Chapter 1

Introduction

The theory of nonlinear analysis has come out as one of the momentous mathematical disciplines during the last 50 years. The fixed point theorem, generally known as the Banach Contraction Mapping Principle, appeared in explicit form in Banach's thesis in 1922 where it was used to establish the existence of a solution for an integral equation. Since then, because of its simplicity and usefulness, it has become a very popular tool in solving existence problems in many branches of mathematical analysis.

Fixed point theorems for single-valued mappings are useful in the existence theory of differential equations, integral equations, partial differential equations, random differential equations, and in other related areas. It has very fruitful applications in eigenvalue problems as well as in boundary value problems, including approximation theory, variational inequality, and complementarity problems. Fixed point theory for a multivalued (set-valued) mapping was originally initiated by von Neumann [48] in the study of game theory. In 1969, the Banach's Contraction Mapping Principle extended nicely to set-valued or multivalued mappings, a fact first noticed by Nadler [47]. The fixed point theory of multivalued nonexpansive mappings is however much more complicated and difficult than the corresponding theory of singlevalued nonexpansive mappings. But some papers have appeared showing some properties which state fixed point results for multivalued mappings.

In 1999, Latif and Tweddle [41] established some coincidence point theorem for I-nonexpansive mappings by using the commutativity condition of mappings. Badshsh and Sayyed [2] studied some common random fixed points of multivalued operators on Polish spaces. In 2004, Shahzad [59] extended and proved some general random coincidence point theorems and, as applications, derived a number of random fixed point results. Afterwards, Shahzad and Hussain [61] improved and established some coincidence point theorems for I-nonexpansive mappings. As applications, invariant approximation theorems are derived. The results of Shahzad and Hussain [61] unify, extend and complement many known results existing in the literature including those of Beg and Shahzad [7, 8, 10], Doston [23], Juncgk and Sessa [30], Jungck [29], Kamran [32], Latif and Bano [40], Latif and Tweddle [41], Shahzad [59], Shahzad and Latif [60], Tan and Yaun [63], and Xu [64].

Recently, Al-Thagafi and Shahzad [1] defined new mapping which generalizes contraction mapping and established some coincidence point results for multivalued mappings satisfying generalized I-contraction. The main results of Al-Thagafi and Shahzad [1] used assumption that I was T-weakly commuting at some $v \in C(I,T)$ in the proof.

In this thesis, we will define property $(W.P.)$ and $(W.P.)^*$ and establish some coincidence point theorems for generalized I-contraction and generalized I-nonexpansive mappings by using the new property. We observe that if we drop the assumption that I is T-weakly commuting at some $v \in C(I,T)$ in Theorem 2.6 of Shahzad and Hussain [61] and, Theorem 2.1 of Al-Thagafi and Shahzad [1], then the theorems are still true. So we will establish some common fixed point theorems for multivalued generalized I-contraction and generalized I-nonexpansive mappings without assumption that I is T-weakly commuting at some $v \in C(I, T)$ and several invariant approximations results are derive. A random coincidence point and common random fixed point results are also proved. Our theorems in this thesis generalize and extend the Banach Contraction Principle, Nadler's Contraction Principle and, the recent theorems of Al-Thagafi and Shahzad [1], Shahzad and Hussain [61] and many authors.

This thesis is organized into 4 chapters as follows:

Chapter 1: We will discuss the basic problem which can be solved by fixed point theory. Also we will introduce some researches related to our work in this thesis.

Chapter 2: We will give some definitions, notation, terminology and some useful results that will be used in later chapters.

Chapter 3: This chapter contains our main results such as the deterministic coincidence point, common fixed point and application to the invariant approximations. Moreover, random version is also proved.

Chapter 4: The last chapter contains the conclusion of the new theorems that we obtain.