Chapter 5

Conclusions and Recommendations

The objective of this study is to compare five new confidence intervals for a ratio of binomial proportions. There are

- The confidence limits using the direct and inverse binomial sampling methods. There are 3 cases in this method as following

  - when the asympotic variance is used,
  - when the true value of the variance is used,
  - when we fix the number of success as in the first experiment.

- The confidence limits using only the direct binomial sampling method.

- The confidence limits using only the inverse binomial sampling method.

The criteria for comparing among methods are the coverage probability median and mean of intervals length. The factor of studying are 3 levels of sample size n (n=30,50,100) ,but for the case of direct and inverse method when we fix the number of success as in the first experiment, the sample size n=30,50,70,100. The reason we chose these particular sample sizes is that because we are planning to compare the performance of the new confidence intervals for three possible categories of sample sizes: small, moderate, and large. We took n=70 for the direct-inverse case to investigate it more. Consider 95% confidence interval.

The experimental data are generated by the simulation technique using program R version 2.6.1. For each situation, the experiment is repeated 10,000 times for calculate the coverage probability median, mean and standard deviation of intervals length. The conclusions of this study are summarized as follows:

5.1 Confidence limits using only the direct binomial sampling method

The results of this model (8) give the coverage probabilities close to nominal level although still lower. The values of the median and mean of the interval length are practically the same. This means that the length of the interval is symmetrically distributed. Most probably this follows from the application of unbiased estimators for our results. We consider the fact that the length distribution is symmetrical as an additional reason for these intervals to be used for practical applications (in biological, medical and social studies). Median, mean, and standard deviation of the length are increasing when the ratio p_1/p_2 increases even in the case when probabilities start to take values more than 0.5. Hence, it is recommended to use this method of interval estimation in the region $p_1 < 0.2$ for any values of p_2.

5.2 Confidence limits using only the inverse binomial sampling method

The results for the interval (9) give low coverage probabilities for $p_2 < 0.5$ and poor performing values when $p_1 < 0.2$ for all $p_2 \geq p_1$. The values of median and mean of the interval length are still practically the same. The median, mean, and standard deviation of the length increase when the the ratio p_1/p_2 increases.

The comparison between the confidence interval (8) and (9) says that the behaviour of all characteristics of this confidence interval interval (9) is nearly the same as for the interval (8). Hence, it is recommended to use this method of interval estimation only in the region of large values of $p_2 > 0.5$ and $p_1 > 0.3$.

5.3 Confidence limits using the direct and inverse binomial sampling method

5.3.1 When the asymptotic variance is used.

As we already know, the confidence interval (3) has the most poor property (among all other discussed in my thesis) for the coverage probability (it should be close to the nominal value 0.95). If the size of the first sample is smaller than the average size of the second, then coverage probability is smaller, and for the inverse case it is much higher than the nominal level. The mean values, medians, and standard deviation of interval (3) length in many cases appear to be too large. Hence, we do not recommend to use this interval for practical applications.

5.3.2 When the true value of the variance is used.

Not looking that the shape of the region of acceptable values of coverage probabilities for confidence interval (4) is somehow similar to the region for the interval (9), the region itself is much wider. The values $p_1 < 0.2$ should be excluded. But even for $n = 30, m = 30p_2$ the interval may be recommended for all $p_2 > 0.3$. Also, it may be recommended for all n and $m \leq 100p_2$ when sample sizes $n = 30,50,100$ and the hence recommended region of the interval (4) applications is increased to the region $p_2, p_1 > 0.1$.

5.3.3 When the number of successes in the second sample is fixed as in the first experiment.

Confidence interval (6) obtains the coverage probability close to the nominal level. Characteristics of the length are similar to the interval (8), is defined by the number of successes in the first sample. We should exclude values $p_1 < 0.2$. Aacceptable sample size starts from $n = 50$ for the first sample. But even for smaller sample size, say, $n = 30$, the interval is still possible to use for $p_1 > 0.3$ and all values p_2.

5.4 Main conclusion

Hence, if we order the intervals in descending order according to the size of the regions and values where we could recommend for their application, then the order is the following: (8), (6), (4), (9). We would like to mention that the formulas are applicable for different sample schemes. For example, if the experiment is conducted as an inverse-inverse sampling, then the only one that can be used is (9).

While if the experiment occurs as in direct-inverse sampling, then we recommend to use formula (6), where we fix the number of successes in the second sample as the number of successes in the first sample. If the experiment is conducted as a classical case, direct-direct sampling, then the only formula we suggest to use is (8).

5.5 Comparison of our confidence intervals with previously known

We first mention that we can compare only direct-direct sampling case. The main novel feature of the current work is in considering inverse binomial sampling scheme, what, up to our knowledge, was not done before. Next, we would like to mention that numerical illustrations provided in papers Koopman (1984) , Bailey(1987) are very poor. These authors presented only the regions where there confidence intervals work ,and do not provide complete information about parameter values. Contrary to this, we calculated coverage probabilities and interval length characteristics for all possible values of parameters. If we compare the coverage probability of our direct-direct confidence interval with known before, then according to numerical illustrations provided in the previously published papers, we can conclude that our formula is better.

5.6 Future Research

1. The inverse – direct case was not considered in my thesis. By this we mean that the first sample is obtained by the means of inverse sampling, while the second one is obtained by the means of direct sampling. We did not consider this case in the thesis because we cannot imagine a situation when it may be of any practical use. But from theoretical point of view, or in order to have a complete picture, this peculiar case may also be considered in our future research. Anyway, we do not expect good performance of our technique for it.

2. As it is possible to see, the normal approximation technique performs poorly for the small values of Binomial proportions p_1 and p_2. We may apply another type of approximation.

3. In this thesis we consided the case when Binomial samples are independent. It may have some theoretical interest to consider dependent samples. The following example is provided in Steel et al.(1997). “We might wish to compare headache remedies or seasickness preventives.It makes sence to assign a particular treatment to a randomly selected individual from each pair and assign the other treatment to the other individual.Sometimes it will be possible to use the same individual for both treatments but at different times. ”