

Chapter 4

Findings and Results

Coverage probabilities, and median lengths, mean values, and standard deviations of all five new confidence intervals were obtained by the Monte-Carlo method. For each interval 10,000 random numbers with Binomial and/or Negative Binomial distributions were generated with parameters (probabilities of success) $p_1, p_2 = 0.1, 0.3, 0.5, 0.7, 0.9$. The tables presented contain only a part of the results for the probability $p_2 \geq p_1$. The nominal confidence level is chosen to be 0.95.

For the direct binomial sampling the sample sizes are chosen to be $n = 30, 50, 100$ for all intervals, except (6), where $n = 30, 50, 70, 100$. For the inverse binomial sampling, in order to obtain a correct comparison of interval's characteristics we use sampling stopped at $m = np_2$ successes, because in this case $Ev = n$. For each of fixed values of n, m, p_1 and $p_2 (\geq p_1)$ a table contains the block in which the following characteristics are provided (from top to bottom): coverage probabilities, median length, mean length, and standard deviation of the length for the confidence interval obtained from the formula with the number presented at the bottom of the table.

In order to estimate the accuracy of confidence interval characteristics of calculations, each table was reproduced 10 times. For all cases and for all characteristics, we observed a difference only in the third digit after the decimal point. Hence a calculation error should not exceed 0.01.

4.1 Comparative analysis of methods

We start with analysis of the modeling results (see Table1) for the confidence interval (8), which is a modification of the classical interval with only direct sampling. After, we compare characteristics of other intervals with this “classical” case.

Assuming that the coverage probability error in comparison with the nominal should not exceed 0.025, it is possible to make a certain conclusion about strictly low coverage probability for the values $p_1 < 0.2$ for any values of p_2 and $n, m \leq 30$. If p_1

becomes bigger than 0.2, then for all values of p_2, n , and m , the coverage probability does not differ too much from the nominal level (still lower), and the value of median and mean of the interval length is practically the same. This means that the distribution of the interval length is symmetrical, nice and pleasant property. The smallest value of the coverage probability always corresponds to the equal proportions ($p_1/p_2 = 1$).

Now it is appropriate to compare the case of only direct binomial sampling with the case of only inverse binomial sampling for both experiments (see Table 2). In this case the region of appropriate values of proportions is smaller. Except poor performing values $p_1 < 0.2$, the coverage probability is low for $p_2 \leq 0.5$ and nearly all sample sizes. The behaviour of all characteristics of the confidence interval (9) is the same as for the interval (8). That is, we have the following tendency: the coverage probability is increasing when the ratio of probabilities increasing. Also, the values of median and mean for the length of the interval are not significantly different (symmetry of the length distribution). But these characteristics for the confidence interval (9) length are somehow better than for the interval (8).

Confidence interval (6) (see Table 3) where the sample size for the second experiment is defined by the number of successes in the first sample possesses good properties according to the coverage probability (close to the nominal level) and characteristics of the length very similar to the interval (8), while the coverage probabilities are a little bit lower and the "good" region is smaller. If we exclude the values $p_1 < 0.2$, then practical applications (according to the nominal level) may start from the size $n = 50$ for the first sample. But even for $n = 30$, the interval is still possible to use when $p_1 > 0.3$ for all p_2 .

Now we discuss the confidence interval (4) (see Table 4), the modification of the ``poor'' interval (3). The shape of the region of acceptable values of coverage probabilities is similar to the region for the interval (9), but the region itself is much wider. As before, we should exclude the values $p_1 < 0.2$, but even for $n = 30, m = 30p_2$ the interval works for all $p_2 > 0.5$. The same is still true for all n and $m = 30p_2$ or $m = 50p_2$. For sample sizes $n = 30, 50, 100$ and $m = 100p_2$, the

“good” region of the interval (4) applications is increased to the region $p_2, p_1 > 0.1$. Probability characteristics of the interval (4) length are practically the same as characteristics of interval (8).

Unfortunately, as the results of stochastic modeling show (see Table 5), the confidence interval (3) possesses poor asymptotic properties according to the correspondence of the coverage probability to the nominal value 0.95 of the confidence level. If the size of the first sample is smaller than the average size of the second, then coverage probability is smaller than the nominal level. Contrary, if the size of the first sample is larger than the average size of the second, then coverage probability seems to be higher than the nominal level. In the last case the mean values, medians, and standard deviation of interval (3) length appear to be too large, especially for small values of p_1 and p_2 .

4.2 Comparative analysis with the previously known

We start discussing our result from direct-direct sampling case with previously known. The main novel feature of the current work is in considering inverse binomial sampling scheme, what, up to our knowledge, was not done before. Next, we would like to mention that Koopman (1984) , Bailey(1987) presented only the regions where there confidence intervals work ,and do not provide complete information about parameter values. Contrary to this, we calculated coverage probabilities and interval length characteristics for all possible values of parameters. If we compare the coverage probability of our direct-direct confidence interval with the paper of Koopman (1984), we found as an example, for $\theta = 1.00, p_1 = 0.5, n = m = 100$ the coverage probability by χ^2 method is 0.94,while we have a better value 0.945.

Table 1
Confidence limits with using only direct binomial sampling method (formula 8)

		n=30,m=30						n=30,m=50											
		P2			P1			P2			P1			P2			P1		
P1 \ P2	0.9	0.7	0.5	0.3	0.1		0.9	0.7	0.5	0.3	0.1		0.9	0.7	0.5	0.3	0.1		
0.1	0.859	0.872	0.874	0.862	0.786		0.844	0.872	0.875	0.880	0.851								
	0.239	0.311	0.444	0.768	2.518		0.1	0.239	0.309	0.438	0.750	2.386							
	0.236	0.314	0.464	0.888	3.100		0.058	0.235	0.308	0.446	0.811	3.042							
0.3	0.058	0.091	0.17	0.558	2.633		0.056	0.081	0.138	0.378	0.378	2.582							
	0.933	0.938	0.932	0.902	0.945		0.930	0.937	0.929	0.920	0.944								
	0.367	0.498	0.744	1.448	0.258	0.9	0.3	0.364	0.486	0.715	1.311	0.294	0.9						
0.367	0.51	0.800	1.673	0.259	0.048		0.362	0.487	0.734	1.423	0.301								
	0.046	0.102	0.249	0.972	0.048		0.039	0.075	0.166	0.518	0.080								
	0.942	0.946	0.935	0.950	0.952		0.939	0.944	0.937	0.943	0.958								
0.5	0.414	0.595	0.978	0.507	0.313	0.7	0.5	0.401	0.561	0.887	0.579	0.335	0.7						
	0.415	0.617	1.046	0.518	0.315		0.405	0.571	0.919		0.604	0.338							
	0.042	0.123	0.334	0.092	0.033		0.027	0.078	0.206		0.157	0.055							
0.7	0.943	0.951	0.945	0.949	0.945		0.942	0.950	0.936	0.950	0.943								
	0.400	0.650	0.769	0.464	0.320	0.5	0.7	0.386	0.586	0.878	0.504	0.332	0.5						
	0.407	0.674	0.801	0.475	0.322		0.386	0.596	0.944		0.527	0.335							
0.9	0.056	0.160	0.192	0.072	0.024		0.038	0.094	0.327		0.119	0.039							
	0.959	0.925	0.941	0.937	0.938		0.950	0.929	1.298	0.650	0.407	0.289	0.3						
	0.332	1.155	0.592	0.388	0.286	0.3	0.9	0.299	1.513	0.688	0.418	0.291							
0.335	0.335	1.256	0.614	0.394	0.286		0.297		0.919	0.221	0.085	0.033							
	0.082	0.482	0.138	0.058	0.027		0.058		0.778	0.898	0.908	0.906	0.891						
	0.863	0.901	0.911	0.900	0.890			2.056	0.625	0.345	0.241	0.182	0.1						
2.085	2.688	0.594	0.339	0.239	0.184	0.1		2.762	0.707	0.365	0.244	0.182							
	2.226	0.246	0.098	0.055	0.037			2.277	0.394	0.124	0.064	0.039							
	0.1	0.3	0.5	0.7	0.9			0.1	0.3	0.5	0.7	0.9							
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Confidence limits with using only direct binomial sampling method (formula 8) (Continued)

		n=30,m=100						n=50,m=100											
		P2			P1			P2			P1			P2			P1		
P1 \ P2	0.9	0.7	0.5	0.3	0.1		0.9	0.7	0.5	0.3	0.1		0.9	0.7	0.5	0.3	0.1		
0.1	0.834	0.861	0.863	0.881	0.879		0.1	0.876	0.900	0.900	0.907		0.884						
0.235	0.238	0.308	0.434	0.733	2.323		0.1	0.183	0.237	0.333	0.575		1.924						
0.055	0.075	0.305	0.436	0.763	2.533		0.1	0.180	0.236	0.334	0.587		2.151						
0.3	0.929	0.930	0.935	0.933	0.909		0.3	0.049	0.080	0.172	1.159								
0.366	0.475	0.688	1.229		0.270	0.9	0.3	0.941	0.942	0.942	0.935		0.947						
0.360	0.473	0.689	1.257		0.270		0.3	0.283	0.375	0.551	1.021		0.224						
0.035	0.058	0.109	0.293		0.083		0.3	0.282	0.376	0.557	1.053		0.226						
0.5	0.938	0.945	0.943	0.936	0.952		0.5	0.022	0.040	0.084	0.247		0.048						
0.5	0.396	0.535	0.800	0.523	0.270	0.7	0.5	0.944	0.943	0.946	0.944		0.954						
0.396	0.537	0.814		0.550	0.273		0.5	0.312	0.431	0.672	0.438		0.245						
0.018	0.045	0.109		0.162	0.057		0.5	0.313	0.435	0.683	0.451		0.246						
0.7	0.932	0.945		0.922	0.941	0.955	0.7	0.942	0.952	0.942	0.950		0.949						
0.377	0.526	0.530		0.782	0.426	0.251	0.7	0.297	0.441	0.666	0.375		0.237						
0.371	0.530			0.852	0.446	0.256	0.7	0.297	0.444	0.696	0.386		0.240						
0.031	0.053			0.319	0.114	0.038	0.7	0.020	0.046	0.181	0.067		0.032						
0.9	0.937		0.907	0.927	0.950	0.949	0.9	0.949	0.922	0.942	0.950		0.949						
0.269	0.265		1.169	0.545	0.320	0.211	0.3	0.9	0.226	0.997	0.479	0.296		0.206					
0.051	0.051		1.354	0.583	0.333	0.215	0.3	0.9	0.224	1.084	0.498	0.302		0.208					
			0.802	0.199	0.076	0.026	0.3	0.9	0.032	0.422	0.118	0.048		0.018					
			1.169	0.545	0.320	0.211	0.3												
			1.835	0.504	0.263	0.178	0.132	0.1											
			2.511	0.578	0.282	0.181	0.132	0.1											
			1.994	0.337	0.096	0.041	0.021	0.1											
			0.1	0.3	0.5	0.7	0.9												

		n=30,m=100						n=50,m=100						P1					
		P2			P1			P2			P1			P2			P1		
P1 \ P2	0.9	0.7	0.5	0.3	0.1		0.9	0.7	0.5	0.3	0.1		0.9	0.7	0.5	0.3	0.1		
0.1	0.834	0.861	0.863	0.881	0.879		0.1	0.876	0.900	0.900	0.907		0.884						
0.235	0.238	0.308	0.434	0.733	2.323		0.1	0.183	0.237	0.333	0.575		1.924						
0.055	0.075	0.305	0.436	0.763	2.533		0.1	0.180	0.236	0.334	0.587		2.151						
0.3	0.929	0.930	0.935	0.933	0.909		0.3	0.037	0.049	0.080	0.172		1.159						
0.366	0.475	0.688	1.229		0.270	0.9	0.3	0.941	0.942	0.942	0.935		0.947						
0.360	0.473	0.689	1.257		0.270		0.3	0.283	0.375	0.551	1.021		0.224						
0.035	0.058	0.109	0.293		0.083		0.3	0.282	0.376	0.557	1.053		0.226						
0.5	0.938	0.945	0.943	0.936	0.952		0.5	0.022	0.040	0.084	0.247		0.048						
0.5	0.396	0.535	0.800	0.523	0.270	0.7	0.5	0.944	0.943	0.946	0.944		0.954						
0.396	0.537	0.814		0.550	0.273		0.5	0.312	0.431	0.672	0.438		0.245						
0.018	0.045	0.109		0.162	0.057		0.5	0.313	0.435	0.683	0.451		0.246						
0.7	0.932	0.945		0.922	0.941	0.955	0.7	0.942	0.952	0.942	0.950		0.949						
0.377	0.526	0.530		0.782	0.426	0.251	0.7	0.297	0.441	0.666	0.375		0.237						
0.371	0.530			0.852	0.446	0.256	0.7	0.297	0.444	0.696	0.386		0.240						
0.031	0.053			0.319	0.114	0.038	0.7	0.020	0.046	0.181	0.067		0.032						
0.9	0.937		0.907	0.927	0.950	0.949	0.9	0.949	0.922	0.942	0.950		0.949						
0.269	0.265		1.169	0.545	0.320	0.211	0.3	0.9	0.226	0.997	0.479	0.296		0.206					
0.051	0.051		1.354	0.583	0.333	0.215	0.3	0.9	0.224	1.084	0.498	0.302		0.208					
			0.802	0.199	0.076	0.026	0.3	0.9	0.032	0.422	0.118	0.048		0.018					
			1.169	0.545	0.320	0.211	0.3												
			1.835	0.504	0.263	0.178	0.132	0.1											
			2.511	0.578	0.282	0.181	0.132	0.1											
			1.994	0.337	0.096	0.041	0.021	0.1											
			0.1	0.3	0.5	0.7	0.9												

n=100,m=30

n=100,m=50

n=100,m=100

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Confidence limits with using only direct binomial sampling method (formula 8) (Continued)

		n=100, m=100				
		P2 0.9	0.7	0.5	0.3	0.1
P1		0.929	0.931	0.928	0.932	0.905
	0.1	0.129	0.169	0.244	0.43	1.579
0.3	0.129	0.170	0.246	0.444	1.798	
	0.019	0.027	0.047	0.114	1.010	
	0.946	0.947	0.943	0.936		
0.5	0.202	0.276	0.424	0.831		
	0.203	0.279	0.432	0.865		
	0.013	0.029	0.068	0.214		
0.7	0.945	0.950	0.945			
	0.227	0.332	0.547			
	0.228	0.335	0.559			
0.9	0.012	0.035	0.091			
	0.950	0.947				
	0.222	0.360				
	0.223	0.364				
	0.016	0.044				
	0.955					
	0.183					
	0.184					
	0.023					

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Table 2
Confidence limits with using only inverse binomial sampling method (formula 9)

n=30p1,m=30p2						
P2 \ P1		0.9	0.7	0.5	0.3	0.1
0.1	0.805	0.8	0.794	0.776	0.683	
	0.187	0.238	0.346	0.566	1.588	
	0.228	0.297	0.428	0.718	2.16	
	0.145	0.196	0.292	0.523	1.96	
0.3	0.914	0.907	0.886	0.837	0.933	
	0.357	0.474	0.678	1.161	0.254	0.9
	0.365	0.49	0.707	1.229	0.253	
	0.085	0.13	0.21	0.439	0.04	
0.5	0.926	0.919	0.882	0.913	0.944	
	0.406	0.571	0.851	0.475	0.312	0.7
	0.408	0.576	0.864	0.476	0.313	
	0.053	0.104	0.19	0.059	0.027	
0.7	0.931	0.907	0.897	0.923	0.938	
	0.397	0.615	0.666	0.447	0.319	0.5
	0.404	0.616	0.674	0.449	0.319	
	0.046	0.1	0.113	0.062	0.031	
0.9	0.935	0.854	0.908	0.927	0.925	
	0.327	0.922	0.539	0.374	0.28	0.3
	0.325	0.955	0.553	0.382	0.285	
	0.071	0.265	0.126	0.078	0.052	
n=50p1, m=50p2						
		0.1	0.3	0.5	0.7	0.9
					P1	P2

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Confidence limits with using only inverse binomial samplings method (formula 9) (Continued)

n=30p1, m=100p2						
P2 \ P1	0.9	0.7	0.5	0.3	0.1	
0.1	0.81	0.805	0.807	0.794	0.762	
	0.185	0.238	0.341	0.561	1.658	
	0.228	0.293	0.419	0.696	2.095	
	0.148	0.191	0.276	0.467	1.525	
	0.915	0.908	0.902	0.887	0.893	
	0.353	0.456	0.645	1.084	0.271	0.9
0.3	0.359	0.466	0.662	1.116	0.258	
	0.079	0.108	0.163	0.303	0.069	
	0.927	0.92	0.911	0.894	0.934	
0.5	0.397	0.521	0.751	0.473	0.268	0.7
	0.394	0.521	0.752	0.474	0.267	
	0.038	0.062	0.106	0.089	0.047	
	0.928	0.925	0.856	0.91	0.938	
0.7	0.377	0.515	0.63	0.394	0.247	0.5
	0.375	0.514	0.636	0.398	0.25	
	0.024	0.042	0.128	0.073	0.035	
	0.908	0.802	0.882	0.917	0.937	
0.9	0.269	0.819	0.459	0.302	0.209	0.3
	0.261	0.845	0.467	0.306	0.212	
	0.051	0.234	0.106	0.061	0.033	
	0.685	0.84	0.888	0.899	0.909	
	1.163	0.405	0.238	0.165	0.124	0.1
	1.346	0.437	0.254	0.175	0.131	
	0.804	0.181	0.092	0.059	0.041	
	0.1	0.3	0.5	0.7	0.9	
					P1	P2

n=50p1, m=100p2

		n=50p1, m=100p2						
		P2						
P1 \ P2		0.9	0.7	0.5	0.3	0.1		
0.1	0.875	0.873	0.866	0.852	0.797			
	0.162	0.212	0.295	0.492	1.431			
	0.182	0.236	0.331	0.554	1.667			
	0.084	0.111	0.157	0.274	0.961			
0.3	0.932	0.926	0.911	0.888		0.924		
	0.279	0.365	0.519	0.878		0.217	0.9	
	0.283	0.37	0.529	0.898		0.217		
	0.049	0.069	0.108	0.208		0.038		
0.5	0.93	0.935	0.916		0.911	0.942		
	0.312	0.42	0.609		0.4	0.241	0.7	
	0.311	0.42	0.613		0.401	0.242		
	0.025	0.044	0.079		0.054	0.026		
0.7	0.936	0.931		0.882	0.921	0.943		
	0.297	0.42		0.548	0.352	0.236	0.5	
	0.297	0.422		0.552	0.354	0.237		
	0.015	0.033		0.089	0.048	0.023		
0.9	0.933		0.838	0.898	0.926	0.935		
	0.222		0.738	0.42	0.284	0.205	0.3	
	0.22		0.756	0.427	0.287	0.206		
	0.029		0.177	0.083	0.048	0.028		
		0.749	0.864	0.899	0.907	0.914		
		1.151	0.393	0.233	0.163	0.123	0.1	
		1.288	0.42	0.246	0.172	0.13		
		0.661	0.159	0.083	0.056	0.039	P1	P2
			0.1	0.3	0.5	0.7	0.9	

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Confidence limits with using only inverse binomial samplings method (formula 9) (Continued)

		n=100p1, m=100p2					
		0.9	0.7	0.5	0.3	0.1	P2
P1 \ P2							
0.1	0.911	0.916	0.904	0.887	0.810		
	0.123	0.161	0.277	0.378	1.127		
	0.130	0.170	0.240	0.402	1.229		
0.3	0.943	0.933	0.914	0.863			
	0.201	0.268	0.390	0.668			
	0.202	0.271	0.394	0.680			
0.5	0.025	0.038	0.064	0.133			
	0.942	0.937	0.902				
	0.226	0.318	0.476				
0.7	0.227	0.318	0.478				
	0.016	0.030	0.057				
	0.945	0.928					
0.9	0.220	0.335					
	0.221	0.335					
	0.013	0.029					

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Table 3

Confidence limits with using direct and inverse binomial samplings method when the number of successes is fixed as the first experiment (formula 6)

N=30							n=70							n=100						
P2 \ P1		0.9	0.7	0.5	0.3	0.1	P2 \ P1		0.9	0.7	0.5	0.3	0.1	P2 \ P1		0.9	0.7	0.5	0.3	0.1
P2	P1	0.826	0.855	0.834	0.816	0.783	P2	P1	0.934	0.9	0.879	0.875	0.867	P2	P1	0.944	0.9	0.867	0.855	0.848
0.1	0.214	0.323	0.500	0.868	2.630		0.1	0.157	0.228	0.326	0.585	1.878		0.9	0.1	0.157	0.228	0.326	0.585	1.878
	0.246	0.347	0.532	0.969	2.882			0.161	0.225	0.342	0.610	1.948			0.1	0.161	0.225	0.342	0.610	1.948
	0.088	0.167	0.293	0.583	1.896			0.039	0.073	0.125	0.242	0.814			0.039	0.073	0.125	0.242	0.814	
	0.939	0.905	0.897	0.896	0.953			0.940	0.932	0.930	0.931		0.944	0.3	0.3	0.251	0.360	0.551	0.996	
0.3	0.382	0.513	0.820	1.485		0.255	0.9	0.3	0.251	0.360	0.551		0.944		0.3	0.251	0.360	0.551	0.996	
	0.381	0.551	0.848	1.523		0.257		0.3	0.252	0.363	0.558		0.944		0.3	0.252	0.363	0.558	0.996	
	0.073	0.151	0.263	0.506		0.046		0.031	0.063	0.112	0.213		0.944		0.3	0.251	0.360	0.551	0.996	
	0.934	0.930	0.917	0.940	0.939			0.5	0.282	0.419	0.653		0.946	0.5	0.5	0.282	0.419	0.653	0.996	
0.5	0.405	0.628	0.977	0.504	0.319	0.7	0.9	0.5	0.283	0.421	0.658		0.946		0.5	0.283	0.421	0.658	0.996	
	0.427	0.637	1.000	0.510	0.322	0.075		0.5	0.026	0.057	0.101		0.946		0.5	0.282	0.419	0.653	0.996	
	0.063	0.134	0.238	0.075	0.038			0.7	0.272	0.428		0.946	0.943	0.7	0.7	0.272	0.428	0.549	0.996	
	0.936	0.930	0.929	0.940	0.945			0.7	0.273	0.432		0.946	0.943		0.7	0.272	0.428	0.549	0.996	
0.7	0.407	0.644	0.772	0.490	0.327	0.5	0.9	0.027	0.054		0.946	0.943	0.7	0.7	0.273	0.432	0.552	0.996		
	0.413	0.656	0.780	0.496	0.334	0.079		0.027	0.054		0.946	0.943		0.7	0.273	0.432	0.552	0.996		
	0.064	0.125	0.143	0.079	0.037			0.027	0.054		0.946	0.943		0.7	0.272	0.428	0.549	0.996		
	0.943	0.917	0.919	0.932	0.945			0.9	0.217		0.836	0.463		0.946	0.935	0.930	0.939	0.942	0.946	
0.9	0.324	1.168	0.648	0.422	0.292	0.3	0.9	0.9	0.218		0.842	0.467		0.946	0.935	0.930	0.939	0.942	0.946	
	0.328	1.188	0.656	0.429	0.297	0.043		0.9	0.032		0.151	0.079		0.946	0.935	0.930	0.939	0.942	0.946	
	0.077	0.302	0.156	0.089	0.043			0.9	0.032		0.151	0.079		0.946	0.935	0.930	0.939	0.942	0.946	
	0.848	0.857	0.874	0.883	0.903			0.9	0.032		0.151	0.079		0.946	0.935	0.930	0.939	0.942	0.946	
P1	2.158	0.687	0.389	0.263	0.189	0.1	P2	0.1	0.3	0.5	0.7	0.9		P1	0.1	0.3	0.5	0.7	0.9	
	2.285	0.723	0.404	0.267	0.190	0.055		0.1	0.139	0.337	0.175	0.103			0.1	0.139	0.337	0.175	0.103	

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Table 4
Confidence limit with using direct and inverse binomial sampling method when the true value of variance is used(formula 4)

		n=30,m=50p2					
		n=30,m=30p2					
P1 \ P2		0.9	0.7	0.5	0.3	0.1	
0.1	0.847	0.864	0.869	0.859	0.751		
	0.241	0.309	0.429	0.72	1.898		
	0.238	0.314	0.454	0.808	2.163		
0.061	0.061	0.093	0.197	0.509	1.398		
	0.934	0.921	0.914	0.856	0.937		
	0.362	0.486	0.701	1.188	0.251	0.9	
0.3	0.365	0.491	0.718	1.241	0.251		
	0.044	0.088	0.168	0.4	0.043		
	0.937	0.927	0.892	0.928	0.936		
0.5	0.404	0.568	0.846	0.465	0.309	0.7	
	0.411	0.578	0.874	0.47	0.31		
	0.039	0.095	0.198	0.065	0.03		
0.7	0.93	0.923	0.9	0.931	0.941		
	0.397	0.593	0.669	0.445	0.316	0.5	
	0.4	0.607	0.676	0.451	0.32		
0.051	0.051	0.112	0.116	0.056	0.022		
	0.943	0.864	0.922	0.928	0.938		
	0.318		0.939	0.551	0.38	0.285	0.3
0.9	0.32		0.964	0.558	0.382	0.285	
	0.075		0.242	0.1	0.052	0.026	
		n=50,m=50p2					
		n=50,m=30p2					
P1 \ P2		0.9	0.7	0.5	0.3	0.1	
		n=50,m=30p2					
		n=50,m=50p2					
P1 \ P2		0.9	0.7	0.5	0.3	0.1	
		n=50,m=30p2					
		n=50,m=50p2					

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Confidence limit with using direct and inverse binomial sampling method
when the true value of variance is used(formula 4) (Continued)

		n=30,m=100p2					n=50,m=100p2					n=100,m=30p2					n=100,m=50p2											
		P1					P2					P1					P2					P1						
P2	P1	0.9	0.7	0.5	0.3	0.1	P1	0.9	0.7	0.5	0.3	0.1	P1	0.9	0.7	0.5	0.3	0.1	P1	0.9	0.7	0.5	0.3	0.1	P2			
0.1	0.841	0.859	0.864	0.871	0.847		0.1	0.89	0.895	0.905	0.896	0.847		0.1	0.893	0.895	0.905	0.906	0.847		0.1	0.893	0.895	0.905	0.906	0.847		
0.239	0.308	0.429	0.711	0.991			0.237	0.183	0.235	0.331	0.549	1.586		0.181	0.233	0.331	0.56	1.663		0.237	0.183	0.235	0.331	0.549	1.586			
0.237	0.308	0.439	0.771	2.068			0.057	0.08	0.146	0.378	0.896		0.057	0.036	0.05	0.078	0.173	0.626		0.057	0.08	0.146	0.378	0.896				
0.3	0.927	0.924	0.921	0.911	0.9		0.3	0.939	0.935	0.929	0.903	0.923		0.3	0.283	0.371	0.529	0.892		0.3	0.939	0.935	0.929	0.903	0.923			
0.363	0.471	0.664	1.12	0.263	0.9		0.358	0.469	0.664	1.126	0.255		0.056	0.095	0.209	0.072	0.022	0.038	0.069	0.161		0.363	0.471	0.664	1.12	0.263	0.9	
0.358	0.469	0.664	1.126	0.255			0.035	0.056	0.095	0.209		0.5	0.946	0.935	0.919	0.915	0.943		0.5	0.946	0.935	0.919	0.915	0.943				
0.397	0.525	0.753	0.461	0.267	0.7		0.397	0.525	0.753	0.461	0.267	0.7	0.5	0.312	0.42	0.614	0.392		0.5	0.946	0.935	0.919	0.915	0.943				
0.397	0.526	0.759	0.468	0.263			0.018	0.04	0.084	0.108	0.048		0.018	0.04	0.084	0.108	0.013	0.033	0.07		0.397	0.525	0.753	0.461	0.267	0.7		
0.934	0.936	0.925	0.897	0.942			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.397	0.525	0.753	0.461	0.267	0.7		0.397	0.526	0.759	0.468	0.263		0.018	0.04	0.084	0.108	0.013	0.033	0.07		0.397	0.525	0.753	0.461	0.267	0.7		
0.5	0.936	0.936	0.925	0.897	0.942		0.5	0.946	0.935	0.919	0.915	0.943		0.5	0.312	0.42	0.614	0.392		0.5	0.946	0.935	0.919	0.915	0.943			
0.525	0.753	0.461	0.267	0.7			0.526	0.759	0.468	0.263		0.018	0.04	0.084	0.108	0.013	0.033	0.07		0.525	0.753	0.461	0.267	0.7		0.525		
0.526	0.759	0.468	0.263				0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93	0.928	0.862	0.912	0.942		0.93			
0.526	0.759	0.468	0.263	0.7			0.93																					

Confidence limits with using direct and inverse binomial samplings method
when the true value of variance is used (formula 4) (Continued)

		n=100 , m=100p2				
P1 \ P2		0.9	0.7	0.5	0.3	0.1
0.1	0.917	0.900	0.905	0.899	0.895	
	0.135	0.187	0.285	0.502	1.611	
	0.135	0.190	0.290	0.516	1.647	
0.3	0.027	0.052	0.088	0.169	0.559	
	0.941	0.937	0.940	0.933		
	0.211	0.301	0.464	0.833		
0.5	0.212	0.305	0.468	0.841		
	0.021	0.044	0.077	0.151		
	0.942	0.942	0.941			
0.7	0.235	0.350	0.550			
	0.237	0.352	0.552			
	0.018	0.040	0.070			
0.9	0.948	0.945				
	0.228	0.359				
	0.229	0.361				
	0.019	0.037				
	0.948					
	0.183					
	0.183					
	0.022					

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Table 5
Confidence limit with using direct and inverse binomial sampling method when the asymptotic variance is used (formula3)

		n=30,m=30p2						n=30,m=50p2						n=30,m=50p2					
		P2			P1			P2			P1			P2			P1		
P1 \ P2	0.9	0.7	0.5	0.3	0.1	P2	P1	0.9	0.7	0.5	0.3	0.1	P2	P1	0.9	0.7	0.5	0.3	0.1
0.1	0.866	0.927	0.935	0.942	0.923	P1	0.1	0.788	0.824	0.888	0.929	0.933	P1	0.1	0.788	0.824	0.888	0.929	0.933
	0.255	0.368	0.583	1.221	5.574		0.1	0.195	0.279	0.453	0.961	4.507		0.1	0.195	0.279	0.453	0.961	4.507
	0.252	0.368	0.597	1.312	6.279		0.06	0.182	0.572	4.099	0.045	0.068	0.125	0.393	0.125	0.045	0.068	0.125	0.393
0.3	0.93	0.943	0.951	0.955	0.637	P1	0.3	0.855	0.864	0.892	0.928	0.703	P1	0.3	0.855	0.864	0.892	0.928	0.703
	0.381	0.536	0.847	1.79	0.125		0.09	0.29	0.396	0.627	1.364	0.191		0.29	0.396	0.627	1.364	0.191	0.191
	0.375	0.535	0.851	1.858	0.126		0.036	0.07	0.156	0.523	0.05	0.023	0.038	0.087	0.3	0.284	0.394	0.629	1.384
0.5	0.919	0.904	0.889	0.757	0.884	P1	0.5	0.84	0.781	0.775	0.873	0.953	P1	0.5	0.84	0.781	0.775	0.873	0.953
	0.4	0.54	0.839	0.317	0.268		0.399	0.542	0.848	0.303	0.262	0.7		0.399	0.542	0.848	0.303	0.262	0.7
	0.02	0.059	0.148	0.072	0.031		0.02	0.072	0.031	0.016	0.052	0.112		0.02	0.072	0.031	0.016	0.052	0.112
0.7	0.879	0.744	0.894	0.909	0.929	P1	0.7	0.741	0.49	0.944	0.963	0.979	P1	0.7	0.741	0.49	0.944	0.963	0.979
	0.349	0.418	0.66	0.419	0.309		0.336	0.396	0.666	0.42	0.309	0.5		0.336	0.396	0.666	0.42	0.309	0.5
	0.053	0.111	0.087	0.033	0.011		0.777	0.965	0.961	0.954	0.94	0.9		0.777	0.965	0.961	0.954	0.94	0.9
0.9	0.188	1.424	0.663	0.414	0.293	P1	0.9	0.658	0.97	0.98	0.985	0.98	P1	0.9	0.658	0.97	0.98	0.985	0.98
	0.184	1.452	0.668	0.414	0.291		0.075	0.318	0.093	0.04	0.02	0.069		0.075	0.318	0.093	0.04	0.02	0.069
	0.075						0.972	0.965	0.953	0.947	0.898	0.1		0.972	0.965	0.953	0.947	0.898	0.1
		4.669	0.951	0.449	0.279	0.192	0.1	5.062	0.978	0.45	0.276	0.19	P1	2.405	0.294	0.108	0.058	0.038	P2
		0.1	0.3	0.5	0.7	0.9	P2						P1	0.1	0.3	0.5	0.7	0.9	P2
		n=50,m=50p2						n=50,m=30p2						n=50,m=30p2					

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Confidence limit with using direct and inverse binomial sampling method when the true value of variance is used (formula 3) (Continued)

		n=30,m=100p2						n=50,m=100p2						
		P2			P1			P2			P1			P2
P1 \ P2	0.9	0.7	0.5	0.3	0.1	P2	P1	0.1	0.9	0.7	0.5	0.3	0.1	P2
0.1	0.738	0.743	0.768	0.851	0.93	P1	0.1	0.837	0.86	0.888	0.938	0.972	P1	P1
	0.136	0.193	0.313	0.676	3.303		0.1	0.136	0.194	0.316	0.673	3.423		
	0.133	0.189	0.312	0.692	3.425		0.1	0.133	0.191	0.314	0.68	3.570		
	0.03	0.042	0.074	0.232	1.494		0.026	0.037	0.067	0.173	1.33			
	0.7	0.69	0.715	0.808	0.875		0.3	0.824	0.842	0.869	0.92	0.978		
	0.196	0.251	0.389	0.871	0.223		0.200	0.202	0.276	0.44	0.964	0.152		
0.3	0.192	0.248	0.385	0.874	0.217	P1	0.217	0.200	0.275	0.441	0.972	0.146	P1	P1
	0.012	0.019	0.045	0.135	0.046		0.217	0.011	0.017	0.04	0.145	0.041		
	0.629	0.494	0.433	0.943	0.99		0.5	0.794	0.743	0.725	0.895	0.966		
	0.193	0.198	0.286	0.52	0.376		0.7	0.209	0.257	0.386	0.379	0.284		
	0.185	0.184	0.277	0.527	0.376		0.046	0.205	0.249	0.375	0.38	0.282		
	0.022	0.06	0.108	0.062	0.025		0.046	0.008	0.032	0.07	0.038	0.017		
0.5	0.468	0.647	0.976	0.992	0.997	P1	0.697	0.414	0.414	0.958	0.977	0.987	P1	P1
	0.128	0.235	0.959	0.596	0.418		0.7	0.161	0.146	0.721	0.449	0.317		
	0.124	0.24	0.972	0.602	0.419		0.5	0.154	0.147	0.728	0.451	0.319		
	0.045	0.109	0.169	0.068	0.023		0.023	0.034	0.063	0.095	0.037	0.012		
	0.793	0.982	0.994	0.997	0.998		0.9	0.673	0.981	0.989	0.992	0.990		
	0.18	1.895	0.891	0.557	0.384		0.3	0.109	1.477	0.690	0.425	0.295		
0.9	0.173	1.955	0.907	0.563	0.386	P1	0.028	0.112	1.5	0.695	0.427	0.296	P1	P1
	0.064	0.553	0.167	0.07	0.028		0.028	0.049	0.323	0.098	0.041	0.018		
	5.928	0.996	0.997	0.995	0.992		0.1	0.985	0.993	0.993	0.988	0.982		
	6.71	1.221	0.583	0.367	0.253		0.135	4.813	0.966	0.456	0.281	0.193		
	3.794	0.395	0.135	0.066	0.038		0.066	5.174	0.989	0.460	0.281	0.193		
	0.1	0.3	0.5	0.7	0.9		0.1	0.1	0.3	0.5	0.7	0.9	P1	P2
		n=100,m=30p2						n=100,m=50p2						P1

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length

Confidence limit with using direct and inverse binomial sampling method when the true value of variance is used (formula 3) (Continued)

		n=100 , m=100p2				
		0.9	0.7	0.5	0.3	0.1
P2 \ P1						
0.1	0.939	0.956	0.972	0.984	0.991	
	0.136	0.197	0.322	0.688	3.496	
	0.135	0.196	0.323	0.696	3.631	
	0.019	0.029	0.053	0.143	1.203	
0.3	0.947	0.952	0.964	0.976		
	0.207	0.292	0.473	1.026		
	0.206	0.292	0.475	1.037		
	0.010	0.019	0.047	0.160		
0.5	0.932	0.911	0.901			
	0.218	0.297	0.473			
	0.219	0.297	0.475			
	0.005	0.016	0.042			
0.7	0.889	0.753				
	0.188	0.221				
	0.186	0.215				
	0.014	0.036				
0.9	0.540					
	0.076					
	0.075					
	0.031					

From top to bottom: Coverage probabilities, median length, mean length and standard deviation of the length