cap B_M(p) \neq \varnothing$. *Moreover, if IIv=Iv for some* $v \in C(I,T)$, *then* $F(I,T) \cap B_M(p) \neq \varnothing$.

8. *Let* (Ω, Σ) *be a measurable space and* D *be a separable, closed, and q-starshaped subset of a normed space* X. *Let* $I : \Omega \times D \to D$ *and* $T : \Omega \times D \to CL(D)$ *be continuous random operators such that* $I(\omega, \cdot)$ *and* $T(\omega, \cdot)$ *satisfy the property (W.P.) on* D, $I(\omega, \cdot)(D) = D$, $T(\omega, \cdot)(D)$ *is bounded, and the pair* $(I(\omega, \cdot), T(\omega, \cdot))$ *satisfies the CPC on* $A \in CL(D)$ *for every* $\omega \in \Omega$. *If* $T(\omega, \cdot)$ *is a generalized I-nonexpansive on* D *for every* $\omega \in \Omega$, *then* $RC(I, T) \neq \varnothing$. *Moreover, if for every* $\omega \in \Omega$, $I(\omega, I(\omega, \mu(\omega))) = I(\omega, \mu(\omega))$ *for some* $\mu \in RC(I, T)$, *then* $RF(I, T) \neq \varnothing$.

The property (W.P.) and (W.P.)*, it is possible to obtain coincidence points, common fixed point, random coincidence points, common random fixed point, and invariant approximations results for the following mappings

- Hybrid strict contraction mappings (see [32]).
- Multivalued weak contraction or multivalued (θ, L)-weak contraction (see [12]).
- Generalized multivalued weak contraction or generalized multivalued (α, L)-weak contraction (see [13]).
- Multivalued f-weak contraction or multivalued (f, θ, L)-weak contraction (see [33]).
- Generalized multivalued f-weak contraction or generalized multivalued (f, α, L)-weak contraction (see [33]).
- Pointwise contraction mappings (see [11, 36]).
- Asymptotic pointwise contraction mappings (see [39]).
- Asymptotic pointwise nonexpansive mappings (see [39]).

- Asymptotic pointwise eventually nonexpansive mappings (see [39]).

We might devote future publications to the study of the previous mappings.