THESIS APPROVAL

GRADUATE SCHOOL, KASETSART UNIVERSITY

Doctor of Philosophy (Statistics)

DEGREE

Statistics — **FIELD**

Statistics — **DEPARTMENT**

TITLE: Statistical Inference Based on a Stochastic Analysis of Insurance Claim Counts

NAME: Miss Uraiwan Jaroengeratikun

THIS THESIS HAS BEEN ACCEPTED BY

THESIS ADVISOR

(Assistant Professor Winai Bodhisuwan, Ph.D.)

THESIS CO-ADVISOR

(Mrs. Ampai Thongteeraparp, Ph.D.)

DEPARTMENT HEAD

(Associate Professor Pasit Payakkapong, M.S.)

APPROVED BY THE GRADUATE SCHOOL ON

DEAN

(Associate Professor Gunjana Theeragool, D.Agr.)

THESIS

STATISTICAL INFERENCE BASED ON A STOCHASTIC ANALYSIS OF INSURANCE CLAIM COUNTS

URAIWAN JAROENGERATIKUN

A Thesis Submitted in Partial Fulfillment of
the Requirements for the Degree of
Doctor of Philosophy (Statistics)
Graduate School, Kasetsart University
2013

Uraiwan Jaroengeratikun 2013: Statistical Inference Based on a Stochastic Analysis of Insurance Claim Counts. Doctor of Philosophy (Statistics), Major Field: Statistics, Department of Statistics. Thesis Advisor: Assistant Professor Winai Bodhisuwan, Ph.D. 110 pages.

The aim of this research is to propose an estimation approach to non-life insurance claim counts related to the claim counting process. The considered processes are a homogeneous Poisson process (HPP) and a non-homogeneous Poisson process (NHPP) with bell-shaped and beta-shaped intensities. The claim counts $N(t)$ and predicting claim counts can be obtained by the estimated parameter $\widehat{\Lambda(t)}$ of process using an estimating function via a zero mean martingale. The model parameters of claim intensity of process are estimated by using the maximum likelihood estimation (MLE) and the Bayesian estimation (BE). Some situations based on estimation and prediction of claim counts are studied by using the simulation technique.

This study bases on the insurance claim counting process. The study found that, for HPP, the $\widehat{\Lambda(t)}$ using the BE provides the best fit to $N(t)$ when the number of observations is slightly larger than a constant intensity rate. For NHPP with a bell-shaped intensity, using BE for predicting $N(t)$ performs very well when the number of observations is slightly larger than an average number of claims over a period of time. Where a beta-shaped intensity, using BE fits $N(t)$ very well when the two parameters p and q of claim intensity are slightly more than 1 and the number of observations is small; however, using MLE fits $N(t)$ very well when p and q are much more than 1 and the number of observations is equal to the value of the peak level of claims over a period of time. In addition, this research presents a procedure of claim count estimation and prediction, which are demonstrated by the examples of sample paths from simulated data. An application of the proposed approach to real insurance claim data from a non-life insurance company in Thailand is illustrated. These procedures can be a clear and useful guide to actuaries and researchers in their works.

____________________ ____________________ ___ / ___ / ___

Student's signature Thesis Advisor's signature

ACKNOWLEDGEMENTS

First of all, I would like to thank all the members of thesis committee for agreeing to serve on it. Acknowledgements are due to the anonymous referees who made various comments and suggestions which led to an improvement of this thesis.

I would like particularly to thank my thesis advisor, Assistant Professor Dr. Winai Bodhisuwan, to whom I am deeply indebted for his excellent advice, guidance, encouragement and valuable suggestions. I would also like to thank Dr. Ampai Thongteeraparp my co-advisor for her valuable comments which are much appreciated.

This thesis could not have been completed without the help of many practical discussions with Thai actuaries and the real data sets provided by an insurance company. I would like to express my deepest thanks to them.

I am sincerely grateful to the Department of Statistics at Kasetsart university for supporting me and providing a helpful environment throughout the period of my study in Ph.D. program. I want to express my warmest thanks to my family and friends for their encouragements during my work on this thesis.

Finally, I am forever grateful to my parents for the education and the opportunity they offered me to become a statistician and a lecturer at King Mongkut's University of Technology North Bangkok.

Uraiwan Jaroengeratikun
January 2013

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (Continued)

Figure **Page**

LIST OF FIGURES (Continued)

Figure **Page**

LIST OF FIGURES (Continued)

Figure **Page**

LIST OF FIGURES (Continued)

Figure **Page**

Appendix Figure

LIST OF FIGURES (Continued)

Appendix Figure **Page**

LIST OF FIGURES (Continued)

LIST OF FIGURES (Continued)

Appendix Figure **Page**

LIST OF ABBREVIATIONS

HPP	=	homogeneous Poisson process
NHPP	=	non-homogeneous Poisson process
MLE	=	maximum likelihood estimation
ZMM	=	zero mean martingale
BE	=	Bayesian estimation
iid	=	independent and identically distributed
GLM	=	generalized linear model
MCMC	=	Markov Chain Monte Carlo
MSE	=	mean squared error

STATISTICAL INFERENCE BASED ON A STOCHASTIC ANALYSIS OF INSURANCE CLAIM COUNTS

INTRODUCTION

Nowadays, insurance is a common way of managing risks and the insurance industry has grown rapidly over time. This business is concerned with financial services. If firms face ruin, this problem can in turn effect a country's economy. The business of insurance is classified into 2 groups, life insurance and non-life insurance. This research is concerned with the latter, the non-life insurance. Its products are only the contract which cover uncertain risks in the future with regard to property and casualty, for example, fire insurance, automobile insurance, crop insurance and other similar things. The task of an actuary is concerned in the assessment of risks, such as pricing, loss reserve, reinsurance planning, and investment planning, etc. These tasks are based on aggregate claim amounts occurring over a specified period and they consist of the amount of claims and the number of claims. The amount of claims and the number of claims (claim counts) can be referred to claim severity and frequency, respectively. Several studies have found that changes in claim amounts or claim counts effected the assessment of risks, and sometimes found changes in the aggregate claim amounts, for example, Jaroengeratikun and Bodhisuwan (2009) studied the parameter estimation method of aggregate claims under Pareto distribution which shows a probability model of the change in aggregate claim amounts. Frees and Wang (2006) developed credibility predictors of aggregate claim amounts and showed that predictors are important in explaining both the change in claim counts and claim amounts. Information of claim counts plays an important component of the aggregate claim amount model. In the past four decades or so, a few researchers have studied the modeling of claim counts for non-life insurance. Walhin and Paris (2000); Denuit *et al.* (2007); Klugman *et al.* (2008) were interested in studying the frequency distribution. Bühlman (1967); Bühlmann (1972) presented the credibility approach in the form of linear function to estimate and predict the expected claim counts in upcoming periods, using past experience of claims as a risk class or related risk classes. Bülhman's approach is interesting and can be extended to other approaches, such as the Bülhman-Straub model, Jewell's model or the Exact credibility approach,etc. (see Ward, 1997;

Bühlmann and Gisler, 2005; Klugman *et al.*, 2008). Calculating the expected claim counts using the credibility approach depends on the information from prior experience of claim counts and does not consider the occurrence behavior of claim counts over time. In these approaches, there may not be sufficient for estimating the claim counts for analyzing the assessment of risks. So, the actuary should know how the occurrence behavior of claim counts changes over time and this is called the claim counting process. Some researchers have considered an approach to claim counts relating to a specified time, for example, Kasozi and Paulsen (2005); Mikosch (2009) viewed the claim counting process as a homogeneous Poisson process (HPP) through the Cramer-Lundberg model, one of the most popular and useful risk models in non-life insurance, and Matsui and Mikosch (2010) also considered a Poisson cluster model for the modeling of total claim amounts by a point of claim counts as an HPP with a constant rate of occurrence called the constant intensity. For some non-life insurance portfolios, the claim counts during a time period are caused by periodic phenomena or seasonality. These claim counts are modeled in term of a non-homogeneous Poisson process (NHPP) with a period time-dependent intensity rate. Morales (2004) presented a periodic risk model consisting of the claim counting process with a bell-shaped intensity function. The unknown parameters of periodic intensity were estimated by the method of maximum likelihood estimation (MLE) and the ruin probability was evaluated through a simulation study. Furthermore, Lu and Garrido (2005) explored the periodic NHPP model with a beta-shaped intensity function.

The precision of estimating and predicting claim counts is the key to running insurance businesses successfully. In this thesis, we present an estimation approach to non-life insurance claim counts related to a specification of the two different claim counting processes, i.e. HPP and NHPP. Our purpose is to estimate the parameter of the non-life insurance claim counting process, claim intensity function $\lambda(t)$ in term of mean value function, $\Lambda(t) = \int_0^t \lambda(u)du$, which makes a complicated distribution function of insurance claim counts. An estimating function which is provided by the martingale method, namely the zero mean martingale (ZMM), is used here as a procedure for the parameter estimation of an insurance claim count model, and the model parameters of claim intensities are estimated by using the method of MLE and Bayesian estimation (BE). In addition, according to this approach, the estimated parameter, $\widehat{\Lambda(t)}$, of process can be interpreted as the insurance claim counts, $N(t)$, during the time interval $(0,t]$. Such an estimate is useful for predicting the claim counts for the further periods. We also illustrate the procedure of estimating and predicting the claim counts by the example of simulated data and real insurance claim data which actuaries and researchers can be applied.

Publications

This thesis was produced under the guidance of my thesis advisor, Dr Winai Bodhisuwan, and my co-thesis advisor, Dr Ampai Thongteeraparp. The results in Part 1.1 and 1.2 in Chapter of Results and Discussion have been published in Applied Mathematics (Jaroengeratikun *et al.*, 2012b) and also the Part 2.2 and 2.3 have been published in Open Journal of Statistics (Jaroengeratikun *et al.*, 2012a).

OBJECTIVES

1. To study an approach to claim count estimation on the non-life insurance claim counting process using an estimating function provided by the martingale method, zero mean martingale (ZMM).

2. To estimate and predict claim counts on the insurance claim counting process.

3. To present some examples of how to estimate and predict claim counts in order to provide a guide to actuaries and researchers.

Scope of the Study

The estimation and the prediction of non-life insurance claim counts are based on the Poisson claim counting process in which the process is as follows:

1. Homogeneous Poisson Process

In this thesis, the parameter of $N(t)$, denoting the cumulative claim counts occurring in time period $t \geq 0$, is estimated using an estimating function which is provided by the martingale method, namely the ZMM.

2. Non-Homogeneous Poisson Process

We study the behavior of claim intensity over time t, and some manners will be investigated. The claim intensities are as follows,

2.1 Bell-shaped intensity

$$\lambda(t) = \frac{\lambda^* \exp\left\{-\frac{1}{2\sigma^2}\left(t-\frac{1}{2}\right)^2\right\}}{\left(\Phi(\frac{1}{2\sigma}) - \Phi(-\frac{1}{2\sigma})\right)\sigma\sqrt{2\pi}}, \tag{1}$$

$t \in [0,1)$, $\lambda(s+t) = \lambda(t)$, $s = 0,1,2,...$, $\lambda^* > 0$ and $\sigma > 0$, where λ^*, σ and s are the model parameters of claim intensity, Φ is the standard normal distribution function. Here s is an initial season defined as $s = 0$, λ^* is an average number of claims over a period and σ is the variability of the season (Morales, 2004).

2.2 Beta-shaped intensity

$$\lambda(t) = \frac{\lambda^* \left(\frac{t-[t]-m_1}{D}\right)^{p-1}\left(1-\frac{t-[t]-m_1}{D}\right)^{q-1}}{\alpha^*}, \tag{2}$$

for $D = m_2 - m_1$, $\alpha^* = \left(\frac{t^*-m_1}{D}\right)^{p-1}\left(1-\frac{t^*-m_1}{D}\right)^{q-1}$ is the scale factor, while $t^* = m_1 + \frac{D(p-1)}{p+q-2}$ is the mode of distribution, $\lambda^* > 0$, p, $q \geq 1$ and $0 \leq m_1 < m_2 \leq 1$, where λ^*, p and q are the parameters, $[\cdot]$ is the greatest integer function, m_1 and m_2 represent the starting and ending point of the occurrence intervals, respectively. Here p and q are set to $p = q$,the m_1 and m_2 are set respectively to 0 and 1 (Garrido and Lu, 2004; Lu and Garrido, 2005).

Both the bell-shaped and the beta-shaped intensities are depicted in Figure 1. In Figure 1(a) the claims occurrence in the tail of the period, i.e. left and right tail of period, changes slowly. While the beta-shaped intensity is shown in Figure 1(b), the claims occurrence in the left and right tail of the period changes quickly.

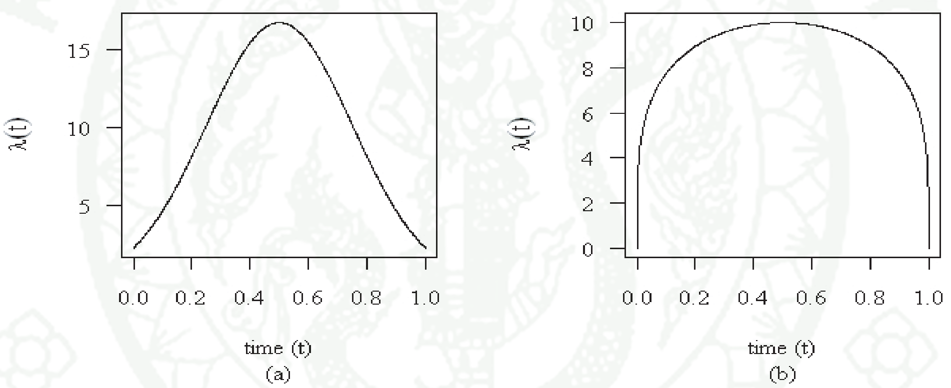


Figure 1 Non-life insurance claim intensity function, (a) bell-shaped intensity $\lambda(t) = \frac{42}{\sqrt{2\pi}}\exp\left\{-8(t-1/2)^2\right\}$ and (b) beta-shaped intensity $\lambda(t) = 10\sqrt{2}(t-[t])^{1/4}(1-t+[t])^{1/4}$ where $p = q$

In this thesis, the model parameters of claim intensities are estimated by using MLE and BE. For each process, the martingale method, such as a ZMM, is used to set up an estimating equations and then this procedure is connected to estimate the important parameter of claim counting process.

LITERATURE REVIEW

This chapter is divided into two topics: the literature review and the statistical methods used in this thesis.

1. Literature Review

The research studies related to the evaluation of the probability model of claim counts, the estimation and the prediction of claim counts in the field of non-life insurance are as follows:

1.1 The credibility approach

Table 1 The schematic relation of the papers on the credibility approach

Approach	Unweighted case	Weighted case	Weighted with a priori differences
Linear credibility	Bühlman (1967)	Ward (1997)	Bühlmann and Gisler (2005)
Exact credibility	Jewell (1974)	Kaas *et al.* (1997)	Ohlsson and Johansson (2006)

From Table 1, it can be seen that Bühlman (1967) presented the credibility approach which is in the form of a linear function to predict the claim counts in the following periods, i.e. the $\mu(\theta_j) = E(N_{j(t+1)}|\theta_j)$ is estimated, where $N_{j(t+1)}$ is a random variable representing the claim counts of the j^{th} insurance contract during the $(t+1)^{th}$ period with the parameter θ_j. The N_j is independent and identically distributed (iid) with mean $\mu(\theta_j) = E(N_{js}|\theta_j)$ and covariance $Cov(N_{jr}, N_{js}|\theta_j) = \delta_{rs}\sigma^2(\theta_j)$, where $\delta_{rs} = \begin{cases} 1 & ;r = s \\ 0 & ;r \neq s. \end{cases}$, $\sigma^2(\theta_j) = Var(N_{js}|\theta_j)$, and the structure parameter is defined as,

1. $m = E(N_{jr}) = E(\mu(\Theta_j)) = E(N_{js}|\Theta_j)$,

2. $a = Var(\mu(\Theta_j)) = Var(N_{js}|\Theta_j)$,

3. $s^2 = E(\sigma^2(\Theta_j))$.

When the risk exposure or the number of policies of j^{th} contract in each period s is 1, where $j = 1,2,3,\ldots,k$ and $s = 1,2,3,\ldots,t$. The best $g^*(N_{j1},\ldots,N_{jt}) = \hat{E}(\mu(\theta_j)|N_{j1},\ldots,N_{jt}) = \hat{E}(N_{j(t+1)}|N_{j1},\ldots,N_{jt})$ is the estimator that minimizes $E\left\{\mu(\theta_j) - g(N_{j1},\ldots,N_{jt})\right\}^2$. The $g^*(N_{j1},\ldots,N_{jt})$ in the form of a linear function is $g^*(N_{j1},\ldots,N_{jt}) = \hat{E}(N_{j(t+1)}|N_{j1},\ldots,N_{jt}) = \hat{\mu}(\theta_j) = (1 - z)m + z\bar{N}_j$, where $z = \frac{at}{s^2+at}$ is the credibility factor, and $\bar{N}_j$ is the sample mean of N_j.

Ward (1997) reviewed the credibility approach which was developed by Bühlmann and Straub in 1970. The Bühlmann-Straub model is the Bühlmann model in which the claim counts relate to the weighted case with the risk exposure or the number of policies of j^{th} contract in each period s, w_{js}, where $j = 1,2,3,\ldots,k$ and $s = 1,2,3,\ldots,t$. The $g^*(N_{j1},\ldots,N_{jt})$ in the form of a linear function is $g^*(N_{j1},...,N_{jt}) = \hat{E}(N_{j(t+1)}|N_{j1},...,N_{jt}) = \hat{\mu}(\theta_j) = (1 - z_j)m + z_j N_{jw}$, where $z = \frac{a \cdot w_{j\cdot}}{s^2 + a \cdot w_{j\cdot}}$, $N_{jw} = \sum_{r=1}^{t} \frac{w_{jr} N_{jr}}{w_{j\cdot}}$, and $w_{j\cdot} = \sum_{r=1}^{t} w_{jr}$.

Bühlmann and Gisler (2005) developed the Bühlmann-Straub model which is the weighted case with a risk exposure or the number of policies of the j^{th} contract in each period s , and w_{js}, is considered where $j = 1,2,3,\ldots,k$ and $s = 1,2,3,...,t$. Also, the weighted case with priori differences where the different contracts may have the different priori means, denoted by a_j. Let $X_{jr} = \frac{N_{jr}}{a_j}$, and the transformation N_{jr} to X_{jr}. The $g^*(N_{j1},...,N_{jt})$ in form of linear function is $g^*(N_{j1},...,N_{jt}) = \hat{E}(N_{j(t+1)}|N_{j1},...,N_{jt}) = a_j\hat{\mu}(\theta_j) = (1 - z_j)m_j + z_j X_{jw}$, where $m_j = a_j m$, $z = \frac{a_j \cdot w_{j\cdot}}{\frac{s^2}{a_j \cdot w_{j\cdot}} + a_j \cdot w_{j\cdot}}$, and $w_{j\cdot} = \sum_{r=1}^{t} w_{jr}$.

Jewell (1974) presented the exact credibility approach when the risk exposure or the number of policies of the j^{th} contract in each period s is equal to 1, where $j = 1,2,3,...,k$ and $s = 1,2,3,...,t$. The $\mu(\theta_j) = E(N_{j(t+1)}|\theta_j)$, estimate is given on the exponential family with a natural conjugate prior. Solving $\mu(\theta_j) =$

$E(N_{j(t+1)}|\theta_j)$ estimate could be rewritten in the form of a linear function, and its result and the Bühlmann model are not different.

Kaas *et al.* (1997) extended Jewell's approach to consider the claim counts in relation to the weighted case with a risk exposure or the number of policies of the j^{th} contract in each period s, w_{js}, where $j = 1,2,3,...,k$ and $s = 1,2,3,...,t$. The result of $\mu(\theta_j) = E(N_{j(t+1)}|\theta_j)$ is an estimate derived in the form of a linear function which is not different from the Bühlmann-Straub model.

Ohlsson and Johansson (2006) extended Jewell's approach to consider the claim counts in relation to a weighted case with a risk exposure or the number of policies of the j^{th} contract in each period s, w_{js}, where $j = 1,2,3,\ldots,k$ and $s = 1,2,3,\ldots,t$. The $\mu(\theta_j) = E(N_{j(t+1)}|\theta_j)$ estimate is given on the exponential family with a natural conjugate prior. There is an important situation where one has a number of ordinary rating factors alongside the factor estimated by the credibility approach. The number of ordinary rating factors might be estimated by the standard generalized linear model (GLM) procedure. For example, the ordinary rating factors in the private automobile insurance would be sex, age of driver, age of car, mileage per year, or power of engine, etc.

1.2 The probability model

Aggoun (2004) estimated and predicted the claim rates for a heterogeneous portfolio or different groups using a recursive estimate under the measured change techniques and the Bayesian method. All processes are defined with a probability space $(\Omega, \mathcal{F}, P)$, the claim rates per policy in the c^{th} group during the n^{th} year, δ_n^c , is a random variable with the assumption that the prior distribution of δ_n^c is a Gamma, the claim counts reported by the c^{th} group during the n^{th} year, Y_n^c, with $Y_n^c \sim Poisson(N_n^c \delta_n^c)$, where N_n^c is the number of policies purchased in the c^{th} group during the n^{th} year, such that $N_n^c \sim Poisson(\mu_n^c)$, μ_n^c is known. So the posterior distribution of the claim rates is a Gamma which is close to a normal distribution when the parameter value of the posterior distribution is large enough. The claim rates for a heterogeneous portfolio is given by $\Lambda_n = \prod_{m=0}^{n} \lambda_m$, where $\lambda_m = \prod_{c=1}^{G} e^{(1-\delta_m^c)}(\delta_m^c)^{y_m^c} e^{(1-\mu_m^c)}(\mu_m^c)^{N_m^c}$, $\lambda_0 = 1$.

On the measure P^{+}, the processes Y_n^c, N_n^c, $c = 1,2,3,\ldots,G$, are sequences of iid Poisson with rate one. The estimation of the claim rates $E^{+}[f(\delta_n^1,\ldots,\delta_n^G)\Lambda_n|\mathscr{F}_n]$, is derived where $f(\delta_n^1,\ldots,\delta_n^G)$ is a test function, and the prediction of the claim rates is $E^{+}[f(\delta_n^1,\ldots,\delta_n^G)\Lambda_{n+1}|\mathscr{F}_n]$.

1.3 The stochastic process

Walhin (2000); Walhin and Paris (2006) presented an evaluation of the probability model for $N(t)$, the claim counting process during the time interval $(0,t]$, using the recursive algorithm or Panjer's algorithm while $N(t) = N_1(t) + N_2(t)$ is an infinitely divisible number mixed with the Poisson process, where $N_1(t)$ is a purely random claim and the HPP claim counting process with mean δt, and $N_2(t)$ is related to the behavior of the driver reported that the claims have a Hofman distribution. Furthermore, their work proposed the claim intensity of process, $E[N(t+h)$ - $N(t)|N(t) = k]$, in which the parameters are estimated by the MLE, and the claim intensity is used the prediction of claim counts.

Morales (2004) studied estimating the claim counts on the two different claim counting processes in the surplus process of risk models, called a ruin probability model, denoted $N(t)$ as the claim counts during the time interval $(0,t]$, and he stated that the first claim counts $N(t)$ is the HPP claim counting process with mean per unit time, λ^*,estimated by an average number of claims over a year $\frac{\Lambda(c)}{c}$, where c is a period, and the second $N(t)$ is an NHPP claim counting process with the periodic bell-shaped intensity of the form

$$\lambda(t) = \frac{\lambda^* \exp\left\{-\frac{1}{2\sigma^2}\left(1-\frac{1}{2}\right)^2\right\}}{\left(\Phi(\frac{1}{2\sigma}) - \Phi(-\frac{1}{2\sigma})\right)\sigma\sqrt{2\pi}},$$

for $t \in [0,1)$, $\lambda(s+t) = \lambda(t)$, s is an initial season, λ^* is an average number of claims over a period, σ is a variable of season and Φ is the standard normal distribution function where $\lambda^* > 0$, $\sigma > 0$ and $s = 0,1,\ldots$. The estimator of the parameters of periodic bell-shaped intensity could be obtained from the raw data using standard techniques for a normal distribution. Another form of the claim intensity which is easier to implement is the integrated claim intensity or the mean value function for an initial season $s = 0$ given by

$$\Lambda(t) = [t]\lambda^* + \frac{\lambda^* \left\{ \Phi\left(\frac{t-[t]-0.5}{\sigma}\right) - \Phi\left(-\frac{1}{2\sigma}\right) \right\}}{\Phi(\frac{1}{2\sigma}) - \Phi(-\frac{1}{2\sigma})}, \tag{3}$$

where $[\cdot]$ is the greatest integer function.

Garrido and Lu (2004); Lu and Garrido (2005) studied the occurrence behavior of claims during a time period caused by periodic phenomena or seasonality and modeled in term of an NHPP with singly and doubly periodic claim intensity rates. These processes have been proposed as claim counts in the risk model. As the periodic claim intensity rate is a function of time, the NHPP in the risk models is more realistic in practice than the HPP claim counting process. Hence, their study considers the periodic claim intensities with beta-type intensity functions and uses the MLE to estimate the parameters of claim intensity. The short-term intensity is the single beta periodic claim intensity given the following:

$$\lambda(t) = \frac{\lambda^* \left(\frac{t-[t]-m_1}{D}\right)^{p-1} \left(1 - \frac{t-[t]-m_1}{D}\right)^{q-1}}{\alpha^*},$$

for $D = m_2 - m_1$, $\alpha^* = \left(\frac{t^*-m_1}{D}\right)^{p-1} \left(1 - \frac{t^*-m_1}{D}\right)^{q-1}$ is the scale factor, while $t^* = m_1 + \frac{D(p-1)}{p+q-2}$ is the mode of function, $\lambda^* \geq 0$, $p, q \geq 1$ and $0 \leq m_1 < m_2 \leq 1$, where λ^*, p and q are the parameters, $[\cdot]$ is the greatest integer function, m_1 and m_2 represent the starting and ending point of the occurrence intervals, respectively, and λ^* is the peak level for claim intensity. So, the mean value function is

$$\Lambda(t) = \frac{\lambda^* D B_p \left(\frac{t-[t]-m_1}{D}\right)^{p-1} \left(1 - \frac{t-[t]-m_1}{D}\right)^{q-1}}{\alpha^*}, \tag{4}$$

where $B_p = [t]B(p,q) + B\left(p,q;\frac{t-[t]-m_1}{D}\right)$, $B(p,q) = \int_0^1 v^{p-1}(1-v)^{q-1}dv$, and

$B(p,q;t) = \int_0^t v^{p-1}(1-v)^{q-1}dv.$

Also, the double periodic claim intensity in form of the beta claim intensity is given by

$$\lambda(t) = \lambda_1(t-[t])\lambda_c\left([t] - \left[\frac{t}{c}\right]c + t_1^*\right); t \geq 0,$$

where $[t]$ is the integer part of t, c is an number of years, and where $\lambda_1(t) = \frac{\left(\frac{t-m_1}{D}\right)^{p_1-1}\left(1-\frac{t-m_1}{D}\right)^{q_1-1}}{\alpha_1^*}$; $0 \leq m_1 \leq t \leq m_2 \leq 1$, would be a beta-type function with parameters $p_1, q_1 \geq 1$, defined on $[0,1]$, $D = m_2 - m_1$, a scale factor is $\alpha_1^* = \left(\frac{t_1^*-m_1}{D}\right)^{p_1-1}\left(1-\frac{t_1^*-m_1}{D}\right)^{q_1-1}$, $t_1^* = m_1 + D\frac{p_1-1}{p_1+q_1-2}$ is the mode of λ_1, so the mode, $\lambda_1(t_1^*) = 1$, is the peak level, and a long-term beta function

$$\lambda_c(t) = a + \frac{b-a}{\alpha_c^*}\left(\frac{t-m_c}{c} - \left[\frac{t-m_c}{c}\right]\right)^{p_c-1}\left[1-\left(\frac{t-m_c}{c} - \left[\frac{t-m_c}{c}\right]\right)\right]^{q_c-1}; t > 0,$$

is the claim intensity with parameters p_c, and $q_c \geq 1$, a and b are the minimum and maximum amplitudes of the peak values, respectively, while $\alpha_c^* = \left(\frac{t_c^*-m_c}{c}\right)^{p_c-1}\left(1-\frac{t_c^*-m_c}{c}\right)^{q_c-1}$, here m_c is the starting point of the complete cycle of the long-term beta intensity and $t_c^* = m_c + c\left(\frac{p_c-1}{p_c+q_c-2}\right)$ denotes the mode of λ_c. This double beta periodic claim intensity can be written in the form of a mean value function as follows:

$$\Lambda(t) = \left[\frac{t}{c}\right]DB(p_1,q_1)\sum_{j=0}^{c-1}\frac{\lambda_c(j+t_1^*)}{\alpha_1^*} + DB(p_1,q_1)\sum_{j=0}^{\left[t-\left[\frac{t}{c}\right]c\right]-1}\frac{\lambda_c(j+t_1^*)}{\alpha_1^*}$$
$$+DB\left(p_1,q_1;\frac{t-[t]-m_1}{D}\right)\frac{\lambda_c\left(\left[t-\left[\frac{t}{c}\right]c\right]+t_1^*\right)}{\alpha_1^*},$$

where $t \geq m_1$ and $\Lambda(t) = 0$ for $0 \leq t \leq m_1$.

The corresponding claim counting process $\{N(t), t \geq 0\}$ or $\{N_t, t \geq 0\}$ with double beta periodic claim intensity can be decomposed as $N_t = M_1 + \ldots + M_{\left[\frac{t}{c}\right]} + N^*_{\frac{t-[t]-m_1}{D}}$, where $\{M_i, i \geq 1\}$ are iid Poisson with mean $DB(p_1,q_1)\sum_{j=0}^{c-1}\frac{\lambda_c(j+t_1^*)}{\alpha_1^*}$.

They are independent of $N_c^{(j)}$ and $N_{\frac{t-[t]-m_1}{D}}^{\left(\left[t-\left[\frac{t}{c}\right]c\right]\right)}$ where $j = 0, 1, ..., \left[t - \left[\frac{t}{c}\right]c\right] - 1$. The $N_c^{(j)}$ and $N_{\frac{t-[t]-m_1}{D}}^{\left(\left[t-\left[\frac{t}{c}\right]c\right]\right)}$ are also Poisson with mean $DB(p_1, q_1)\frac{\lambda_c(j+t_1^*)}{\alpha_1^*}$ and $DB\left(p_1, q_1; \frac{t-[t]-m_1}{D}\right)\frac{\lambda_c\left(\left[t-\left[\frac{t}{c}\right]c\right]+t_1^*\right)}{\alpha_1^*}$, respectively.

In this thesis, we propose an estimation approach to claim counts related to a specified time or their behavior over time with two different counting processes, i.e. the HPP and the NHPP claim counting processes. The NHPP is characterized by bell-shaped and beta-shaped intensities. Then, the estimating function provided by the theory of martingale is used to estimate and make an inference of the claim counts.

2. Statistical Methods

The focus of this thesis has presented an estimation approach to the claim counts on the non-life insurance claim counting process, using both the HPP and the NHPP claim counting processes. We apply the statistical methods, including the Poisson process, estimating function, theory of martingale and the parameter estimation of claim intensity as these methods are useful for a stochastic analysis of claim counts on the claim counting process.

2.1 The Poisson process

There are two features of the Poisson process, which are the homogeneous Poisson process and the non-homogeneous Poisson process. These are defined as follows:

2.1.1 The homogeneous Poisson process

Definition 1: The stochastic process is a collection of random variables $\{N(t), t \in T\}$ indexed by the real-valued parameter t taking values in the index set T. Usually, T represents a set of observation times. The stochastic process $\{N(t), t \geq 0\}$ is said to be a counting process if $t \mapsto N(t)$ is right-continuous and $N(t) - N(t-)$ is 0 or 1. Intuitively speaking, $N(t)$ represents the total number of events that have occurred up to time t. Such a counting process $N(t)$ must satisfy: (i) $N(t) \geq 0$, (ii) $N(t)$ is the integer value, (iii) if $s < t$, then $N(s) \leq N(t)$, and (iv) for $s < t$, then $N(t) - N(s)$ equals the number of events that have occurred in the interval $(s,\ t]$. The counting process $\{\ N(t), t \geq 0\}$ is said to be a Poisson process with the intensity rate λ, $\lambda > 0$, if (Ross, 1993; Denuit *et al.*, 2007; Mikosch, 2009)

(a) $N(0) = 0$,

(b) The process has stationary increments, that is,

$$Pr\left[N(t+\Delta) - N(t) = k\right] = Pr\left[N(s+\Delta) - N(s) = k\right],$$

for any integer k, instant $s \leq t$, and increment $\Delta > 0$.

This assumption implies that the probability of causing an accident is assumed to be the same for every day during any given period.

(c) The process has independent increments, that is, for any integer $k > 0$, and instants $0 \leq t_0 < t_1 < t_2 < \cdots < t_k$, the random variables $N(t_1) - N(t_0), N(t_2) - N(t_1), \ldots, N(t_k) - N(t_{k-1})$ are mutually independent.

The common assumption is that the occurrence of an accident at one point in time is independent of all accidents that might have occurred before, reporting one accident does not increase nor decrease the probability of causing an accident in the future. This is supported by the fact that traffic accidents occur randomly in time,

(d) $Pr\left[N(h) = k\right] = \begin{cases} 1 - \lambda h + o(h) & \text{if } k = 0, \\ \lambda h + o(h) & \text{if } k = 1, \\ o(h) & \text{if } k \geq 2, \end{cases}$

where $o(h)$ is a function of that which tends to be 0 faster than the identity, that is, $\lim\limits_{h \to 0} \frac{o(h)}{h} = 0$.

This assumption indicates that the probability of two or more claims of the policyholder in a sufficiently small time interval is negligible when compared to the probability that the zero or only one claim is reported.

(e) The number of events in any time interval of length t is Poisson distributed with the mean $\Lambda = \int_0^t \lambda du = \lambda t$, that is, for all $s, t \geq 0$,

$$Pr\left[N(t+s) - N(s) = k\right] = \frac{(\lambda t)^k \exp(-\lambda t)}{k!}, k = 0, 1, 2, \ldots$$

Note that it has stationary increments and $E\left[N(t)\right] = \lambda t$.

Exposure to risk

Setting the Poisson process is useful when one wants to analyze policyholders that have been observed during periods of unequal lengths. Assume that the claims occur according to a Poisson process with the intensity rate λ. If the policyholder is covered by the company for a time period of length d then the number N of claims reported to the company has a probability mass function (Denuit *et al.*, 2007)

$$Pr\left(N = k\right) = \frac{(\lambda d)^k \exp(-\lambda d)}{k!}, k = 0, 1, 2, \ldots$$

That is, $N \sim Pois(\lambda d)$, d is referred to as the exposure to the risk.

Time between accidents or inter-arrival time

The Poisson distribution arises for events occurring randomly and independently in time. Indeed, this is denoted as W_1, W_2, $W_3, \ldots$ for the times between two consecutive accidents. The $W_k = T_k - T_{k-1}$ is called the inter-arrival time as the T_k is the time interval from time 0 until event number k or the k^{th} event time which is called the arrival time. Assume further that these accidents occur according to the Poisson process with an intensity rate λ. Then, the W_k's are independent and identically distributed and $Pr\left(W_k > t\right) = Pr\left(W_1 > t\right) = Pr\left(N(t) = 0\right) = \exp(-\lambda t)$ so that W_1, W_2, $W_3, \ldots$ have a common Exponential distribution. Consequently,

$$Pr\left(W_k > s+t \middle| W_k > s\right) = \frac{Pr\left(W_k > s+t\right)}{Pr\left(W_k > s\right)} = Pr\left(W_k > t\right),$$

where $s,t \geq 0$, the so-called memoryless property is related to the fact that the increments of the process $\{N(t), t \geq 0\}$ are independent and stationary. Assuming that the claims occur according to the Poisson process, equivalently, the time between two consecutive events has an Exponential distribution (Denuit *et al.*, 2007).

2.1.2 The non-homogeneous Poisson process

The first generalization of the Poisson process is the non-homogeneous, also called the nontationary. The Poisson process has an intensity rate which is a function $t \mapsto \lambda(t)$ of time t.

Definition 2: The counting process $\{N(t), t \geq 0\}$ is said to be the NHPP with the intensity function $\lambda(t), t \geq 0$, and is assumed to be non-decreasing, right continuous, and integer valued, so called Càdlàg, if (Ross, 1993; Denuit *et al.*, 2007)

(a) $N(0) = 0$,

(b) the process $\{N(t), t \geq 0\}$ has independent increments,

(c) and $Pr[N(t+h) - N(t) = k] = \begin{cases} 1 - \lambda(t)h + o(h) & \text{if } k = 0, \\ \lambda(t)h + o(h) & \text{if } k = 1, \\ o(h) & \text{if } k \geq 2. \end{cases}$

The number of events in the time interval $(s,t]$, $s \leq t$, is Poisson distributed with the mean $\Lambda(s,t) = \int_s^t \lambda(u)du$, and $\Lambda(s,t)$ is called the mean value function of the process, that is

$$Pr[N(t) - N(s) = k] = \frac{(\Lambda(s,t))^k \exp\left(-\Lambda(s,t)\right)}{k!}, \ k = 0,\ 1, \ldots$$

The second, the intensity function $\lambda(t)$, is bounded. Think of the NHPP as a random sample from the HPP. Specifically, let λ be such that $\lambda(t) \leq \lambda$ for all $t \geq 0$ and consider the Poisson process with an intensity rate λ. Now if we suppose that an event of the Poisson process that occurs at a certain time is counted with probability $\frac{\lambda(t)}{\lambda}$, then the process of counted events are the NHPP with the intensity function $\lambda(t)$. From Definition 2 one event can be written in the following form (Ross, 1993),

$$\begin{aligned} Pr[N(t+h)-N(t)=1] &= Pr[\text{one event in } (t,t+h)]\frac{\lambda(t)}{\lambda}+o(h) \\ &= \lambda h\frac{\lambda(t)}{\lambda}+o(h) \\ &= \lambda(t)h+o(h). \end{aligned}$$

2.2 Estimating function

The general problem of the estimation method is dealing with the consideration of the estimator $T(X)$ for the parameter θ and its properties, as an unbiased, sufficient, uniformly minimum variance. The estimator $T(X)$ is obtained by some classical methods, such as the MLE, the least squares method, the method of moment, etc. In actuarial science, sometimes the classical techniques of parameter estimation are too difficult to tackle and the model consists of complicated form. For example, the claim intensity function in the claim counts model makes a complicated distribution function of claim counts. In order to solve the parameter estimation, one needs to use the numerical method. A simpler method of statistical inference is the estimating function. Also, the advantage of using the estimating function is that the form of the function is unchanged as time varies in the case that the model specification depended on time.

On a probability space $(\Omega, \mathscr{F}, P_\theta)$, where $\theta \in \Theta$, Θ is an open interval on the real line, $P_\theta = p(x;\theta)$, suppose that the observations or data $\mathbf{x} = (x_1,x_2,\ldots,x_\tau)'$ as a realization of a random vector $\mathbf{X} = (X_1,X_2,\ldots,X_\tau)'$. The estimating function, $g(x;\theta)$, is a function of $\mathbf{X}$ and parameter θ. By solving $g(x;\theta) = 0$, called an estimating equation, an estimate of θ is obtained. Then $g(x;\theta)$ is an unbiased estimating function if $E\left(g(X;\theta)\right) = 0$ (Heyde, 2001; Mukhopadhyay, 2004).

A function $g(x;\theta)$ is called a regular estimating function if it satisfies the following conditions (Mukhopadhyay, 2004):

(i) $E(g(X;\theta)) = 0$ for all $\theta \in \Theta$,

(ii) for almost all $x \in \Omega$, $\frac{\partial}{\partial\theta} g(x;\theta)$ exists for all $\theta \in \Theta$,

(iii) $\int g(x;\theta)p(x;\theta)dx$ is differentiable with respect to θ under the integral sign,

(iv) $E(\partial g(X;\theta)/\partial\theta)^2 > 0$ for all $\theta \in \Theta$,

(v) $E(g(X;\theta))^2 = Var(g(X;\theta)) < \infty$.

Let $\mathscr{G}$ denote the class of all regular estimating functions. A function $g^* \in \mathscr{G}$ is said to be an optimal estimating function if $E\left(\frac{g^*}{E(\partial g^*/\partial\theta)}\right)^2 \leq E\left(\frac{g}{E(\partial g/\partial\theta)}\right)^2$ for all $g \in \mathscr{G}$ and all $\theta \in \Theta$. A function $g(x;\theta)$ is usually scaled to be standard, the standardized form of an estimating function is defined as $g_s = \frac{g}{E(\partial g/\partial\theta)}$, that is $E(\partial g_s/\partial\theta) = 1$. Under the standardization, the optimal estimating function $g^* \in \mathscr{G}$ becomes $E(g_s^{*2}) \leq E(g_s^2)$ or $Var(g_s^*) \leq Var(g_s)$ for all $\theta \in \Theta$.

2.3 The theory of martingale

The martingale methods are useful for constructing estimating functions. The martingales are random processes relating in time.

On a probability space $(\Omega, \mathscr{F}, P)$, suppose the increasing family $\{\mathscr{F}_t,\ t \geq 0\}$, called a filtration or history, is such that $\mathscr{F}_s \subset \mathscr{F}_t$ for all $0 \leq s < t$ and the history $\mathscr{F}_t$ which is the available data at time t. $M = \{\ M(t), t \geq 0\}$ is an arbitrary sequence of real-valued random variables. We say that (Rolski *et al.*, 2000)

(a) a sequence M is called $\mathscr{F}$-adapted if $M(t)$ is measurable with respect to $\mathscr{F}_t$ for $t \geq 0$,

(b) a sequence M is called increasing if $0 \leq M(t^-) = \lim\limits_{s \to t, s < t} M(s) < M(t)$ for all $s,\ t \geq 0$,

(c) an increasing sequence M is called integrable if $\sup E[M(t)] < \infty$.

A $\mathscr{F}_t$-adapted stochastic sequence M is called a martingale with respect to $\mathscr{F}$ if (Yip, 1995; Rolski *et al.*, 2000)

(i) $E[M(t)] < \infty$, exists,

(ii) $E[M(t+s)|\mathscr{F}_t] = M(t)$, a.s. for all $s,t \geq 0$ and $E[M(t+s)] = \mathrm{E}[M(t)]$, it is called a submartingale if $E[M(t+s)|\mathscr{F}_t] \geq M(t)$, a.s. for $s,t \geq 0$,

(iii) $\sup E[M^2(t)] < \infty$, then M is called square integrable martingale .

A ZMM is as a result of the properties of the martingale $E[M(t)] = E[M(0)]$ for all $t \geq 0$, then $E[M(0)] = 0$ (Yip, 1995). If $\{M(t)\}$ is a zero mean square integrable martingale, $\{M^2(t)\}$ is a submartingale, and these exist as a unique predictable integrable increasing sequence which is denoted by $\{\langle M\rangle(t)\}$ or $\langle M\rangle$ such that $\{M^2(t) - \langle M\rangle(t)\}$ is a ZMM. Then $\langle M\rangle$ is called the predictable variation of $\{M(t)\}$ (Andersen *et al.*, 1992).

2.4 The claim counting process

Suppose a filtration $\{\mathscr{F}_t,\ t \geq 0\}$, is an increasing right continuous on a probability space $(\Omega, \mathscr{F}, P)$ and $N(t) = \#\ \{i \geq 1 : T_i \leq t\};\ t \geq 0$, is the total number of insurance claims or the claim counts that have occurred during time interval $(0,t]$, where $T_n = W_1 + \ldots + W_n;\ n \geq 1$ is a claim arrival time and W_i is an iid with an Exponential whose parameter is $\lambda(w_i)$, called the claim intensity rate. For convenience, in this thesis we consider the claim arrival time to vary in a finite time interval in which time interval (0,1] is taken or one-year period (Yip, 1988). $N = \{N(t);\ t \geq 0\}$ represents a claim counting process such that N is a non-decreasing, right continuous step function as 0 at time $t = 0$, jumps of size 1, $\lim\limits_{t\to\infty} N(t) < \infty$ a.s. A typical claim counting process N is illustrated in Figure 2 N is governed by the claim intensity $\lambda(t)$ for all $t \geq 0$, or parameter of N in terms of the mean value function $\Lambda(t) = \int\limits_0^t \lambda(u)du = E(N(t))$. $\{\lambda(t) = \alpha(t)\cdot k(t);\ t \geq 0\}$ is called the multiplicative intensity process, where $\alpha(t)$ and $k(t)$ are defined as the claim intensity rate and the exposure risk, respectively (Norberg, 1993). In this study, we define $k(t) = 1$ and

$$P = p(N(t);\Lambda) = \Pr\{N(t) = n\} = \frac{\Lambda(t)^n exp(-\Lambda(t))}{n!}; n = 0,1,2,\ldots$$

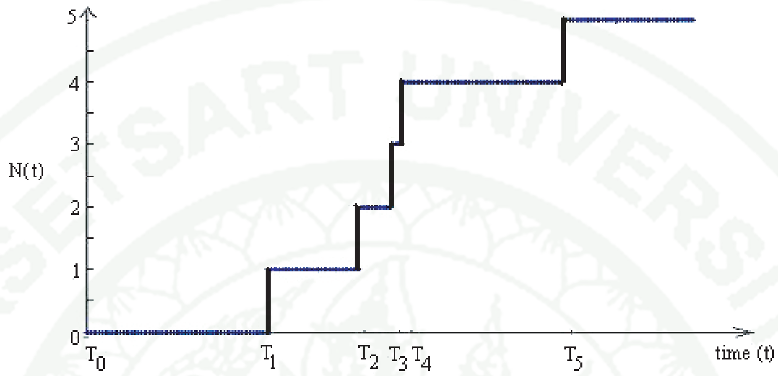


Figure 2 A sample path of a claim counting process

Let $dN(t)$ be an increment of N over a small fraction time $(t, t+dt]$, $N(t)$ can be written as $\int_0^t dN(u)$ and

$$\lambda(t)dt = E\{dN(t)|\mathscr{F}_{t^-}\} = \Pr\{dN(t) = 1|\mathscr{F}_{t^-}\},$$

this equation implies that if we define the process $M = \{\, M(t), t \geq 0\}$, a martingale, it can be written as the martingale-difference given the history,

$$dM(t) = dN(t) - \lambda(t)dt.$$

Then, the martingale is

$$\int dM(t) = \int dN(t) - \int \lambda(t)dt,$$

equivalently $M(t) = N(t)$ - $\Lambda(t)$. According to the properties of martingale, i.e. $M(t = 0) = 0$ and $E[dM(t)|\mathscr{F}_{t-}] = 0$ or equivalently $E[M(t+s)|\mathscr{F}_t] = M(t)$, for all $s,t \geq 0$, the claim counting process martingale given the history is $M(t) = N(t)$ - $\Lambda(t)$, a ZMM.

Based on ZMM, the process $M(t) = N(t)$ - $\Lambda(t)$, we obtain

$$E\left(M(t)\right) = 0 = E\left(N(t) - \Lambda(t)\right).$$

Thus, $N(t)$ - $\Lambda(t) = 0$ is an estimating equation for the parameter estimation of the claim counting process. Also, as a result of the parameter estimate, this can be interpreted as an $N(t)$ estimate. In other words, $\Lambda(t)$ is called the compensator of $N(t)$, i.e. fitting the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$. This estimate is useful for predicting the claim counts or the time of occurrence of insurance claim counts (Yip, 1995). We can depict the behavior of $\Lambda(t)$ relating to a specified $N(t)$, and the associated martingale $N(t) - \Lambda(t) = M(t)$ in Figures 3(a) and 3(b), respectively, based on a sample of 15 independent random times of claim occurrence in the NHPP with an intensity of $\lambda(t) = 31\exp\left\{-3\left(t-\frac{1}{2}\right)^2\right\}$

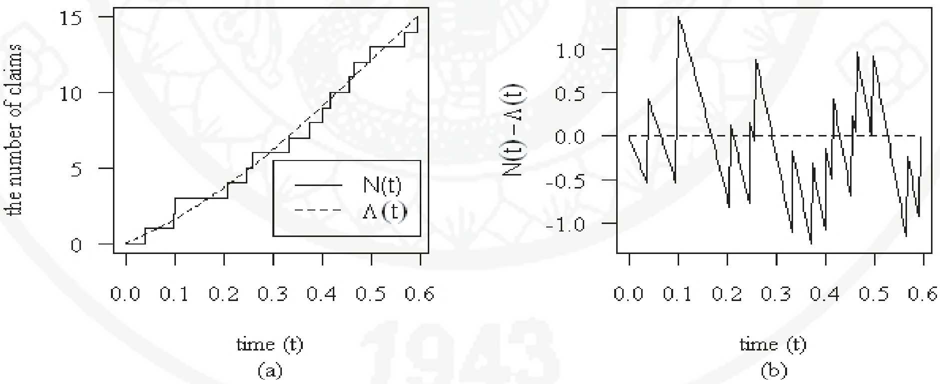


Figure 3 In a sample of 15 independent random times of claim occurrence with the claim intensity $\lambda(t) = 31\exp\left\{-3\left(t-\frac{1}{2}\right)^2\right\}$, (a) the behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the non-life insurance claim counting process, and (b) the martingale $M(t) = N(t) - \Lambda(t)$

2.5 Parameter estimation methods for the model of claim intensity

In this thesis the stochastic analysis of claim counts has been based on the modeling of the non-life insurance claim counting process. The MLE and the BE are used to estimate the model parameters of claim intensity in order that we can get the estimate of the compensator, $\Lambda(t)$, of $N(t)$. Each method has the following details:

2.5.1 Maximum likelihood estimation

The MLE is a method of estimation and inference for parametric models. Specifically, given $N(\mathrm{t}) = \mathrm{n}$, we suppose that the order statistics, t_1, t_2, $t_3, \ldots,$ $t_{N(t)}$, are the arrival times of claims over time interval $(0,t]$ and $t_0 = 0$ with a cumulative distribution function

$$F(t) = 1 - \exp(-\Lambda(t)), \tag{5}$$

which is the general order statistics model and the likelihood function (Cid and Achcar, 1999; Mikosch, 2009) is given by

$$\ell(\boldsymbol{\theta};\boldsymbol{\xi}) = \left(\prod_{i=1}^{n}\lambda(t_i)\right)\exp\left(\int_0^{t_i}\lambda(u)du - \int_0^{t_{i-1}}\lambda(u)du\right),$$

hence,

$$\ell(\boldsymbol{\theta};\boldsymbol{\xi}) = \prod_{i=1}^{n}\lambda(t_i)exp(-\Lambda(t_n)), \tag{6}$$

where $\boldsymbol{\theta}$ is a vector of the parameters of claim intensity, $\boldsymbol{\xi}$ denotes the observed data set of claim arrival times, the $\lambda(t_i)$ of HPP is a constant λ and the $\lambda(t_i)$ of NHPP with bell-shaped and beta-shaped intensities as given in Equations 1 and 2, respectively. While $\theta = \lambda$ a constant claim occurrence rate, $\boldsymbol{\theta} = (\lambda^*, \sigma)'$ of a bell-shaped intensity function and $\boldsymbol{\theta} = (\lambda^*, p, q)'$ of a beta-shaped intensity function are unknown, the estimate of $\boldsymbol{\theta}$

can be simply obtained if we take the logarithm of the likelihood function as

$$\log \ell(\boldsymbol{\theta};\boldsymbol{\xi}) = L(\boldsymbol{\theta};\boldsymbol{\xi}) = \sum_{i=1}^{n} \log\lambda(t_i) - \Lambda(t_n),$$

where $\Lambda(t_n)$ of HPP is equal to λt_n, $\Lambda(t_n)$ of NHPP with bell-shaped and beta-shaped intensities is given in Equations 3 and 4, respectively. Solving the value of $\boldsymbol{\theta}$ that maximizes the log likelihood function, the first derivative of $L(\boldsymbol{\theta};\boldsymbol{\xi})$ with respect to $\boldsymbol{\theta}$ is called Fisher's score, $U = \frac{\partial}{\partial\boldsymbol{\theta}}L(\boldsymbol{\theta},\boldsymbol{\xi})$. We can find the estimate of $\boldsymbol{\theta}$ by setting

$$U = \frac{\partial}{\partial\boldsymbol{\theta}}L(\boldsymbol{\theta},\boldsymbol{\xi}) = 0,$$

then solving the system of equations. The following parameter estimate of claim intensity of HPP claim counting process is $\hat{\theta} = \hat{\lambda} = \frac{n}{t_n}$.

For the NHPP claim counting process, the calculation of the MLE estimator of the model parameter of claim intensity, which is a complicated system of equations, requires an iterative procedure, e.g. the Newton-Raphson algorithm, to solve these equations. Let H denotes the Hessian, or matrix of second derivatives, of the log likelihood function $H(\boldsymbol{\theta}) = \frac{\partial^2}{\partial\boldsymbol{\theta}}L(\boldsymbol{\theta};\boldsymbol{\xi}) = \frac{\partial}{\partial\boldsymbol{\theta}}U$. For $\boldsymbol{\theta}^*$ close enough to $\boldsymbol{\theta}$, a first order Taylor expansion gives $0 = U(\hat{\boldsymbol{\theta}}) \approx U(\boldsymbol{\theta}^*) + H(\boldsymbol{\theta}^*)(\hat{\boldsymbol{\theta}}$ - $\boldsymbol{\theta}^*)$ yielding $\hat{\boldsymbol{\theta}} \approx \boldsymbol{\theta}^*$ - $H^{-1}(\boldsymbol{\theta}^*)U(\boldsymbol{\theta}^*)$. Starting from the appropriate initial value $\boldsymbol{\theta}^{(0)}$, the Newton-Raphson algorithm is based on the recurrence relation

$$\hat{\boldsymbol{\theta}}^{(r+1)} \approx \hat{\boldsymbol{\theta}}^{(r)} - H^{-1}(\hat{\boldsymbol{\theta}}^{(r)})U(\hat{\boldsymbol{\theta}}^{(r)}),$$

to obtain an improved estimate and the process is repeated until the elements of the vector of the first derivatives are sufficiently close to zero.

Noting that $J(\boldsymbol{\theta}) = -E(H(\boldsymbol{\theta}))$, an alternative procedure is to replace the minus of Hessian by its expected value, i.e. the Fisher information matrix. The resulting procedure takes as an improved estimate $\hat{\boldsymbol{\theta}}^{(r+1)} \approx \hat{\boldsymbol{\theta}}^{(r)} + J^{-1}(\hat{\boldsymbol{\theta}}^{(r)})U(\hat{\boldsymbol{\theta}}^{(r)})$ and is known as Fisher Scoring (Denuit *et al.*, 2007).

2.5.2 Bayesian estimation

The BE approach is based on the Bayes' theorem that combines the prior information and likelihood in its approach. Given $N(t) = n$, we suppose that the order statistics, $t_1,\ t_2,\ t_3, \ldots,\ t_{N(t)}$, are the arrival time of claims over time interval $(0,t]$ and $t_0 = 0$ with a distribution function as given in Equation 5 and its likelihood function in the form of Equation 6. The approach treats all unknown parameters in Equation 5 as random variables under their prior distribution and derives their conditional distribution upon the known information from the sample data of claim arrival times. In this study, the parameters estimate of claim intensity $\theta = \lambda$, a constant claim occurrence rate, of HPP, $\boldsymbol{\theta} = (\lambda^*, \sigma)'$ of a bell-shaped intensity function and $\boldsymbol{\theta} = (\lambda^*, p, q)'$ of a beta-shaped intensity function of NHPP are unknown. Especially, the estimate of $\boldsymbol{\theta}$ can be obtained by the Bayesian analysis by applying the prior density as follows,

1. for $\theta = \lambda$: the conjugate prior distribution is

$$\lambda \sim Gamma(\alpha_0,\ \lambda_0), \tag{7}$$

2. for $\boldsymbol{\theta} = (\lambda^*, \sigma)'$:

$$\begin{aligned} \lambda^* &\sim Gamma(a_1, b_1), \\ \sigma^2 &\sim Gamma(a_2, b_2), \end{aligned} \tag{8}$$

3. for $\boldsymbol{\theta} = (\lambda^*, p, q)'$:

$$\begin{aligned} \lambda^* &\sim Gamma(c_1, d_1), \\ p &\sim Gamma(c_2, d_2)\mathrm{I}(1,), \\ q &\sim Gamma(c_3, d_3)\mathrm{I}(1,), \end{aligned} \tag{9}$$

where $\alpha_0,\ \lambda_0,\ a_1,\ b_1,\ a_2,\ b_2,\ c_1,\ d_1,\ c_2,\ d_2,\ c_3$ and d_3 are known parameters, $Gamma(a,b)$ denotes a Gamma distribution with mean a/b and variance a/b^2. The restriction $p,\ q \geq 1$, is imposed with the construct I(1,). Here we suppose that the parameters $\boldsymbol{\theta}$ are independent, then the estimated parameters are based on the joint posterior density as (Klugman *et al.*, 2008)

$$h(\boldsymbol{\theta}|\boldsymbol{\xi}) \propto h(\boldsymbol{\theta})\ell(\boldsymbol{\theta};\boldsymbol{\xi}), \tag{10}$$

where $h(\boldsymbol{\theta})$ is the joint prior density and $\ell(\boldsymbol{\theta};\boldsymbol{\xi})$ is the likelihood function of $\boldsymbol{\xi}$ as given in Equation 6. For the HPP claim counting process, the formulation of the joint posterior density in Equation 10 is in a closed form that we can find directly by inference and by estimating the parameter of claim intensity. The estimate of $\theta = \lambda$ is equal to $\hat{\theta} = \hat{\lambda} = \frac{n+\alpha_0}{t_n+\lambda_0}$. For the NHPP with bell-shaped and bata-shaped intensities, the joint posterior density in Equation 10 is not in a closed form or it is so complicated that we need to derive approximately the inference $E(\mathrm{g}(\boldsymbol{\theta})) = \int g(\boldsymbol{\theta})h(\boldsymbol{\theta}|\boldsymbol{\xi})d\boldsymbol{\theta}$ on the basis of the Markov Chain Monte Carlo (MCMC) algorithm. Considering MCMC is used to set up a Markov chain in $\boldsymbol{\xi}$ with $h(\boldsymbol{\theta}\,|\,\boldsymbol{\xi}\,)$, we can simulate h transitions under this Markov chain by starting with an initial state $\boldsymbol{\theta}^{(0)}$ and by recording the simulated states $\boldsymbol{\theta}^{(j)}$, $j = 1,2,\ldots,s$. Then we get the sample average as $\hat{g} = \frac{1}{s}\sum_{j=1}^{s} g(\boldsymbol{\theta}^{(j)})$ that converges to $E(g(\boldsymbol{\theta}))$, i.e., $\hat{g}$ is equal to an approximate value of $E(g(\boldsymbol{\theta}))$. The standard error of $\hat{g}$ of the Monte Carlo estimate is given by $SE(\hat{g}) = \sqrt{\frac{1}{s(s-1)}\sum_{j=1}^{s}\left(g(\boldsymbol{\theta}^{(j)}) - \hat{g}\right)^2}$.

The simplest implemented sampling method for MCMC is Gibbs sampling. When we need to set $\boldsymbol{\theta} = (\theta_1,\ldots,\theta_d)$, we can sample from the conditional distributions (Zhao *et al.*, 2008; Ntzoufras, 2009)

$$\begin{gathered} h(\theta_1|\theta_2,\theta_3,\ldots,\theta_d,\xi), \\ h(\theta_2|\theta_1,\theta_3,\ldots,\theta_d,\xi), \\ \vdots \\ h(\theta_d|\theta_1,\theta_2,\ldots,\theta_{d-1},\xi), \end{gathered}$$

where $\boldsymbol{\xi}$ is the observed data set of claim inter-arrival times. Starting with an initial point in parameter space, and by generating a random sequence as, $\{(\theta_1{}^{(0)},\theta_2{}^{(0)},\ldots,\theta_d{}^{(0)}),\ldots,(\theta_1{}^{(\mathrm{j})},\theta_2{}^{(\mathrm{j})},\ldots,\theta_d{}^{(\mathrm{j})}),\ldots\}$. The Gibbs sampling algorithm is shown as follows, where we can generate

$$
\begin{aligned}
\text{Step 1. } \theta_1^{(j)} &\sim h(\theta_1|\theta_2^{(j-1)},\ldots,\theta_d^{(j-1)},\boldsymbol{\xi}),\\
\text{Step 2. } \theta_2^{(j)} &\sim h(\theta_2|\theta_1^{(j)},\theta_3^{(j-1)},\ldots,\theta_d^{(j-1)},\boldsymbol{\xi}),\\
\text{Step 3. } \theta_3^{(j)} &\sim h(\theta_3|\theta_1^{(j)},\theta_2^{(j)},\theta_4^{(j-1)},\ldots,\theta_d^{(j-1)},\boldsymbol{\xi}),\\
&\vdots\\
\text{Step d. } \theta_d^{(j)} &\sim h(\theta_d|\theta_1^{(j)},\theta_2^{(j)},\ldots,\theta_{d-1}^{(j)},\boldsymbol{\xi}),
\end{aligned}
$$

for $j = 1,2,\ldots$, Step 1 to Step d define a Markov chain $\boldsymbol{\theta}^{(j)} = (\theta_1^{(j)},\theta_2^{(j)},...,\theta_d^{(j)})$ which converges to $h(\theta_1,\theta_2,\ldots,\theta_d\,|\boldsymbol{\xi}\,)$ as desired.

2.6 Criterion of comparison

In the modeling of the claim counting process, the mean squared error (MSE) is the criterion of comparing the claim count estimation or the measure error of fitting the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$. The appropriate estimation will have a small MSE value. The MSE of a compensator estimate $\widehat{\Lambda(t)}$ with respect to $N(t)$ is defined as,

$$MSE(\widehat{\Lambda(t)}) = E\left(\widehat{\Lambda(t)} - N(t)\right)^2,$$

or it can be written in the form of

$$MSE = \sum_{j=1}^{p} \frac{\int_0^t \left(\widehat{\Lambda_j(u)} - N_j(u)\right)^2 du}{S_p} \tag{11}$$

where S_p denotes the number of sample paths.

MATERIALS AND METHODS

This chapter is divided into 2 sections, firstly, the materials which consist of the computer specifications and the software. Secondly, the details of the methods used for the simulation models for this study. Each section is described as follows:

Materials

The following materials and equipment were used for this study.

1. A personal computer, specifically an Intel Core2 Duo CPU 32bit with 2 GB of RAM running on Windows XP operating system.

2. The statistical computation in this study using R language (R Development Core Team, 2008), WinBUGS (Lunn *et al.*, 2000) and R2WinBUGS (Sturtz *et al.*, 2005).

Methods

In this study, a simulation technique is used to investigate the estimation and the prediction of claim counts based on the Poisson claim counting process, including the HPP and the NHPP with bell-shaped and beta-shaped intensities. The proposed approach needs to estimate the parameters of processes, i.e. claim intensity $\lambda(t)$ or in term of mean value function $\Lambda(t)$ called the compensator of $N(t)$, using the estimating function provided by the martingale method with the ZMM. This approach also includes the MLE and the BE analysis for the estimation of the model parameters of claim intensities. The simulation procedures are as follows:

1. Simulation Procedures of the HPP Claim Counting Process

1. Generate the simulated data, the claim counts during the time intervals of claims occurrence $(0,t]$ governed by Poisson law can be simulated via the claim arrival times $t_1,\ t_2,\ t_3,...,\ t_{N(t)}$ or the claim inter-arrival times $W_i = t_i - t_{i-1}$ where

$i = 1,2,3,...,N(t)$, and follow an Exponential law with claim intensity λ, i.e. with mean $\frac{1}{\lambda}$ as $\lambda = 0.1, 5$ and 10. The number of observations, $N(t) = n$, is composed of 5, 10, 15 and 20. Hence, the claim arrival times of the process are generated as the following algorithm: (Burnecki *et al.*, 2004)

Step 1: Set $t_0 = 0$.

Step 2: For $i = 1,2,3,...,n$ do

generate an Exponential random variate E with claim intensity λ

then set $t_i = t_{i-1} + E$.

2. Set up the estimating function which is provided by the ZMM to estimate the parameter of the process, $\Lambda(t)$ or the compensator of $N(t)$.

3. Estimate the parameter of the claim intensities by using the methods of the MLE and the BE by applying the prior density of λ, i.e. $\lambda \sim Gamma(\alpha_0 = 0.001, \lambda_0 = 0.001)$ where the $Gamma(a,b)$ denotes a Gamma distribution with mean $\frac{a}{b}$ and variance $\frac{a}{b^2}$.

4. Carry out 5000 sample paths.

5. Measure error of fitting the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$ by using the *MSE* in Equation 11.

2. Simulation Procedure of the NHPP Claim Counting Process

1. Generate the simulated data of the process with bell-shaped and beta-shaped intensities, the claim counts during the time intervals of claim occurrences $(0,t]$ in which their observation involves the claim arrival times, t_1, t_2, t_3,..., $t_{N(t)}$. The claim arrival times can be simulated by using the mean value function $\Lambda(t)$, the $\Lambda(t)$ of the bell-shaped and the beta-shaped intensities are in Equation 3and 4, respectively.

According to the inter-arrival times of process, $W_i = t_i - t_{i-1}$, are exponentially distributed with mean $\frac{1}{\lambda(t)}$ where $i = 1,2,3,...,N(t)$ or which can be written by $W_i \sim Exp(\lambda(t))$. Let $F_W(t) = \Pr(W \leq t) = 1 - \exp(-\Lambda(t))$ be the distribution function, which can be rewritten to $\Lambda(t) = -\ln(1 - F_W(t))$ where $1 - F_W(t)$ is uniformly distributed on $(0,1)$. By using a transformation technique of univariate random variable, we found that $\Lambda(t)$ is the Exponential distribution with mean one as a claim arrival time of the HPP with mean one. It implies that E_1, E_2, E_3,..., $E_{N(t)}$ are iid Exponential random variables with mean one where $E_i = \Lambda(t_i) - \Lambda(t_{i-1})$, for all $i = 1,2,3,...,N(t)$ (Morales, 2004; Mikosch, 2009).

Given the process, the number of observations , $N(t) = n$, is composed of $5, 10, 15$ and 20, the model parameters of bell-shaped intensity consist of $\lambda^* = 0.1, \sigma = 0.25$; $\lambda^* = 0.1, \sigma = 5$; $\lambda^* = 5, \sigma = 0.25$; $\lambda^* = 5, \sigma = 5$; $\lambda^* = 10, \sigma = 0.25$ and $\lambda^* = 10, \sigma = 5$ and the model parameters of beta-shaped intensity consist of $\lambda^* = 0.1$, $p = q = 1.25$; $\lambda^* = 0.1$, $p = q = 2$; $\lambda^* = 0.1$, $p = q = 3$; $\lambda^* = 5$, $p = q = 1.25$; $\lambda^* = 5$, $p = q = 2$; $\lambda^* = 5$, $p = q = 3$; $\lambda^* = 10$, $p = q = 1.25$; $\lambda^* = 10$, $p = q = 2$ and $\lambda^* = 10$, $p = q = 3$.

Hence, the claim arrival times t_1, t_2, t_3,..., t_n, are generated based on the following algorithm:

Step 1: Set $t_0 = 0$.

Step 2: For $i = 1,2,3,...,n$ do

generate an Exponential random variate E_k with mean one

then set $t_i = \Lambda^{-1}\left(\sum_{k=1}^{i} E_k\right)$, where $\Lambda^{-1}(E_k)$ is the invertible function of $\Lambda(t)$.

2. Set up estimating function which is provided by the ZMM to estimate the model parameter of process, $\Lambda(t)$ or the compensator of $N(t)$.

3. Estimate the parameters of the bell-shaped and the beta-shaped intensities by using the methods of the MLE and the BE by applying the prior density of $\theta = (\lambda^*, \sigma)'$ for the bell-shaped intensity, i.e. $\lambda^* \sim Gamma(a_1 = 0.01, b_1 = 0.01)$, $\sigma^2 \sim Gamma(a_2 = 5, b_2 = 1)$ and by applying the prior density of $\theta = (\lambda^*, p, q)'$ for the beta-shaped intensity, i.e. $\lambda^* \sim Gamma(c_1 = 0.1, d_1 = 0.1)$, $p \sim Gamma(c_2 = 5, d_2 = 1)I(1,)$ and $q \sim Gamma(c_3 = 5, d_3 = 1)I(1,)$ where the $Gamma(a,b)$ denotes a Gamma distribution with mean $\frac{a}{b}$ and variance $\frac{a}{b^2}$, and the restriction, p, $q \geq 1$, is imposed with the construct $I(1,)$.

4. Carry out 5000 sample paths of each process.

5. Measure error of fitting the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$ by using the MSE in Equation 11.

RESULTS AND DISCUSSION

In this chapter, we show the results and discussion of a simulation study of an estimation approach to non-life insurance claim counts. The insurance claim counts relates to the claim counting process during the time interval (0,t], including HPP, NHPP with bell-shaped and beta-shaped intensities. For this approach, the technique of the estimating function which is provided by the martingale, namely a ZMM, is used to estimate the parameter $\Lambda(t)$ of the claim counting process or the compensator $\Lambda(t)$ of claim counts $N(t)$, i.e. fitting the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$. The estimate of the parameters of claim intensity are computed by using the MLE and BE methods.

Results

Simulation results are divided into 5 parts: the estimation of claim counts using the estimating function via a ZMM and the MLE for estimating the model parameters of claim intensity, the estimation of claim counts using the estimating function via a ZMM and the BE for estimating the model parameters of claim intensity, a comparison of the estimation of claim counts based on the Poisson claim counting processes with the approach of MLE and BE for estimating the model parameters of claim intensity, the prediction of claim counts based on the Poisson claim counting process, and application.

Some results in Part 1.1 and 1.2 have been published in Applied Mathematics (Jaroengeratikun *et al.*, 2012b) and the results in Part 2.2 and 2.3 have been published in Open Journal of Statistics (Jaroengeratikun *et al.*, 2012a). All the original results of the simulation study are shown as follows:

1. The Estimation of Claim Counts Using the Estimating Function via a ZMM and the MLE for Estimating the Model Parameters of Claim Intensity

Following the examples of some sample paths of 5, 10, 15, and 20 claims, shown in Part 1.1- 1.3, illustrate the behavior of $\Lambda(t)$ relating to a specified $N(t)$. The illustrated behavior satisfies the parameter estimation approach of the claim counting process using an estimating function via a ZMM, i.e. $N(t)-\Lambda(t)=0$ and then $\Lambda(t)=N(t)$, where

$M(t) = N(t) - \Lambda(t)$ is the martingale and the MLE is provided for estimating the model parameter of claim intensity.

1.1 The estimation of claim counts based on the HPP

The sample paths of 5, 10, 15 and 20 claims based on the HPP with a small claim intensity $\lambda = 0.1$, are shown in Figure 4. These illustrate the behavior of $\Lambda(t)$ relating to a specified $N(t)$. Notice that the $\Lambda(t)$ fits to $N(t)$ while the number of observations is small. As, the behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the processes with $\lambda = 5$ and $\lambda = 10$ is illustrated respectively in Figure 5 and Figure 6, the $\Lambda(t)$ fits well with $N(t)$ where the number of observations is slightly larger than the value of λ.

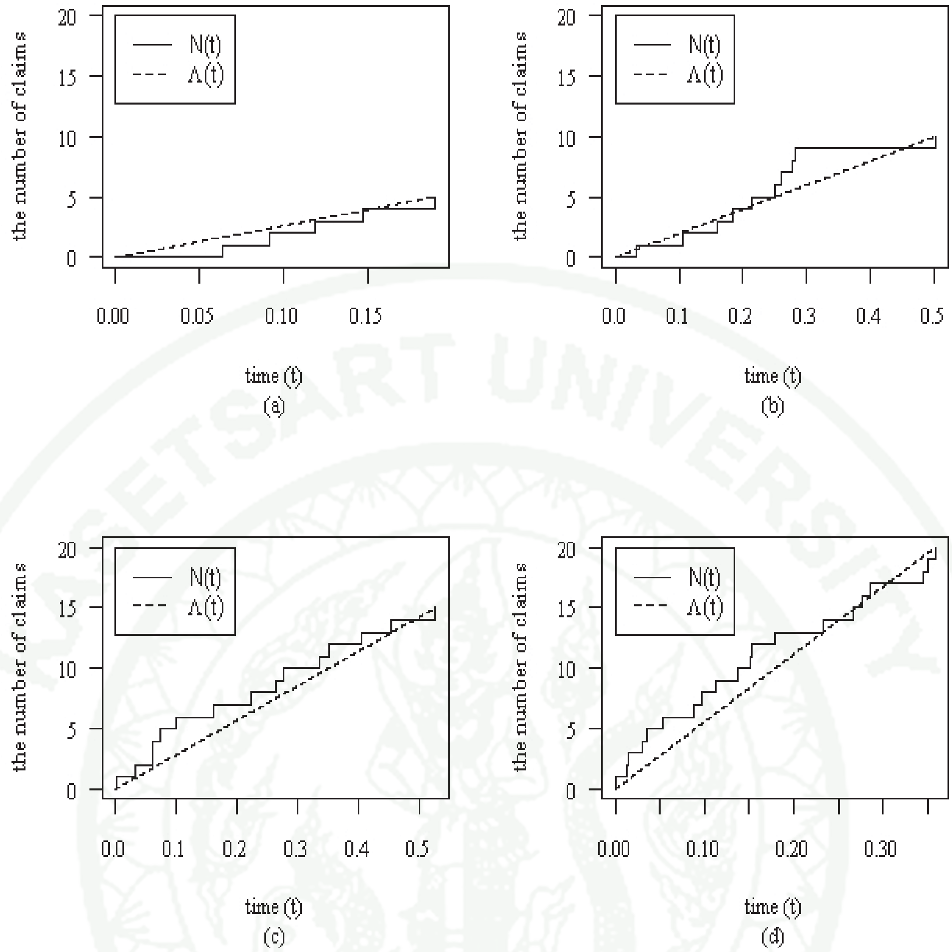


Figure 4 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the HPP with the claim intensity $\lambda = 0.1$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

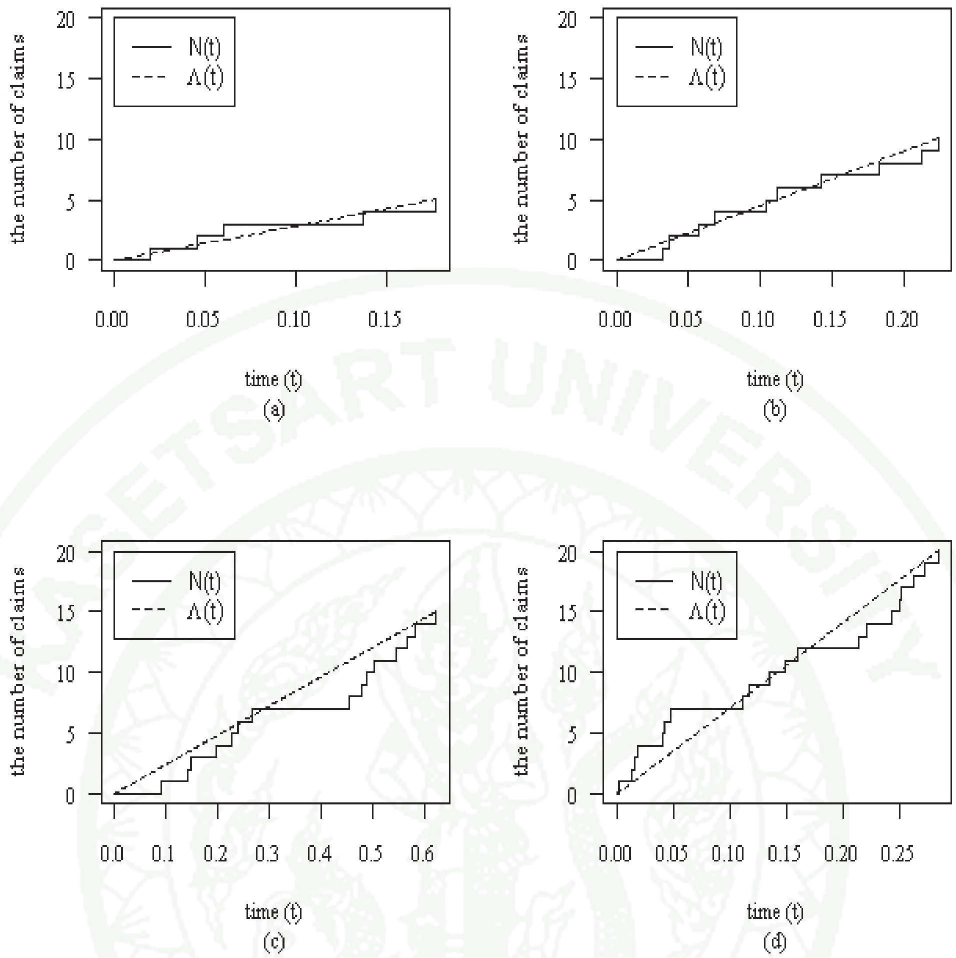


Figure 5 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the HPP with the claim intensity $\lambda = 5$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

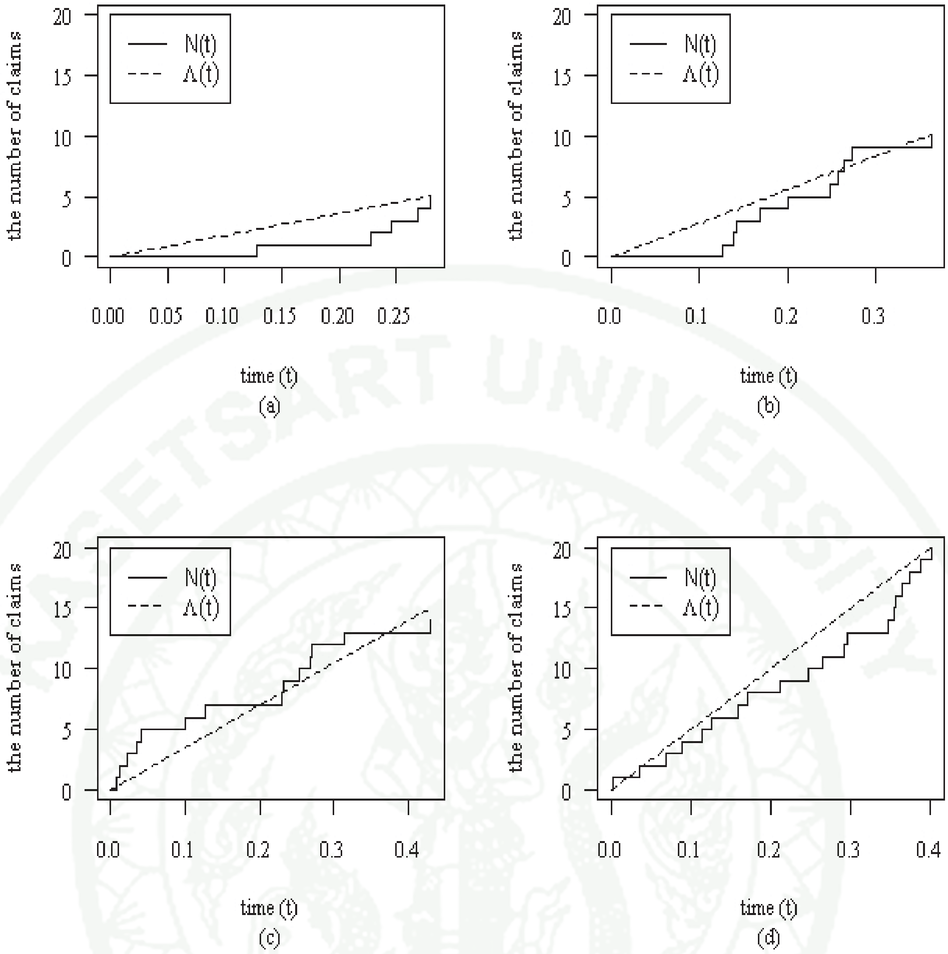


Figure 6 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the HPP with the claim intensity $\lambda = 10$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

1.2 The estimation of claim counts based on the NHPP with a bell-shaped intensity

Some samples of 5, 10, 15 and 20 claims based on the NHPP with a bell-shaped intensity are shown in Figure 7 - 9. The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the process with the parameters of claim intensity $\lambda^* = 0.1$, $\sigma = 5$, a small number of average claims, is illustrated in Figure 7. The $\Lambda(t)$ is near to $N(t)$ where the number of observations is small. For the parameters of claim intensities $\lambda^* = 5, \sigma = 5$ and $\lambda^* = 10, \sigma = 5$, the behavior of $\Lambda(t)$ relating to a specified $N(t)$ is illustrated respectively in Figure 8 and Figure 9. The $\Lambda(t)$ is a good fit to $N(t)$ while the number of observations is slightly larger than the value of λ^*.

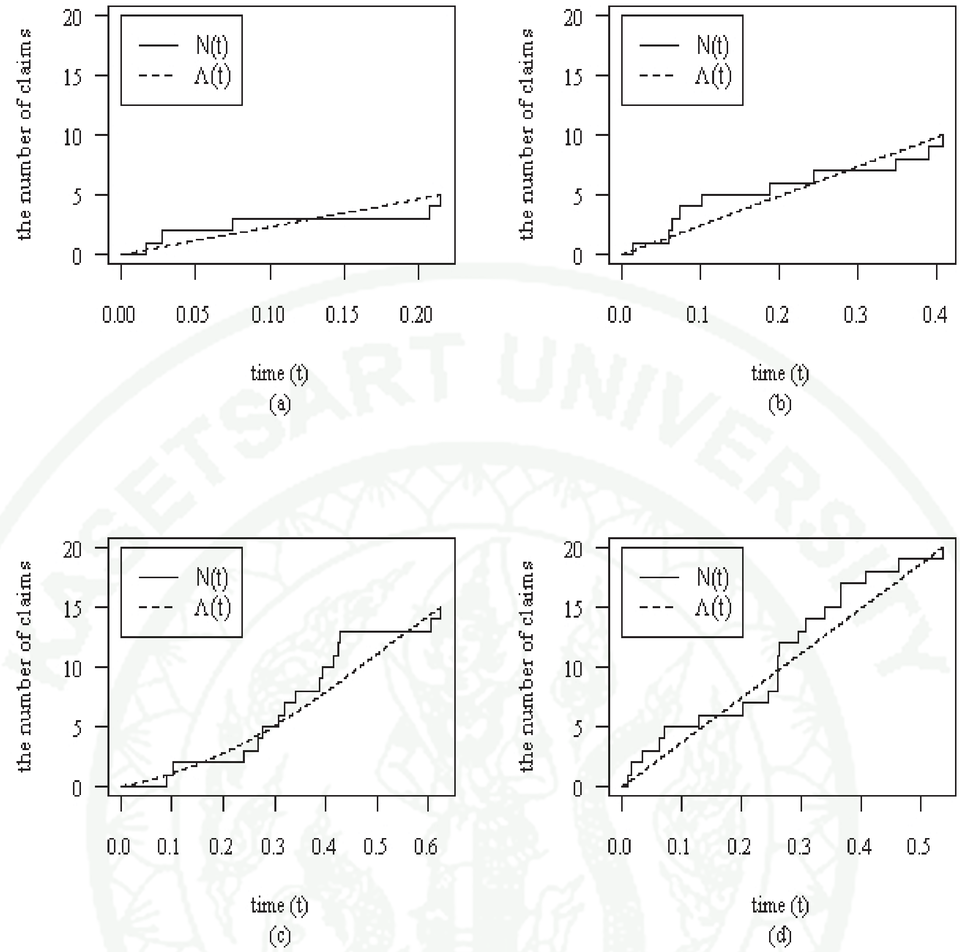


Figure 7 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 0.1, \sigma = 5$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

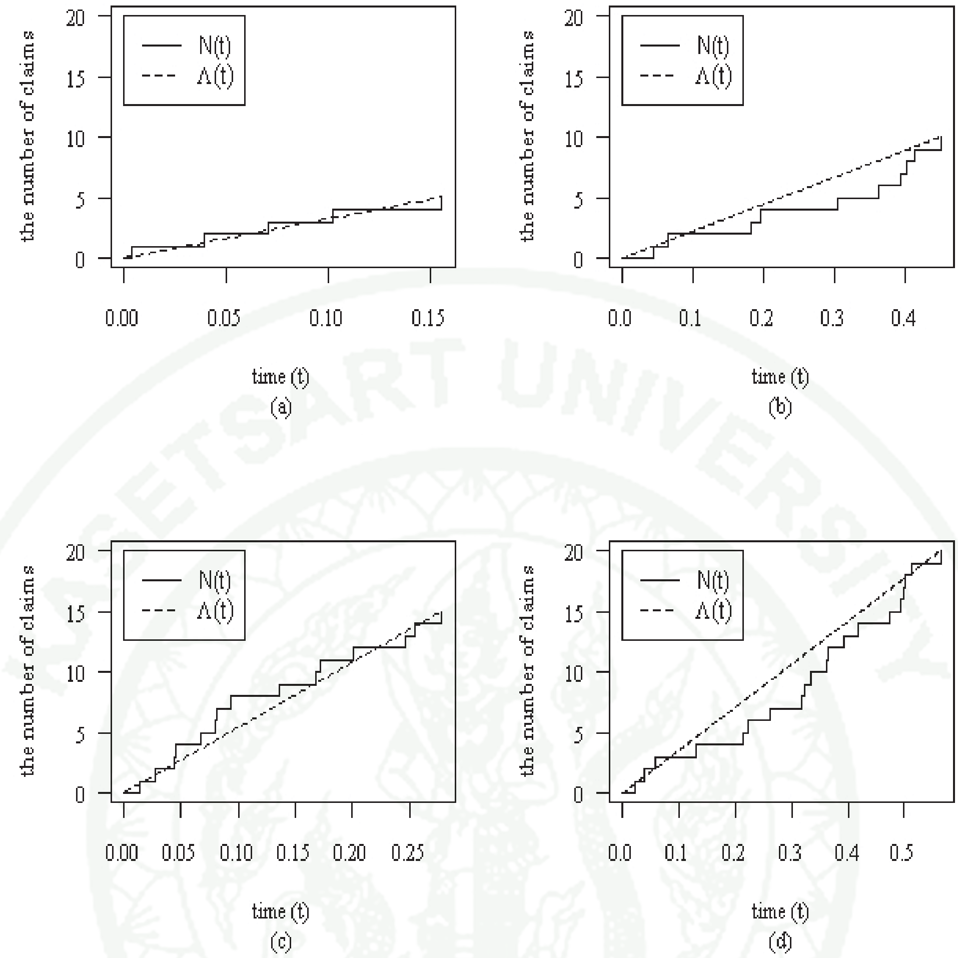


Figure 8 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 5, \sigma = 5$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

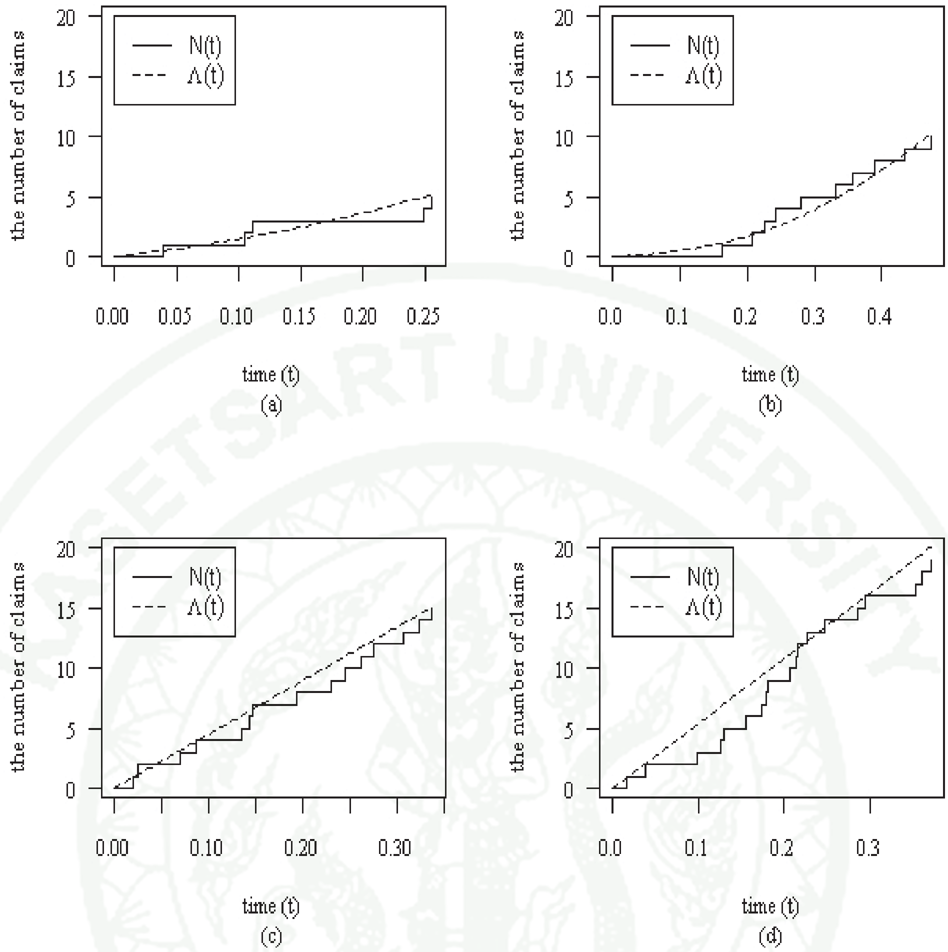


Figure 9 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 10, \sigma = 5$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

1.3 The estimation of claim counts based on the NHPP with a beta-shaped intensity

In Figure 10-12, some samples of 5, 10, 15 and 20 claims based on the NHPP with a beta-shaped intensity are shown and these considered processes have the parameters of beta-shaped intensities $\lambda^* = 0.1, p = q = 1.25$; $\lambda^* = 5, p = q = 1.25$ and $\lambda^* = 10, p = q = 1.25$. The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the process with $\lambda^* = 0.1, p = q = 1.25$, a small peak level or almost no peak level of claim intensity, is illustrated in Figure 10, we found that the $\Lambda(t)$ is around the $N(t)$ where the number of observations is small. As shown in the following Figure 11 and Figure 12, the behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the processes with $\lambda^* = 5, p = q = 1.25$ and $\lambda^* = 10, p = q = 1.25$, is a good fit to $N(t)$ where the number of observations is equal to the value of λ^*.

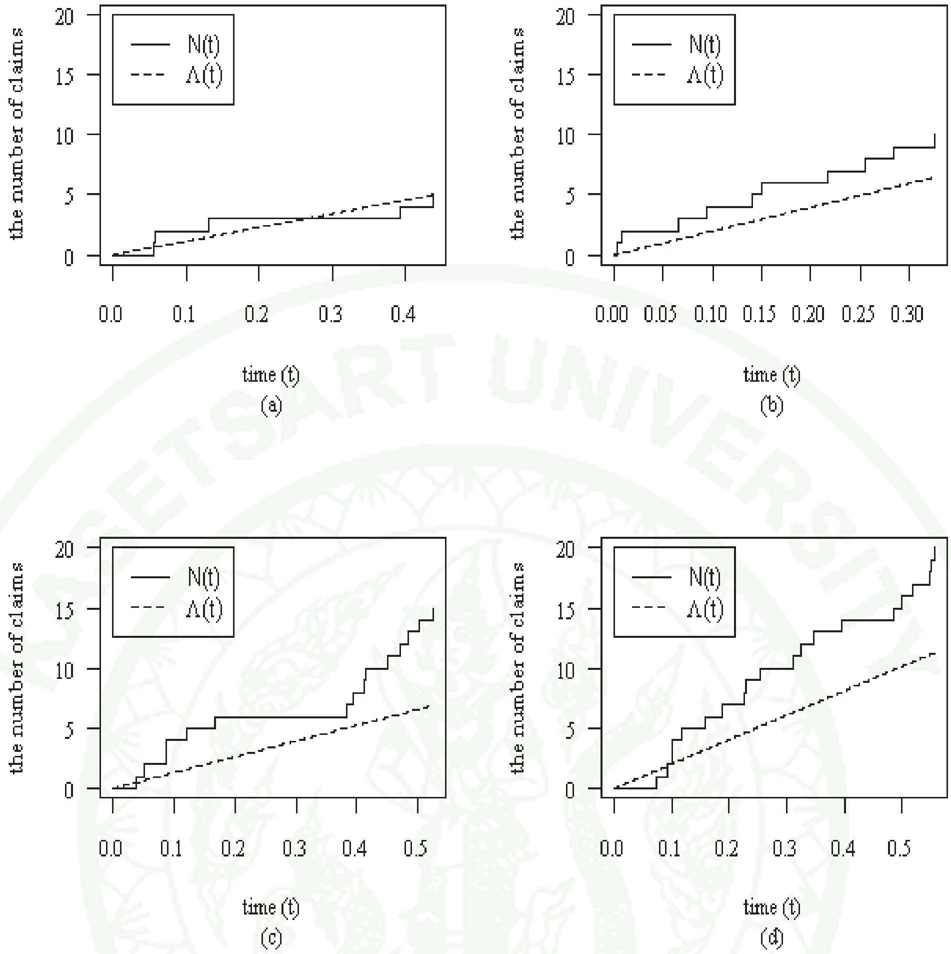


Figure 10 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 0.1, p = q = 1.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

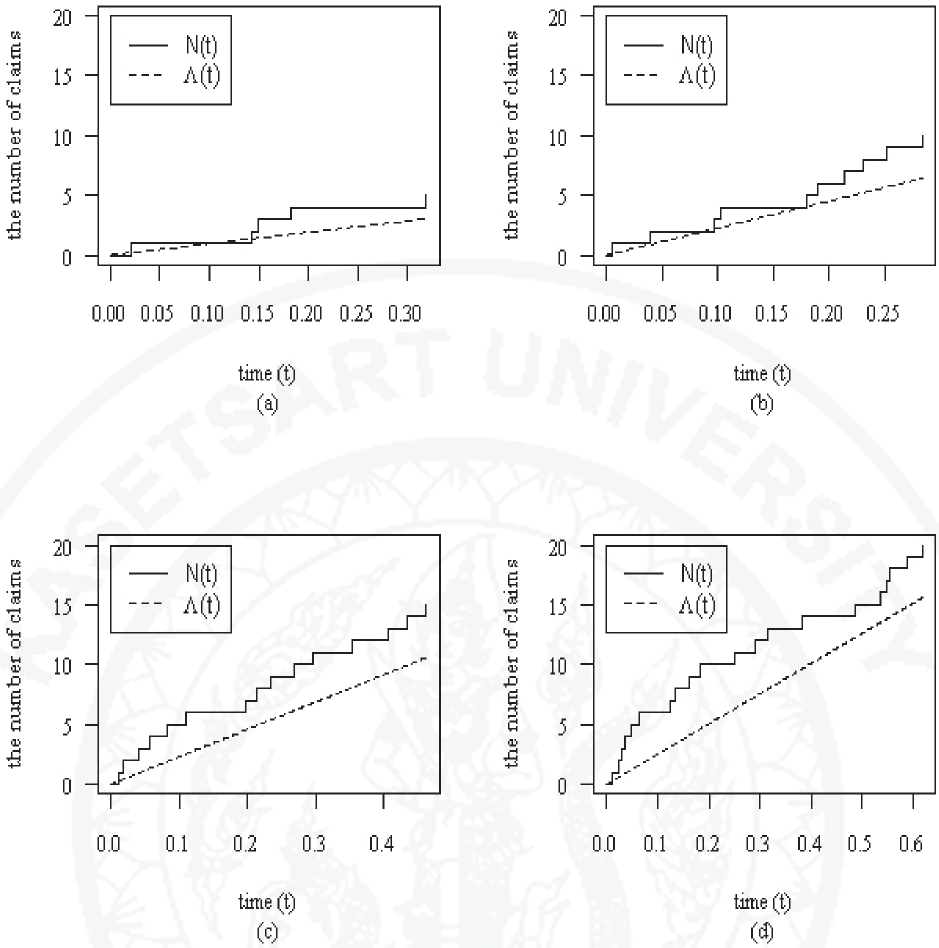


Figure 11 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 5, p = q = 1.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

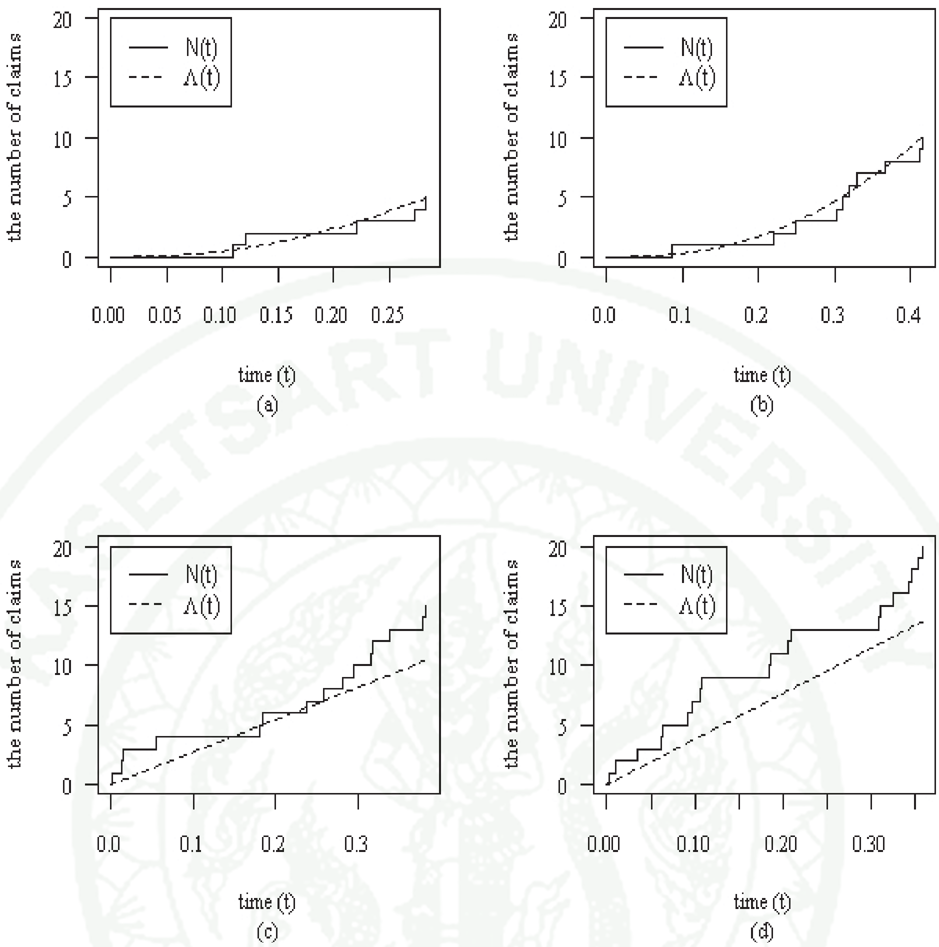


Figure 12 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 10, p = q = 1.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

2. The Estimation of Claim Counts Using the Estimating Function via a ZMM and the BE for Estimating the Model Parameters of Claim Intensity

In Part 2.1-2.3, some samples of 5, 10, 15 and 20 claims, illustrate the behavior of $\Lambda(t)$ relating to a specified $N(t)$ where the parameters of claim intensity of the process are estimated by the BE. These illustrated behaviors satisfy the parameter estimation approach of the claim counting process using the estimating function via a ZMM, i.e. $N(t)-\Lambda(t)=0$ and then $\Lambda(t)=N(t)$, where $M(t)=N(t)-\Lambda(t)$ is the martingale.

2.1 The estimation of claim counts based on the HPP

Some samples of 5, 10, 15 and 20 claims based on the HPP are shown in Figure 13-15. The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the process with a small claim occurrence rate $\lambda=0.1$, is illustrated in Figure 13. The $\Lambda(t)$ fits to $N(t)$ while while the number of observations is small. As, the behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the processes with $\lambda=5$ and $\lambda=10$ is depicted respectively in Figures 14 and 15, the $\Lambda(t)$ fits well with $N(t)$ where the number of observations is slightly larger than the value of λ.

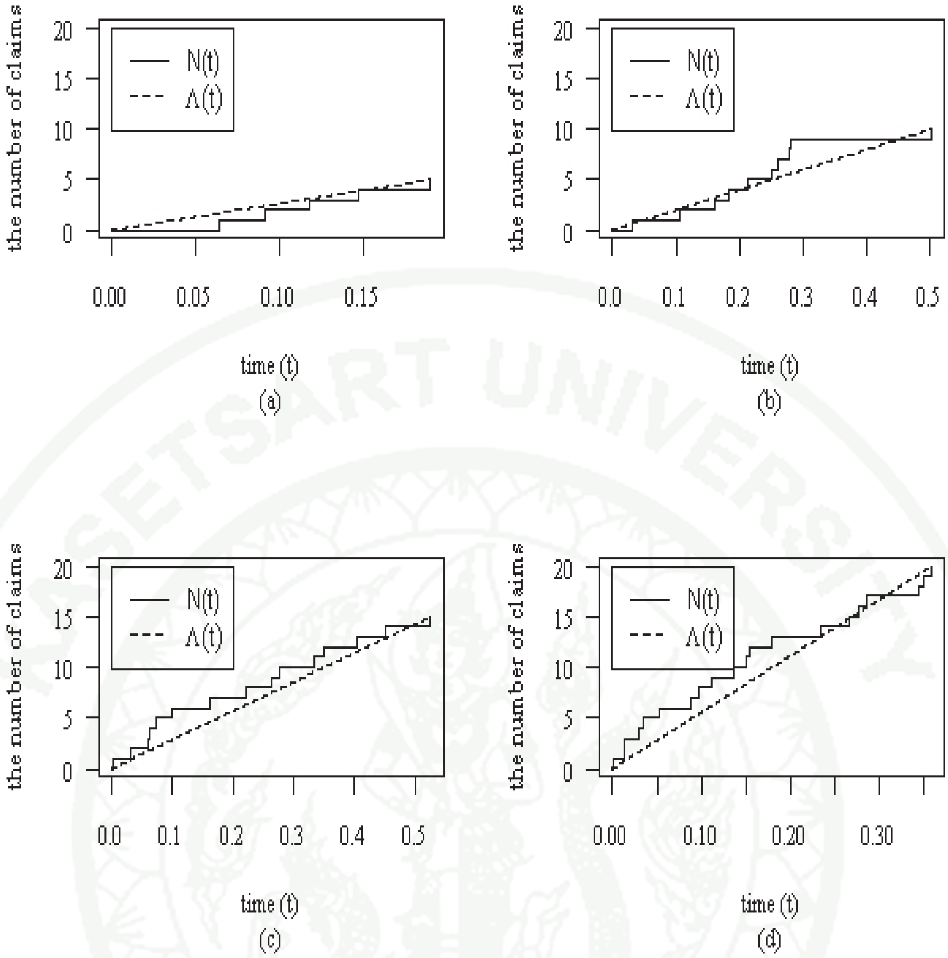


Figure 13 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the HPP with the claim intensity $\lambda = 0.1$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

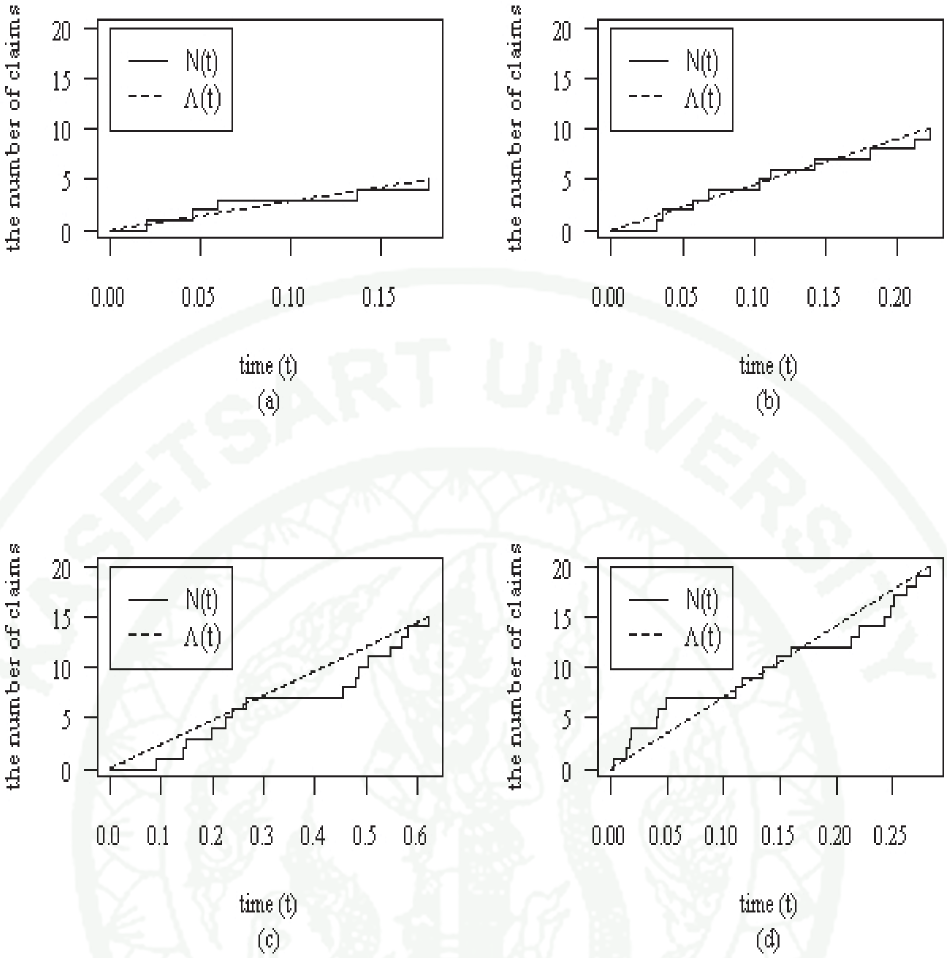


Figure 14 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the HPP with the claim intensity $\lambda = 5$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

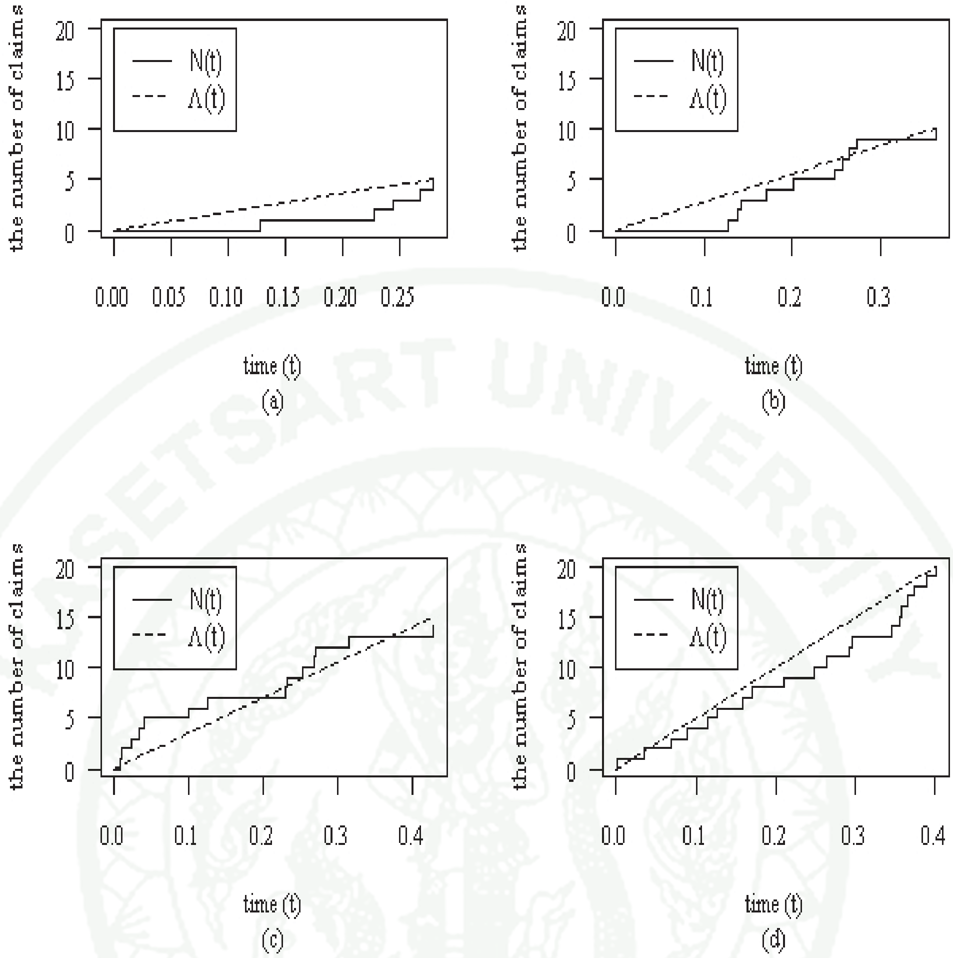


Figure 15 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the HPP with the claim intensity $\lambda = 10$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

2.2 The estimation of claim counts based on the NHPP with a bell-shaped intensity

Some samples of 5, 10, 15 and 20 claims based on the NHPP with a bell-shaped intensity are shown in Figure 16-18. The parameters of bell-shaped intensity are estimated by using the BE. The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the process with $\lambda^* = 0.1, \sigma = 5$, a small number of average claims or almost no average claims over a period, is depicted in Figure 16. The $\Lambda(t)$ is a good fit to $N(t)$ with a small number of observations. On the processes with $\lambda^* = 5, \sigma = 5$ and $\lambda^* = 10, \sigma = 5$, the behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on these processes is illustrated respectively in Figure 17 and 18, the $\Lambda(t)$ fits well with $N(t)$ where the number of observations is slightly larger than λ^*.

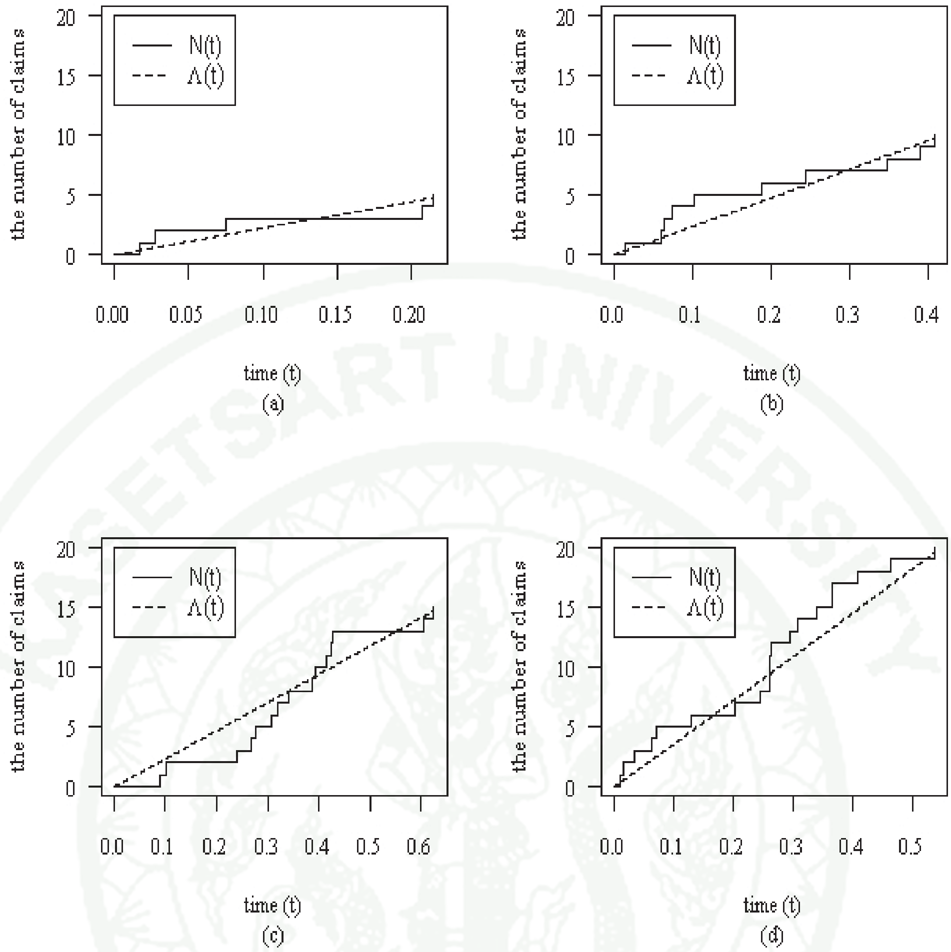


Figure 16 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 0.1, \sigma = 5$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

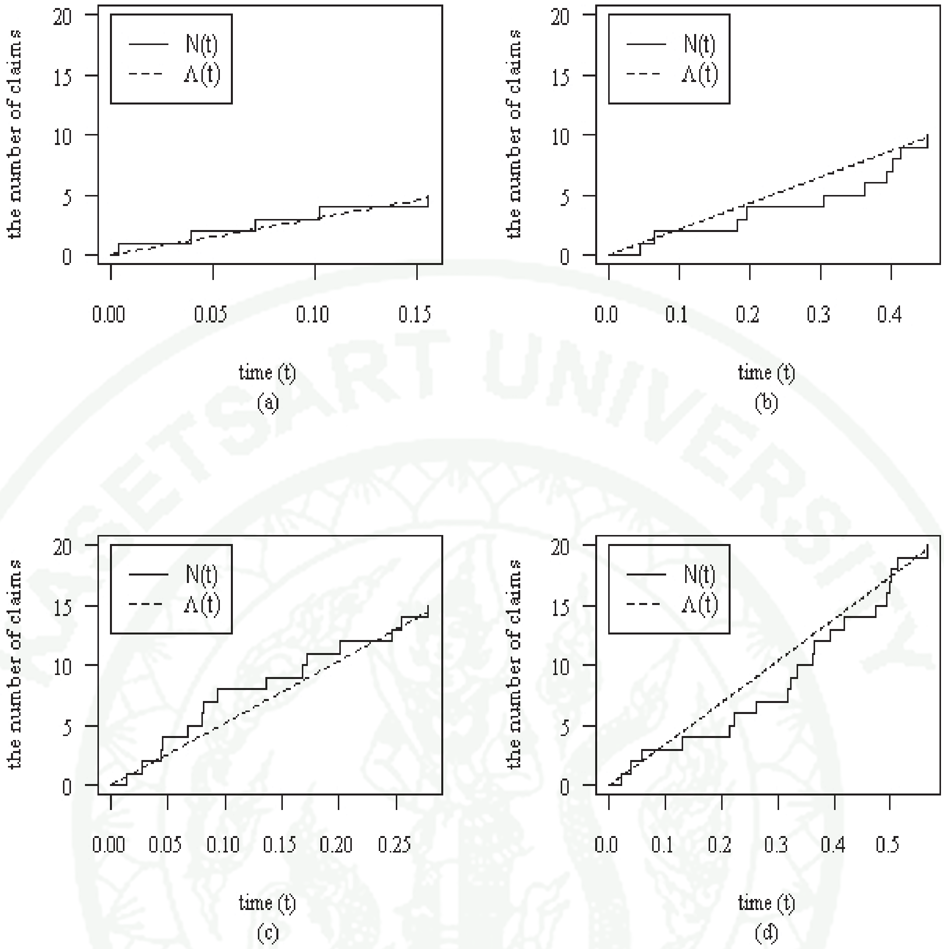


Figure 17 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 5, \sigma = 5$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

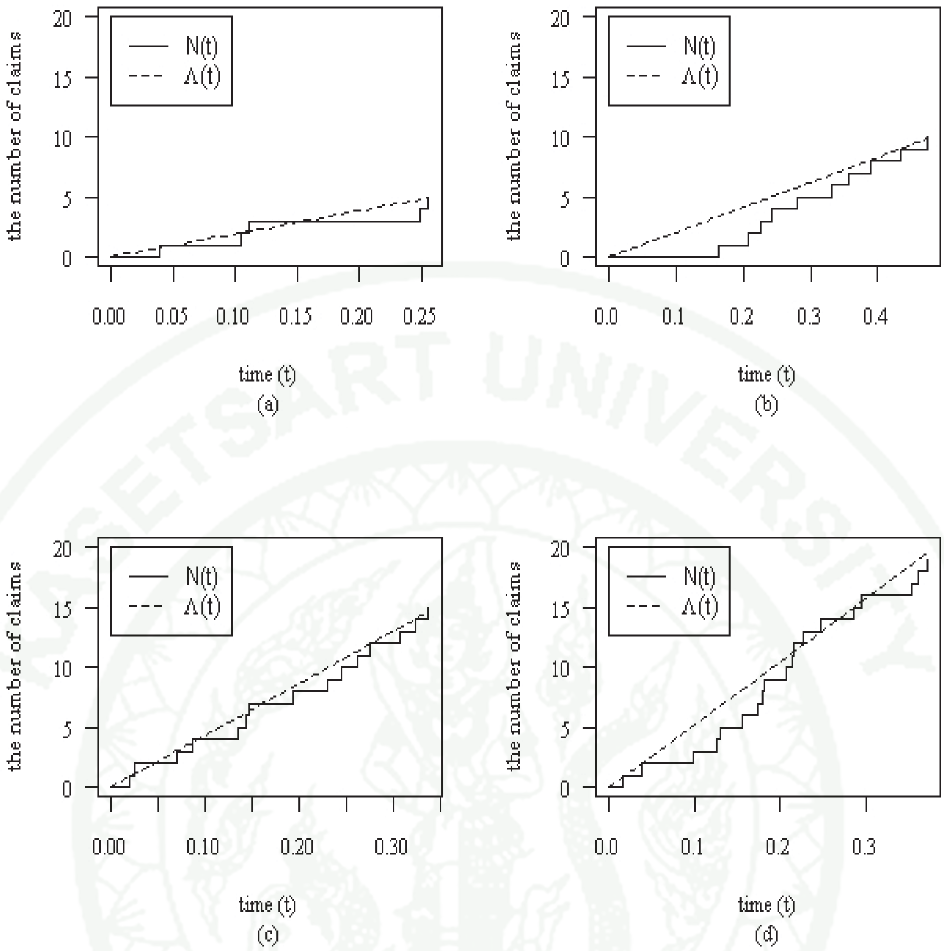


Figure 18 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 10, \sigma = 5$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

2.3 The estimation of claim counts based on the NHPP with a beta-shaped intensity

In this part, the sample paths of 5, 10, 15 and 20 claims are some examples of the NHPP with a beta-shaped intensity. In Figure 19-21, the behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the processes with $\lambda^* = 0.1, p = q = 1.25$; $\lambda^* = 5, p = q = 1.25$ and $\lambda^* = 10, p = q = 1.25$, is illustrated respectively. Notice that the $\Lambda(t)$ fits well with $N(t)$ while the number of observations is small.

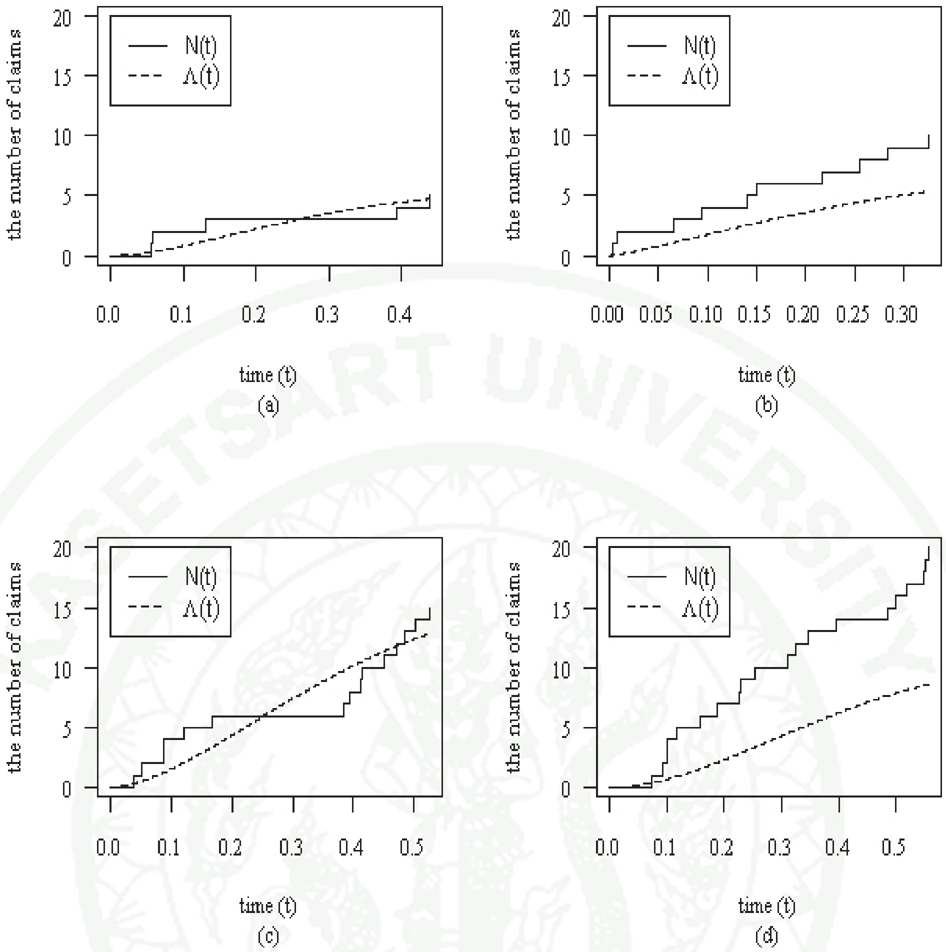


Figure 19 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 0.1, p = q = 1.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

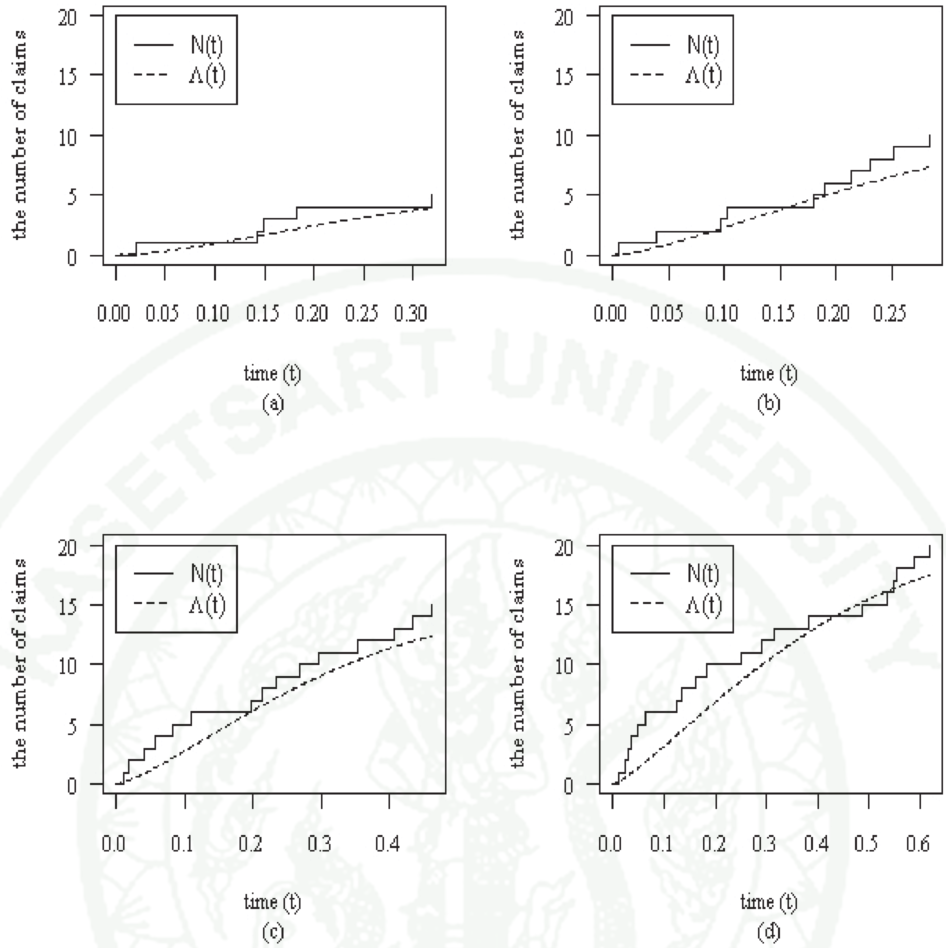


Figure 20 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 5, p = q = 1.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

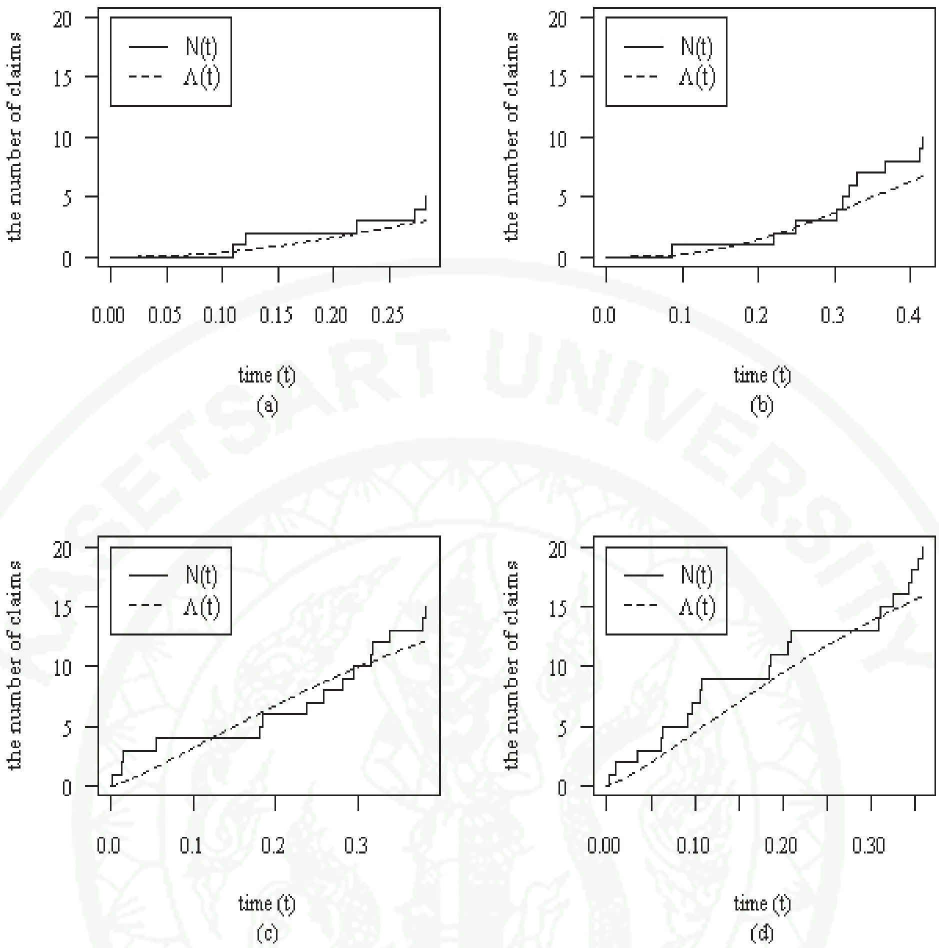


Figure 21 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 10, p = q = 1.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

Some results will be appeared in Appendix A.

3. A Comparison of the Estimation of Claim Counts Based on the Poisson Claim Counting Processes with the Approach of MLE and BE for Estimating the Model Parameters of Claim Intensity

In this part, we show the results of the relationship between the sample path $N(t)$ and the compensator estimate $\widehat{\Lambda(t)}$ with the MSE which consists of 5000 sample paths. Then, we consider a comparison of the estimation of claim counts with the approach of the MLE and the BE for estimating the model parameters of claim intensity.

3.1 The estimation of claim counts based on the HPP

On the HPP using the MLE and the BE for estimating the model parameters of claim intensity, the MSE of the $\widehat{\Lambda(t)}$ to $N(t)$ is illustrated respectively in Table 2 and Table 3. It should be noticed that the MSE relates to the characteristic of claim occurrence rate or the claim intensity of process. On the process with $\lambda = 0.1$, a small claim intensity or almost no claims, the $\widehat{\Lambda(t)}$ of $N(t)$ has a small MSE where the number of observations is small. As the processes with $\lambda = 5$ and $\lambda = 10$, the $\widehat{\Lambda(t)}$ of $N(t)$ based on these processes has a small MSE where the number of observations is slightly larger than a constant claim intensity rate λ.

On the HPP with $\lambda = 0.1$, $\lambda = 5$ and $\lambda = 10$, the $\widehat{\Lambda(t)}$ of $N(t)$ using the BE fits $N(t)$ very well and it has a slightly smaller MSE than the $\widehat{\Lambda(t)}$ using the MLE.

Table 2 The MSE of the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$ of the HPP, using the MLE for estimating the model parameters of claim intensities

λ	$N(t)$	MSE	λ	$N(t)$	MSE	λ	$N(t)$	MSE
0.1	5	1.0038	5	5	2.8385	10	5	11.902
	10	1.8341		10	2.1861		10	5.7769
	15	2.6659		15	2.7163		15	4.9041
	20	3.5288		20	3.3388		20	5.0125

Table 3 The MSE of the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$ of the HPP, using the BE for estimating the model parameters of claim intensities

λ	$N(t)$	MSE	λ	$N(t)$	MSE	λ	$N(t)$	MSE
0.1	5	0.9889	5	5	2.7457	10	5	11.579
	10	1.8160		10	2.1429		10	5.6641
	15	2.6451		15	2.6792		15	4.8124
	20	3.5020		20	3.2979		20	4.9137

3.2 The estimation of claim counts based on the NHPP with a bell-shaped intensity

On the NHPP with bell-shaped intensities using the MLE and the BE for estimating the model parameters of claim intensities, the MSE of the $\widehat{\Lambda(t)}$ to $N(t)$ is illustrated respectively in Table 4 and Table 5. Then, we found that the MSE relates to the characteristic of claim occurrence rate or the parameters of claim intensity. On the processes with the parameters of claim intensities $\lambda^* = 0.1, \sigma = 0.25$ and $\lambda^* = 0.1, \sigma = 5$, a small number of average claims over a period, the $\widehat{\Lambda(t)}$ of $N(t)$ has a small MSE where the number of observations is small. It should be noticed that the $\widehat{\Lambda(t)}$ of the process with $\lambda^* = 0.1, \sigma = 5$, has a slightly smaller MSE than the process with $\lambda^* = 0.1, \sigma = 0.25$ where the number of observations is small. For the parameters of claim intensities $\lambda^* = 5, \sigma = 0.25$; $\lambda^* = 5, \sigma = 5$; $\lambda^* = 10, \sigma = 0.25$ and $\lambda^* = 10, \sigma = 5$, the $\widehat{\Lambda(t)}$ has a small MSE where the number of observations is slightly larger than λ^*. The $\widehat{\Lambda(t)}$ of the processes with $\lambda^* = 5, \sigma = 5$ and $\lambda^* = 10, \sigma = 5$ has a smaller MSE than the processes with $\lambda^* = 5, \sigma = 0.25$ and $\lambda^* = 10, \sigma = 0.25$, respectively, where the number of observations is slightly larger than λ^*.

Table 4 The MSE of the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$ of the NHPP with a bell-shaped intensity, using the MLE for estimating the model parameters of claim intensities

λ^*	σ	$N(t)$	MSE	λ^*	σ	$N(t)$	MSE	λ^*	σ	$N(t)$	MSE
0.1	0.25	5	0.8719	5	0.25	5	2.6902	10	0.25	5	75.963
		10	1.7107			10	2.1129			10	7.6316
		15	2.4583			15	2.6879			15	5.3518
		20	3.2679			20	3.1563			20	5.3790
0.1	5	5	0.8316	5	5	5	5.1256	10	5	5	23.741
		10	1.0593			10	2.1150			10	7.6030
		15	1.6256			15	2.4772			15	4.7581
		20	2.4606			20	3.3408			20	4.7384

On the NHPP with the parameters of bell-shaped intensities $\lambda^* = 0.1, \sigma = 0.25$ and $\lambda^* = 0.1, \sigma = 5$, the MSE of the $\widehat{\Lambda(t)}$ using the MLE and the BE is almost the same and the $\widehat{\Lambda(t)}$ fits well with $N(t)$ where the number of observations is small. For the processes with $\lambda^* = 5, \sigma = 0.25$; $\lambda^* = 5, \sigma = 5$; $\lambda^* = 10, \sigma = 0.25$ and $\lambda^* = 10, \sigma = 5$, the $\widehat{\Lambda(t)}$ using the BE fits well with $N(t)$ and it has a smaller MSE than the $\widehat{\Lambda(t)}$ using the MLE, where the number of observations is slightly larger than λ^*.

Table 5 The MSE of the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$ of the NHPP with a bell-shaped intensity, using the BE for estimating the model parameters of claim intensities

λ^*	σ	$N(t)$	MSE	λ^*	σ	$N(t)$	MSE	λ^*	σ	$N(t)$	MSE
0.1	0.25	5	0.8732	5	0.25	5	2.0310	10	0.25	5	5.9688
		10	1.6634			10	1.9013			10	5.6767
		15	2.4288			15	2.5718			15	4.6307
		20	3.1416			20	3.1726			20	4.8802
0.1	5	5	0.8219	5	5	5	2.2731	10	5	5	9.8067
		10	1.5358			10	1.7573			10	4.7974
		15	2.4374			15	2.2822			15	4.0117
		20	3.1801			20	3.0815			20	4.1293

3.3 The estimation of claim counts based on the NHPP with a beta-shaped intensity

On the NHPP with beta-shaped intensities using the MLE for estimating the model parameters of claim intensity, the MSE of the $\widehat{\Lambda(t)}$ to $N(t)$ is illustrated in Table 6. It should be noticed that the MSE relates to the characteristic of the claim occurrence rate or the model parameters of claim intensity. On the processes with the parameters of claim intensities $\lambda^* = 0.1$, $p = q = 1.25$; $\lambda^* = 0.1, p = q = 2$ and $\lambda^* = 0.1, p = q = 3$, a small peak level or almost no peak level of claim intensity, the $\widehat{\Lambda(t)}$ of $N(t)$ has a small MSE where the number of observations is small. The $\widehat{\Lambda(t)}$ of the process with $\lambda^* = 0.1$, $p = q = 1.25$, has a smaller MSE than the processes with $\lambda^* = 0.1$, $p = q = 2$ and $\lambda^* = 0.1$, $p = q = 3$. For the processes with $\lambda^* = 5$, $p = q = 1.25$; $\lambda^* = 5$, $p = q = 2$; $\lambda^* = 5, p = q = 3$; $\lambda^* = 10$, $p = q = 1.25$; $\lambda^* = 10$, $p = q = 2$ and

$\lambda^* = 10, p = q = 3$, the $\widehat{\Lambda(t)}$ of $N(t)$ has a small MSE where the number of observations is equal to the value of λ^*. The $\widehat{\Lambda(t)}$ of process with $\lambda^* = 5, p = q = 3$, has a smaller than the processes with $\lambda^* = 5, p = q = 1.25$ and $\lambda^* = 5, p = q = 2$ where the number of observations is equal to the value of λ^*. Similarly, the $\widehat{\Lambda(t)}$ of process with $\lambda^* = 10, p = q = 3$, has a smaller than the processes with $\lambda^* = 10, p = q = 1.25$ and $\lambda^* = 10, p = q = 2$ where the number of observations is equal to the value of λ^*.

Table 6 The MSE of the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$ of the NHPP with a beta-shaped intensity, using the MLE for estimating the model parameters of claim intensities

λ^*	p,q	$N(t)$	MSE	λ^*	p,q	$N(t)$	MSE	λ^*	p,q	$N(t)$	MSE
0.1	1.25	5	0.7610	0.1	2	5	0.9748	0.1	3	5	0.9424
		10	5.0783			10	3.7001			10	5.1379
		15	12.357			15	9.0656			15	11.423
		20	24.932			20	20.783			20	17.230
5	1.25	5	1.6018	5	2	5	0.7067	5	3	5	0.5981
		10	3.8040			10	2.6543			10	2.0261
		15	8.2539			15	7.3443			15	7.3047
		20	14.965			20	16.599			20	17.880
10	1.25	5	3.1825	10	2	5	2.4405	10	3	5	1.7362
		10	2.4588			10	1.7389			10	1.5504
		15	5.1993			15	5.1547			15	5.3016
		20	18.603			20	15.5505			20	15.829

On the NHPP with beta-shaped intensities using the BE for estimating the model parameters of claim intensity, the MSE of the $\widehat{\Lambda(t)}$ to $N(t)$ is shown in Table 7. It should be noticed that the MSE relates to the characteristic of the parameters of claim intensity. The considered processes consist of the parameters of claim intensities $\lambda^* = 0.1$, $p = q = 1.25$; $\lambda^* = 0.1$, $p = q = 2$; $\lambda^* = 0.1$, $p = q = 3$; $\lambda^* = 5$, $p = q = 1.25$; $\lambda^* = 5$, $p = q = 2$; $\lambda^* = 5, p = q = 3$; $\lambda^* = 10$, $p = q = 1.25$; $\lambda^* = 10$, $p = q = 2$ and $\lambda^* = 10, p = q = 3$. The $\widehat{\Lambda(t)}$ of these processes has a small MSE where the number of observations is small. The MSE of the processes with $\lambda^* = 0.1$, $p = q = 1.25$; $\lambda^* = 0.1$, $p = q = 2$ and $\lambda^* = 0.1$, $p = q = 3$, is almost the same. Where the number of observations is small, the $\widehat{\Lambda(t)}$ of the processes with $\lambda^* = 5$, $p = q = 2$, has a smaller MSE than the processes with $\lambda^* = 5$, $p = q = 1.25$ and $\lambda^* = 5$, $p = q = 3$. Similarly, the

$\widehat{\Lambda(t)}$ of the processes with $\lambda^* = 10$, $p = q = 2$, has a smaller MSE than the processes with $\lambda^* = 10$, $p = q = 1.25$ and $\lambda^* = 10$, $p = q = 3$.

Table 7 The MSE of the compensator estimate $\widehat{\Lambda(t)}$ to $N(t)$ of the NHPP with a beta-shaped intensity, using the BE for estimating the model parameters of claim intensities

λ^*	p,q	$N(t)$	MSE	λ^*	p,q	$N(t)$	MSE	λ^*	p,q	$N(t)$	MSE
0.1	1.25	5	0.8771	0.1	2	5	0.8321	0.1	3	5	0.8876
		10	1.8831			10	2.0764			10	2.0686
		15	2.8220			15	3.0083			15	2.9132
		20	4.1709			20	4.1431			20	4.1007
5	1.25	5	0.9662	5	2	5	1.0353	5	3	5	1.2388
		10	2.7814			10	2.7365			10	1.9901
		15	3.6521			15	3.7148			15	3.3715
		20	4.7933			20	4.7861			20	4.8915
10	1.25	5	0.8860	10	2	5	0.2963	10	3	5	0.9786
		10	1.9716			10	2.4980			10	3.3801
		15	4.4699			15	3.7115			15	3.5939
		20	5.2463			20	6.1628			20	5.1463

On the NHPP with the parameters of beta-shaped intensities $\lambda^* = 0.1, p = q = 1.25$; $\lambda^* = 5, p = q = 1.25$ and $\lambda^* = 10, p = q = 1.25$, the $\widehat{\Lambda(t)}$ using the BE fits $N(t)$ very well and it has a smaller MSE than the $\widehat{\Lambda(t)}$ using the MLE where the number of observations is equal to the value of λ^*. While the processes with $\lambda^* = 0.1, p = q = 2$; $\lambda^* = 0.1, p = q = 3$; $\lambda^* = 5, p = q = 2$; $\lambda^* = 5, p = q = 3$; $\lambda^* = 10, p = q = 2$ and $\lambda^* = 10, p = q = 3$, the $\widehat{\Lambda(t)}$ using the MLE fits $N(t)$ very well and it has a smaller MSE than the $\widehat{\Lambda(t)}$ using the BE where the number of observations is equal to the value of λ^*.

4. The Prediction of Claim Counts Based on the Poisson Claim Counting Process

The task of actuaries in the non-life insurance industry is involved in the assessment of risks, such as setting the loss reserve and pricing, investment and reinsurance planning, and underwriting, etc. Then, both estimating and predicting claim counts play a very important role in the actuary's work. In this part we propose a prediction procedure which is a useful implement for actuaries and researchers.

We know that the accuracy of predicting claim counts depends on the available data from the historical claim period. Also, actuaries need to know which the historical claim period is the most suitable for a claim count prediction. In this part, the focus is on precise claim count prediction, the predicted claim counts are

$$N(t,t+p] = N(t+p) - N(t), \;\; t,p > 0,$$

i.e. for the history $\mathscr{F}_t$ which is the available data over a time interval $(0,t]$, we will calculate

$$E\left(N(t,t+p]|\mathscr{F}_t\right),$$

or it can be written by

$$E\left(N(t,t+p]|N(t)\right), \tag{12}$$

which is the predicted claim counts $N(t,t+p]$ given $N(t)$. For convenience in this study, an example of prediction procedure is illustrated through a simulation study of a sample path based on the HPP claim counting process which is an occurrence behavior of claims over any one year or a 360 day-period. We consider the prediction of claim counts during the prediction interval $(t,t+p]$ where p = 30, 60, 90, 120, 150, 180, 210, 240, 270, 300 and 330 day-period, and the predicted claim counts, $\widehat{\Lambda(t)}_{\mathrm{Pr}ed}$, is calculated by using the different historical claim periods of t = 30, 60, 90, 120, 150, 180, 210, 240, 270, 300 and 330 days. The parameter estimator $\widehat{\Lambda(t)}$ of the process, called the compensator estimate $\widehat{\Lambda(t)}$ of $N(t)$, is used to predict claim counts in upcoming periods. It is now provided by an estimating function, a ZMM, and the MLE provides the parameter estimate of claim intensity. Then the $\widehat{\Lambda(t)}$ is interpreted as the insurance claim counts during the historical claim period of $(0,t]$, and the $MSE = E\left(\int_{t}^{t+p}\left(\widehat{\Lambda(u)}_{\mathrm{Pr}ed} - N(u)\right)^2 du\right)$, is implemented to measure the prediction error.

4.1 Illustrated example

In this study, we follow the results of part 3.1. Both the MLE and the BE methods are used for the parameter estimation of the claim intensity of the process with the constant intensity $\lambda = 10$, and we found that the $\widehat{\Lambda(t)}$ of $N(t)$ has a minimum MSE where the number of observations is slightly larger than the value of λ. Thereby, we propose the prediction procedure through a simulated sample path of 15 claims, $N(t) = n$ =15, based on the HPP with a constant claim rate $\lambda = 10$. According to the occurrence behavior of claims of a sample path over one year as shown in Figure 22, this sample path is an example showing a prediction procedure which will be of interest to practicing actuaries. In this sample path, we consider the predicted claim counts given the different historical claim periods and then the prediction error of each prediction interval is measured by using the MSE.

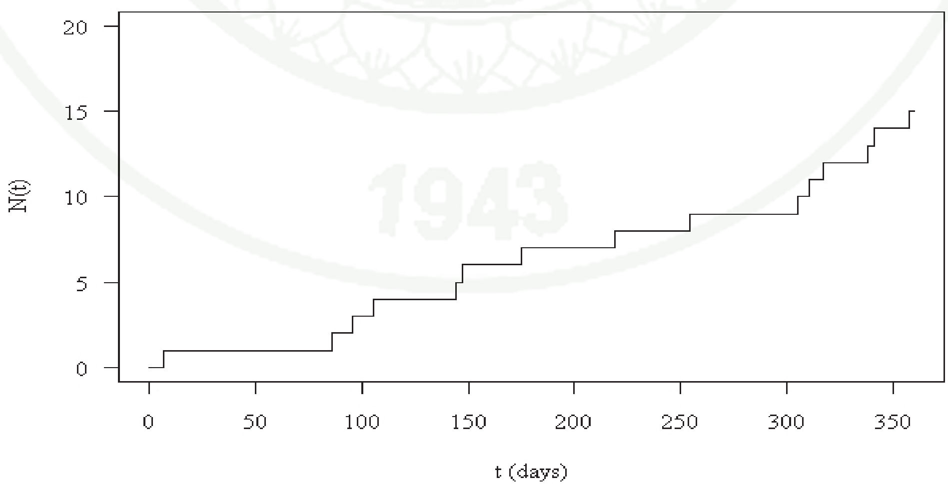


Figure 22 The occurrence behavior of claims over one year or a 360 day-period of a sample of 15 claims based on the HPP claim counting process with $\lambda = 10$

4.2 Simulation study

In Table 8 and Figure 23, we illustrate the MSE of predicting claim counts given the different historical claim periods. Notice that the historical claim period of 120 days is the most suitable for predicting claim counts in these cases because there are the smallest number of prediction errors. However, this historical claim period will be suitable for predicting claim counts over a shorter the period of time. Therefore, the claim count prediction in the next 90 day-period given the historical claim period of 120 days can be depicted as in Figure 24, which shows that the predicted claim counts, $\widehat{\Lambda(t)}_{Pred}$ shown by the dotted line, are very close to the actual claim counts, $N(t)$.

Table 8 The MSE of predicting claim counts of a sample path based on the HPP claim counting process

Prediction Interval (days)	Historical Claim Period of (t: days)										
	30	60	90	120	150	180	210	240	270	300	330
$(t,t+30]$	48	135	1.29	0.84	0.40	1.22	2.03	0.69	1.66	1.56	0.58
$(t,t+60]$	91	174	1.92	0.46	0.98	1.64	2.53	1.63	0.99	4.48	-
$(t,t+90]$	133	229	3.22	0.49	1.52	2.10	3.99	1.16	1.73	-	-
$(t,t+120]$	185	291	3.76	0.61	2.08	3.31	3.44	1.37	-	-	-
$(t,t+150]$	243	370	4.31	0.80	3.29	3.00	2.79	-	-	-	-
$(t,t+180]$	317	470	4.80	1.38	3.15	2.53	-	-	-	-	-
$(t,t+210]$	411	575	4.88	1.26	2.72	-	-	-	-	-	-
$(t,t+240]$	510	697	6.48	1.20	-	-	-	-	-	-	-
$(t,t+270]$	626	815	9.77	-	-	-	-	-	-	-	-
$(t,t+300]$	739	930	-	-	-	-	-	-	-	-	-
$(t,t+330]$	851	-	-	-	-	-	-	-	-	-	-

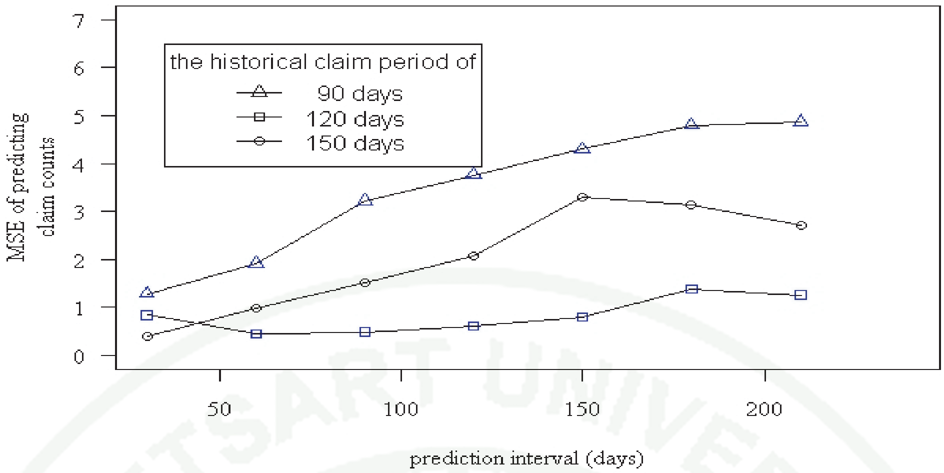


Figure 23 Comparing the MSE of predicting claim counts which is given by the historical claim periods of 90, 120 and 150 days of a sample path based on the HPP claim counting process

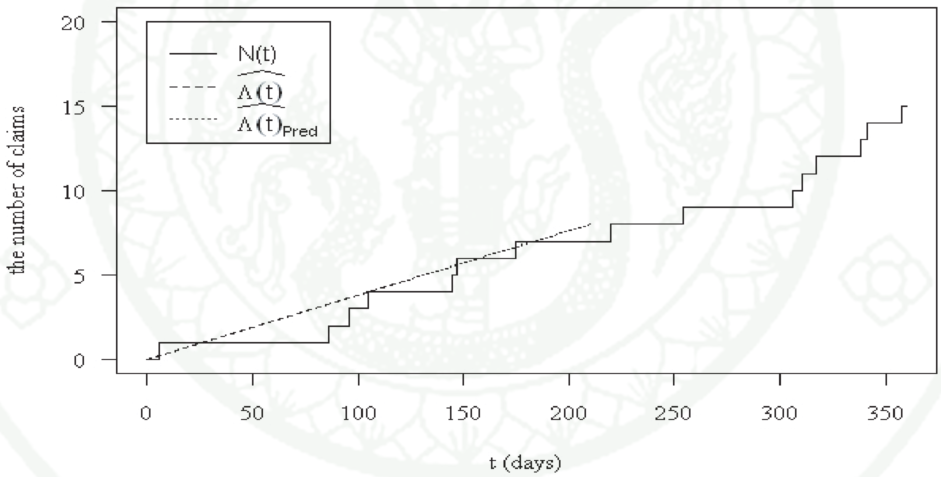


Figure 24 $N(t)$, its compensator estimate $\widehat{\Lambda(t)}$ and the predicted claim counts $\widehat{\Lambda(t)}_{Pred}$ during the prediction interval (120,210] or in the next 90 day-period, given the historical claim period of 120 days of a sample path based on the HPP claim counting process

5. Application

In this part, we illustrate the application of our approach to a real data set of claim counts of motor insurance from a non-life insurance company in Thailand. The data set of claim counts shown in Figure 25 is the individual claim data which these claims occurred during the year 2008. According to the occurrence behavior of claims over the one year, we analyze the estimation and prediction of claim counts based on the HPP claim counting process and the estimating function which is provided by the ZMM, is used to estimate the parameter of the process. Then, the MLE and the BE provide the parameter estimate of claim intensity. For the BE, the prior density of the parameter of claim intensity in the BE approach is defined as mentioned in section of METHODS.

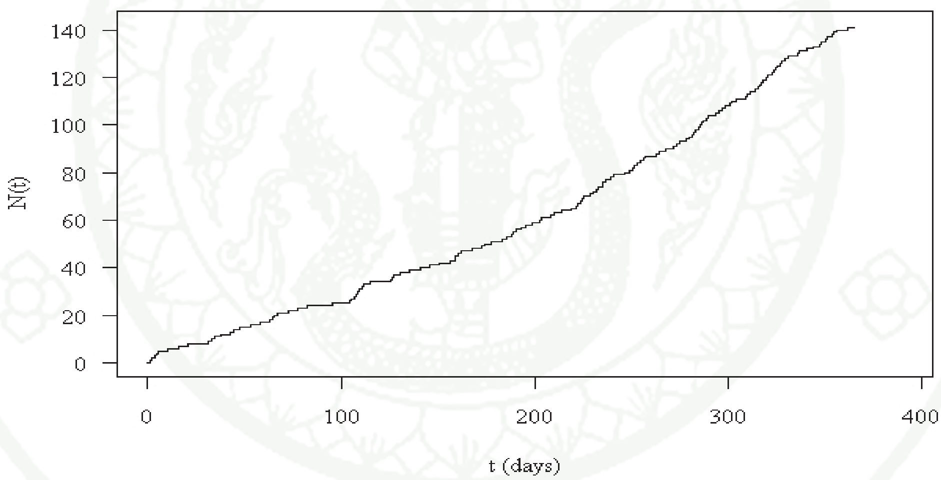


Figure 25 The occurrence behavior of claims during the year 2008 of motor insurance from a non-life insurance company in Thailand

We present prediction procedure with the claim data set. We need to find which the historical claim period is the most suitable for the claim count prediction and focus on precise claim count prediction. We will calculate

$$E\left(N(t,t+p]|N(t)\right),$$

and consider the prediction of claim counts during the prediction interval $(t,t+p]$ where $p = 30$, 60, 90, 120, 150, 180, 210, 240, 270, 300 and 330 day-period, and the predicted claim counts, $\widehat{\Lambda(t)}_{Pred}$, is calculated by using the different historical claim periods of $t =$ 30, 60,90, 120, 150, 180, 210, 240, 270, 300 and 330 days. The parameter estimator $\widehat{\Lambda(t)}$ of the process, called the compensator estimate $\widehat{\Lambda(t)}$ of $N(t)$, is used to predict claim counts in upcoming periods.

In Tables 9 and 10, we illustrate the MSE of predicting claim counts, where the parameter of claim intensity is estimated by using the MLE and the BE, respectively. We found that the historical claim period of 30 days is the most suitable because the prediction error is the smallest. But this historical claim period will be not suitable for predicting claim counts over a longer the period of time. Comparing the MLE and the BE, the MSE of predicting claim counts is slightly different when the predictive period of time is close to the historical claim period of 30 days. Then, the MLE has a much smaller MSE than the BE where the predictive period of time is very far from the historical claim period of 30 days.

Furthermore, the prediction of claim counts in the next (a) 30 day-period, (b) 60 day-period, (c) 90 day-period, (d) 120 day-period, (e) 150 day-period and (f) 180 day-period, are given the historical claim periods of 30 and 60 days, can be depicted respectively in Figures 26- 27. We found that the predicted claim counts in the next 30 day-period, given the historical claim period of 30 days, $\widehat{\Lambda(t)}_{\mathrm{Pr}ed}$, shown by the dotted line in Figure 26(a), are very close to the actual claim counts, $N(t)$.

Table 9 The MSE of predicting claim counts of motor insurance based on the HPP, using the MLE for estimating the parameter of claim intensity

Prediction Interval (days)	Historical Claim Period of (t: days)										
	30	60	90	120	150	180	210	240	270	300	330
$(t, t+30]$	0.32	1.80	7.50	6.05	1.93	4.40	28.9	2.19	23.7	27.4	1.08
$(t, t+60]$	0.91	5.42	5.09	5.88	8.96	34.1	67.2	40.0	95.2	37.3	-
$(t, t+90]$	3.12	5.09	4.05	4.05	40.6	82.2	164	126	132	-	-
$(t, t+120]$	3.03	4.77	3.32	10.5	93.4	187	323	177	-	-	-
$(t, t+150]$	2.08	3.88	12.0	29.7	202	356	422	-	-	-	-
$(t, t+180]$	2.53	9.32	33.8	84.8	376	465	-	-	-	-	-
$(t, t+210]$	8.88	25.3	91.0	188	490	-	-	-	-	-	-
$(t, t+240]$	25.7	71.1	195	259	-	-	-	-	-	-	-
$(t, t+270]$	71.5	158	268	-	-	-	-	-	-	-	-
$(t, t+300]$	157	220	-	-	-	-	-	-	-	-	-
$(t, t+330]$	219	-	-	-	-	-	-	-	-	-	-

Table 10 The MSE of predicting claim counts of motor insurance based on the HPP, using the BE for estimating the parameter of claim intensity

Prediction Interval (days)	Historical Claim Period of (t: days)										
	30	60	90	120	150	180	210	240	270	300	330
$(t, t+30]$	0.40	1.60	6.99	5.48	2.21	4.84	30.0	2.51	24.9	28.7	1.29
$(t, t+60]$	0.85	4.89	4.60	5.25	9.60	35.3	69.0	41.3	97.6	38.9	-
$(t, t+90]$	2.65	4.43	3.56	3.60	41.9	84.2	167	129	135	-	-
$(t, t+120]$	2.39	4.02	3.08	10.7	95.7	190	328	180	-	-	-
$(t, t+150]$	2.08	3.34	12.5	30.8	206	361	427	-	-	-	-
$(t, t+180]$	2.20	9.67	35.5	87.4	381	471	-	-	-	-	-
$(t, t+210]$	10.1	27.1	94.7	192	496	-	-	-	-	-	-
$(t, t+240]$	29.4	75.3	201	264	-	-	-	-	-	-	-
$(t, t+270]$	79.4	165	276	-	-	-	-	-	-	-	-
$(t, t+300]$	171	230	-	-	-	-	-	-	-	-	-
$(t, t+330]$	236	-	-	-	-	-	-	-	-	-	-

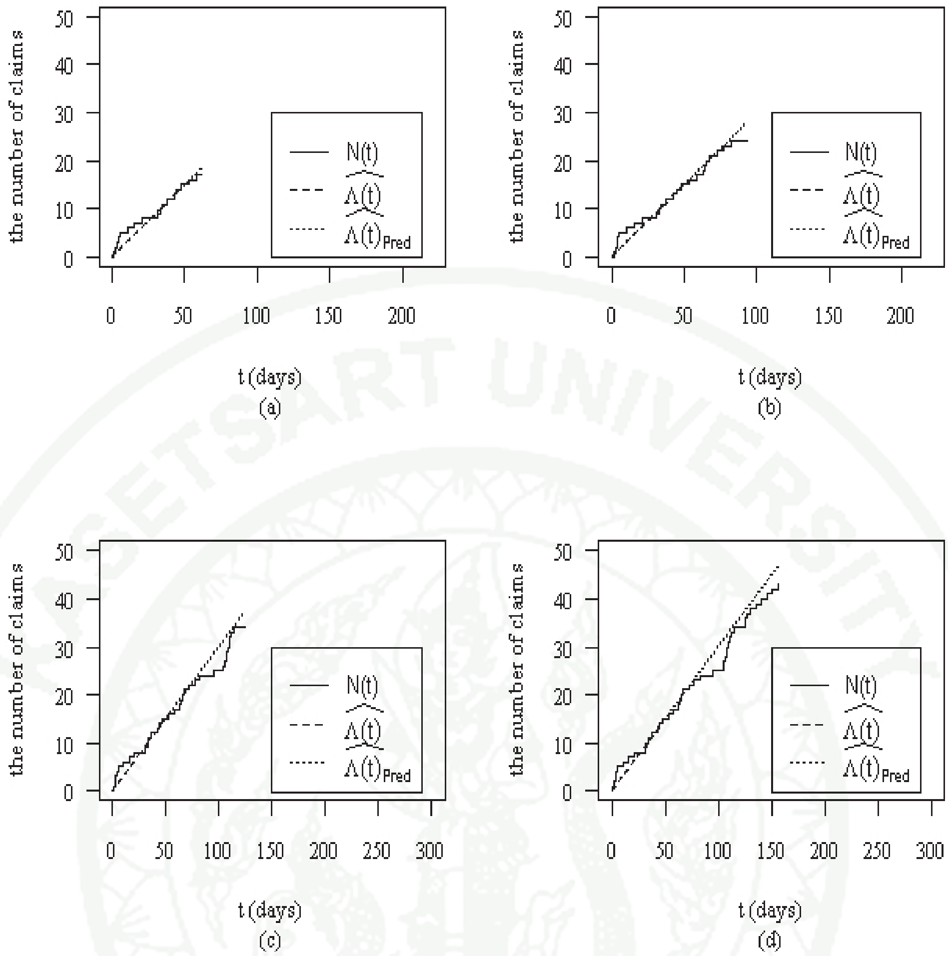


Figure 26 $N(t)$, its compensator estimate $\widehat{\Lambda(t)}$ and the predicted claim counts $\widehat{\Lambda(t)}_{Pred}$ in the next (a) 30 day-period, (b) 60 day-period, (c) 90 day-period, and (d) 120 day-period, given the claim historical period of 30 days, using the MLE for estimating the parameter of claim intensity

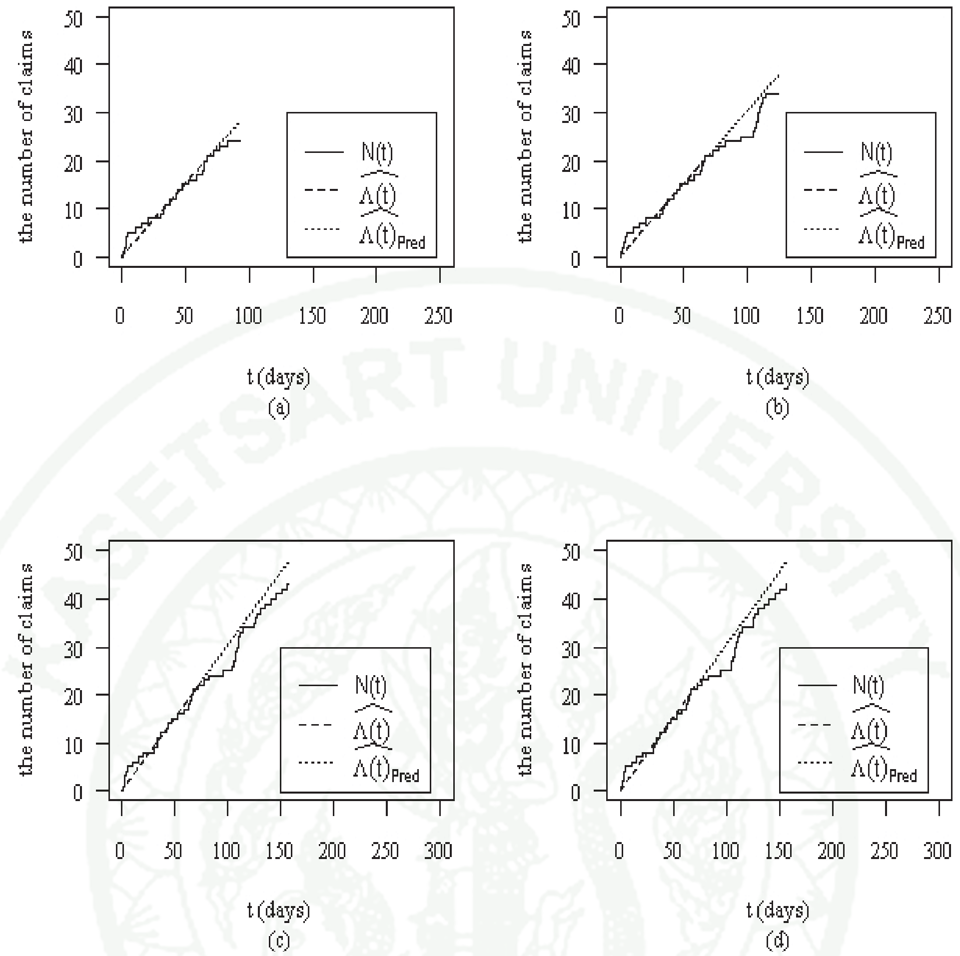


Figure 27 $N(t)$, its compensator estimate $\widehat{\Lambda(t)}$ and the predicted claim counts $\widehat{\Lambda(t)}_{Pred}$ in the next (a) 30 day-period, (b) 60 day-period, (c) 90 day-period, and (d) 120 day-period, using the MLE for estimating the parameter of claim intensity

Discussion

This study is an estimation approach to claim counts on the claim counting process. The parameter $\Lambda(t)$ of the process is estimated by using the estimating function which is provided by the martingale method, such as a ZMM. Then, the MLE and the BE are used to estimate the model parameters of claim intensities. A result of the parameter estimate $\widehat{\Lambda(t)}$ of the process can be interpreted as $N(t)$, called the compensator estimate of $N(t)$. We illustrate an example shown in Figure 28 and Figure 29 to depict a comparison of two estimation approaches to non-life insurance claim counts, including this insurance claim counting process and the Jewell's credibility approach. The approach of Jewell has been mentioned in section of Literature Review where $m = E(\Lambda) = \frac{\gamma}{\beta}$, $z = \frac{t}{\beta+t}$, $\gamma = 0.001$ and $\beta = 0.001$. In Figure 28, the compensator estimate $\widehat{\Lambda(t)}$ of $N(t)$ on the process (the dashed line), is a good fit to $N(t)$ over time $(0,t]$; on the other hand, the procedure of the credibility estimate $\widehat{\Lambda(t)}_{CRED}$ (+) on the credibility model does not consider the occurrence behavior of claims over the time. Furthermore, the estimation approach to claim counts on the claim counting process is also sufficient for analyzing the assessment of risks in the field of non-life insurance.

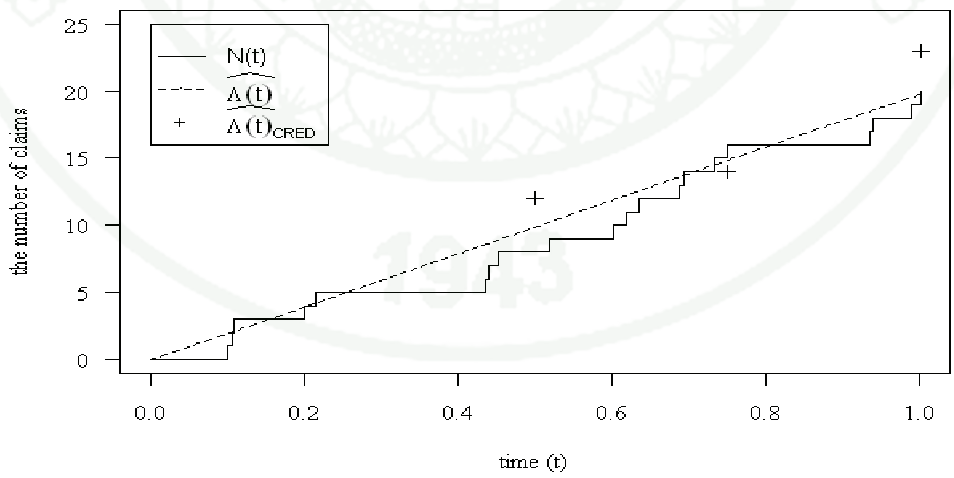


Figure 28 $N(t)$, $\widehat{\Lambda(t)}$ and $\widehat{\Lambda(t)}_{CRED}$ on the non-life insurance claim counting process

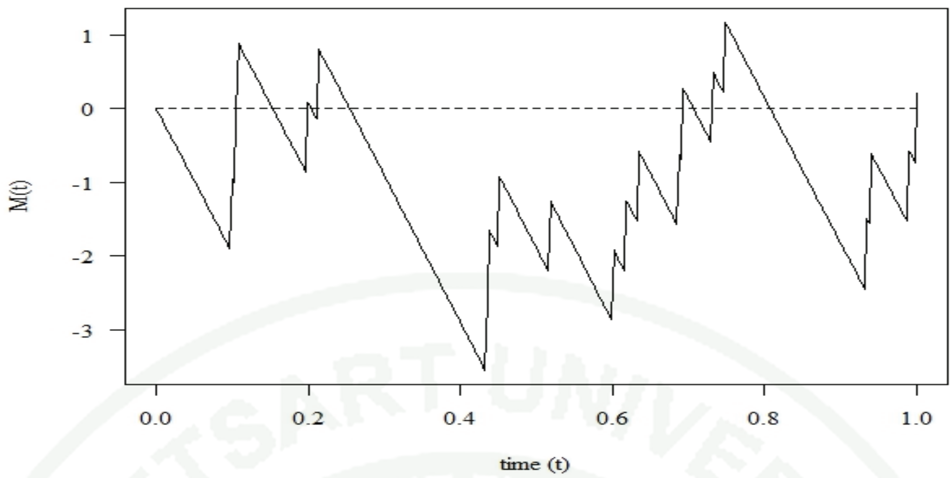


Figure 29 The martingale $M(t) = N(t) - \widehat{\Lambda(t)}$ based on the same data as in Figure 28

CONCLUSION AND RECOMMENDATION

Conclusion

In the field of non-life insurance industry, actuaries need to know how the occurrence behavior of claim counts changes over time. A knowledge of this occurrence behavior of claim counts can be useful for the assessment of risks which is the task of an actuary. Then, the precision of estimating and predicting claim counts on the claim counting process is the key to running an insurance business successfully. In this thesis, the approach of an estimating function provided by the martingale method, namely the ZMM, is proposed as a procedure for the parameter estimation of the claim counting process, claim intensity function $\lambda(t)$ in term of the mean value function $\Lambda(t)$. In this approach, the parameter $\Lambda(t)$ of the claim counting process can be interpreted as the claim counts $N(t)$ over the time interval $(0,t]$, which is called the compensator $\Lambda(t)$ of $N(t)$. The processes in this study consist of HPP, NHPP claim counting process with bell-shaped and beta-shaped intensities. Also, the model parameters of claim intensities are estimated by using the MLE and the BE methods. We present this approach through a simulation study.

From the results of the HPP claim counting process, the compensator estimate $\widehat{\Lambda(t)}$ is a good fit to $N(t)$ with a small MSE when the number of observations is slightly larger than a constant intensity rate λ. Comparing the $\widehat{\Lambda(t)}$ of $N(t)$ using the MLE and the BE for estimating claim intensity, the $\widehat{\Lambda(t)}$ which uses the BE method for estimating claim intensity provides a slightly better fit to $N(t)$.

Similarly, fitting the $\widehat{\Lambda(t)}$ to $N(t)$ on the NHPP claim counting process with a bell-shaped intensity relates to the characteristic of claim occurrence rate or the model parameters of claim intensities, i.e., the $\widehat{\Lambda(t)}$ is a good fit to $N(t)$ where the number of observations is slightly larger than the value of parameter λ^* of claim intensity. When we compare $\widehat{\Lambda(t)}$ of $N(t)$ using the MLE and the BE for estimating claim intensity, the $\widehat{\Lambda(t)}$ which uses the BE for estimating parameters of claim intensity provides a much better fit to $N(t)$.

With regard to the results of the NHPP claim counting process with a beta-shaped intensity, which fits $\widehat{\Lambda(t)}$ to $N(t)$ using the MLE and the BE for estimating parameters of claim intensity, the model parameters of the claim intensities are as follows: firstly, for the beta-shaped intensity with the parameter λ^* a small peak level occurs over a period, and every $p = q$, the $\widehat{\Lambda(t)}$ fits well with $N(t)$ where the number of observations is small. The $\widehat{\Lambda(t)}$ using the MLE is slightly better fit to $N(t)$ than the the $\widehat{\Lambda(t)}$ using the BE where the model parameters of claim intensity $p = q$ are slightly more than 1. Where the model parameters of beta-shaped intensities λ^* is a peak level occurs over a period, and $p = q$ are slightly more than 1, the $\widehat{\Lambda(t)}$ using the BE is a better fit to N(t) than the $\widehat{\Lambda(t)}$ using the MLE and the best fit to $N(t)$ where the number of observations is equal to the value of λ^*. Lastly, where the model parameters of beta-shaped intensities λ^* is a peak level occurs over a period, and $p = q$ are much more than 1, the $\widehat{\Lambda(t)}$ using the MLE is the best fit and a better fit to $N(t)$ than the $\widehat{\Lambda(t)}$ using the BE where the number of observations is equal to the value of λ^*.

Additionally, this study shows a procedure for the estimation and the prediction of claim counts by using the examples of sample paths from simulated data and real insurance claim data from a non-life insurance company in Thailand. These procedures can also provide a clear and useful guide to actuaries and researchers in their works.

Recommendation

The occurrence of claims for non-life insurance over time has different characteristics. These are described by the claim intensity, i.e., the claim intensity is always a constant and varies with time (Morales, 2004; Lu and Garrido, 2005). In this research, the occurrence of claims with the period time-dependent intensity rate includes the forms of bell-shaped and beta-shaped intensities. Both bell-shaped and beta-shaped intensities depicted in Figure 1 have a high level of claims in the middle of the period.

A knowledge of the occurrence behavior of insurance claim counts over time, called the insurance claim counting process, will be useful for the assessment of risk which is the task of an actuary. Actuaries always face the problems of a complicated model of claim counts in the insurance claim counting process. For instance, the parameter of the NHPP claims counting process, claim intensity function $\lambda(t)$ in term of mean value function $\Lambda(t) = \int_0^t \lambda(u)du$, makes a complicated distribution function of insurance claim counts and it is not easy to estimate the claim counts. So, an estimating function, such as the ZMM which we propose in this research, is an alternative estimation approach as a procedure of estimating the parameter of the NHPP model. This approach provides an estimated parameter, $\widehat{\Lambda(t)}$, of the NHPP. The $\widehat{\Lambda(t)}$ can be interpreted as the claim counts $N(t)$ over a time interval $(0,t]$ and it is also useful for predicting the time of claim occurrences or the claim counts in the further periods. Therefore, the procedure for estimating function, such as the ZMM, will povide an approach to the estimation of claim counts in the claim counting process and this technique is easy and practical for actuaries in the non-life insurance industry.

Further Research

The insurance claim counts is a component of a aggregate claim amounts or a sum of claims occurring a specified period. A compound Poisson process serves as the modeling of the aggregate claim amounts. We may study the estimation approach which is proposed in this research to find the aggregate claim amounts with respect to a compound Poisson process. Furthermore, we may investigate the ruin probability model that concerns both the claim counting process and the compound Poisson process.

LITERATURE CITED

Aggoun, L. 2004. Recursive estimation of the claim rates and sizes in an insurance model. **Journal of Applied Mathematics and Stochastic Analysis** 3: 245–259.

Andersen, P. K., Ø. Borgen, R. D. Gill and N. Keiding. 1992. **Statistical Models Based on Counting Processes**. Springer-Verlag, New York.

Bühlman, H. 1967. Introduction report experience rating and credibility. **ASTIN Bullentin** 4(3): 199–207.

Bühlmann, H. 1972. Credibility procedures, pp. 515–525. *In* **Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability**, vol. 1. Berkeley, United States.

Bühlmann, H. and A. Gisler. 2005. **A Course in Credibility Theory and its Applications**. Springer-Verlag, Berlin.

Burnecki, K., W. Härdle and R. Weron. 2004. Simulation of risk processes. **Encyclopedia of Actuarial Science** 1: 1564–1570.

Cid, J. E. R. and J. A. Achcar. 1999. Bayesian inference for non-homogeneous Poisson processed in software reliability models assuming nonmonotonic intensity functions. **Computational Statistics & Data Analysis** 32(2): 147–159.

Denuit, M., X. Maréchal, S. Pitrebois and J. F. Walhin. 2007. **Actuarial Modelling of Claim Counts**. John Wiley & Sons, Hoboken.

Frees, E. W. and P. Wang. 2006. Copula credibility for aggregate loss models. **Insurance: Mathematics and Economics** 38: 360–373.

Garrido, J. and Y. Lu. 2004. On double pariodic non-homogeneous Poisson processes. **Bulletin of the Association of Swiss Actuaries** 2: 195–212.

Heyde, C. C. 2001. Shifting paradigms in inference. **Lecture Notes-Monograph Series** 37: 9–21.

Jaroengeratikun, U. and W. Bodhisuwan. 2009. A comparison of parameter estimation of aggregate claims under Pareto distribution, pp. 509–518. *In* **National Conference of Applied Statistics**. School of Applied Statistics, National Institute of Development Administration, Pattaya, Chonburi, Thailand.

Jaroengeratikun, U., W. Bodhisuwan and A. Thongteeraparp. 2012a. A Bayesian inference of non-life insurance based on claim counting process with periodic claim intensity. **Open Journal of Statistics** 2(2): 177–183.

Jaroengeratikun, U., W. Bodhisuwan and A. Thongteeraparp. 2012b. A statistical analysis of intensities estimation on the modeling of non-life insurance claim counting process. **Applied Mathematics** 3(1): 100–106.

Jewell, W. S. 1974. Credible means are exact bayesian for exponential families. **ASTIN Bulletin** 8(1): 77–90.

Kaas, R., D. Dannenburg and M. Goovaerts. 1997. Exact credibility for weighted observations. **ASTIN Bulletin** 27(2): 287–295.

Kasozi, J. and J. Paulsen. 2005. Numerical Ultimate Ruin Probabilities under Interest Force. **Journal of Mathematics and Statistics** 1(3): 246–251.

Klugman, S. A., H. H. Panjer and G. E. Willmot. 2008. **Loss Models from Data to Decisions**. 3rd ed. John Wiley & Sons, Hoboken.

Lu, Y. and J. Garrido. 2005. Doubly periodic non-homogeneous Poisson models for hurricane data. **Statistical Methodology** 2(1): 17–35.

Lunn, D. J., A. Thomas, N. Best and D. Spiegelhalter. 2000. WinBUGS–A Bayesian Modelling Framework: Concepts, Structure, and Extensibility. **Statistics and Computing** 10: 325–337.

Matsui, M. and T. Mikosch. 2010. Prediction in a Poisson cluster model. **Journal of Applied Probability** 47(2): 350–366.

Mikosch, T. 2009. **Non-Life Insurance Mathematics**. 2nd ed. Springer-Verlag, Berlin.

Morales, M. 2004. On a surplus process under a periodic environment: a simulation approach. **North American Actuarial Journal** 8(2): 76–87.

Mukhopadhyay, P. 2004. **An Introduction to Estimating Functions**. Alpha Science International Ltd., Harrow.

Norberg, R. 1993. Prediction of Outstanding Liabilities in Non-Life Insurance. **ASTIN Bulletin** 23(1): 95–115.

Ntzoufras, I. 2009. **Bayesian Modeling Using WinBUGS**. John Wiley & Sons, Hoboken.

Ohlsson, E. and B. Johansson. 2006. Exact credibility and Tweedie models. **ASTIN Bulletin** 36(1): 121–133.

R Development Core Team. 2008. **R: A Language and Environment for Statistical Computing**. R Foundation for Statistical Computing. Vienna, Austria. ISBN 3-900051-07-0.

Rolski, T., H. Schmidli, V. Schmidt and J. Teugels. 2000. **Stochastic Processes for Insurance and Finance**. John Wiley & Sons, Hoboken.

Ross, S. M. 1993. **Introduction to Probability Models**. 5 thed. Academic Press, Inc, San Diego.

Sturtz, S., U. Ligges and A. Gelman. 2005. R2WinBUGS: A Package for Running WinBUGS from R. **Journal of Statistical Software** 12(3): 1–16.

Walhin, J. F.. 2000. **Recursions for Actuaries and Applications in the Field of Reinsurance and Bonus-Malus Systems**. Ph.D. Thesis, Universite ´ Catholique de Lavain Institut de Statistique.

Walhin, J. F. and J. Paris. 2000. The True Claim Amount and Frequency Distribution within a Bonus-Malus System. **ASTIN Bulletin** 30(2): 391–403.

Walhin, J. F. and J. Paris. 2006. A mixed Poisson model with varying element sizes. **Statistical Methodology** 3: 456–463.

Ward, C. 1997. **Robust Theory Applied to Jewell's Hierarchical Credibility Theory Model**. M.S. Thesis, Concordia University, Canada. *Cited* H. Bühlman and E. Straub. 1970. Glaubwürdigkeit für Schadensätze. **Mitt. Ver. Schweiz. Ver.** 70(1): 111-133.

Yip, P. 1995. Estimating the number of error in a system using a martingale approach. **IEEE Transactions on Reliability** 44(2): 322–326.

Yip, S. F. P. 1988. **Statistical Inference via Martingale Theory in Biomedical Applications**. Ph.D. Thesis, La Trobe University.

Zhao, X., C. Yu and H. Tong. 2008. A Bayesian approach to Weibull survival model for clinical randomized censoring trail based on MCMC simulation, pp. 1181–1184. *In* **the Second International Conference on Bioinformatics and Biomedical Engineering (ICBBE 2008)**. Shanghai, China.

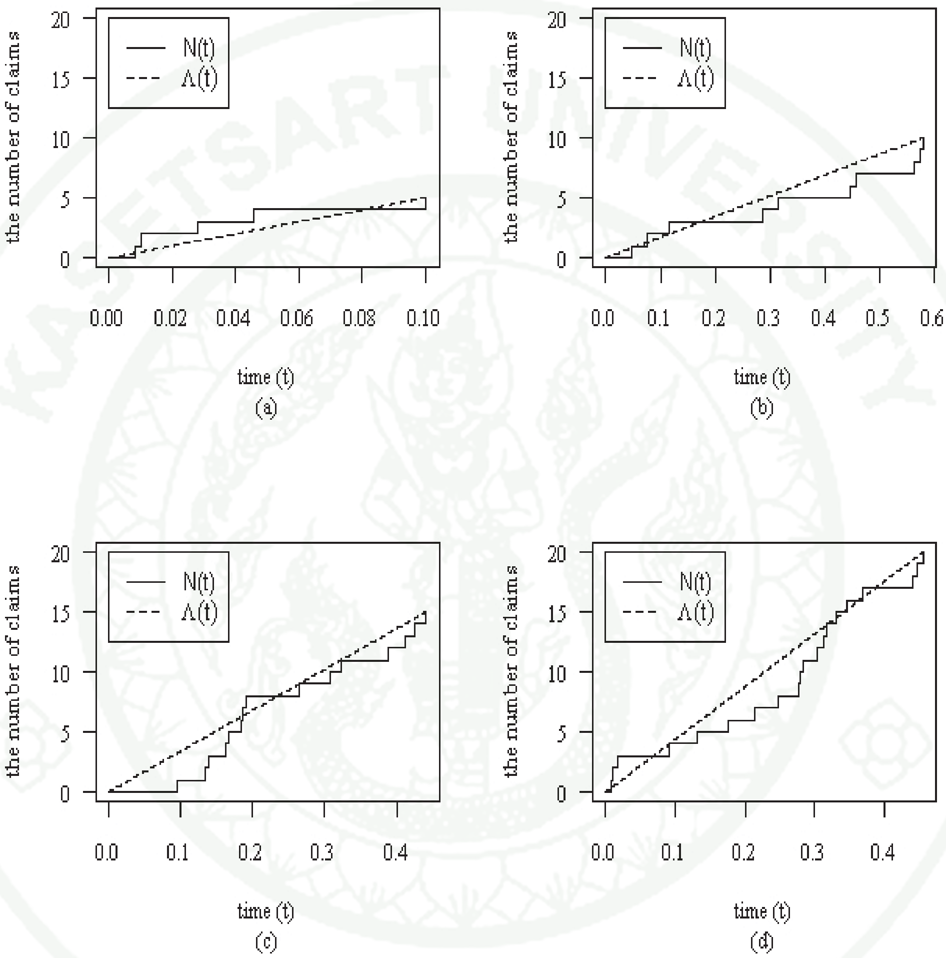
APPENDICES

Appendix A

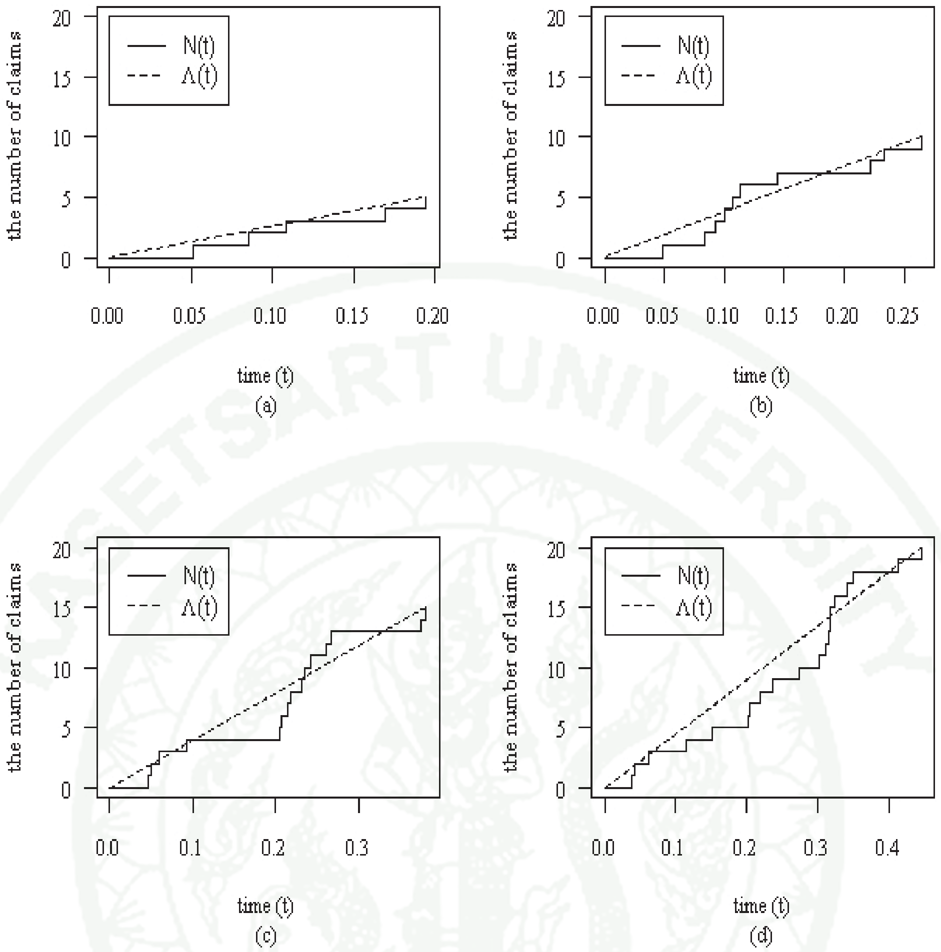
Simulation results of sample paths in the NHPP

1. The MLE for Estimating the Model Parameters of Claim Intensity

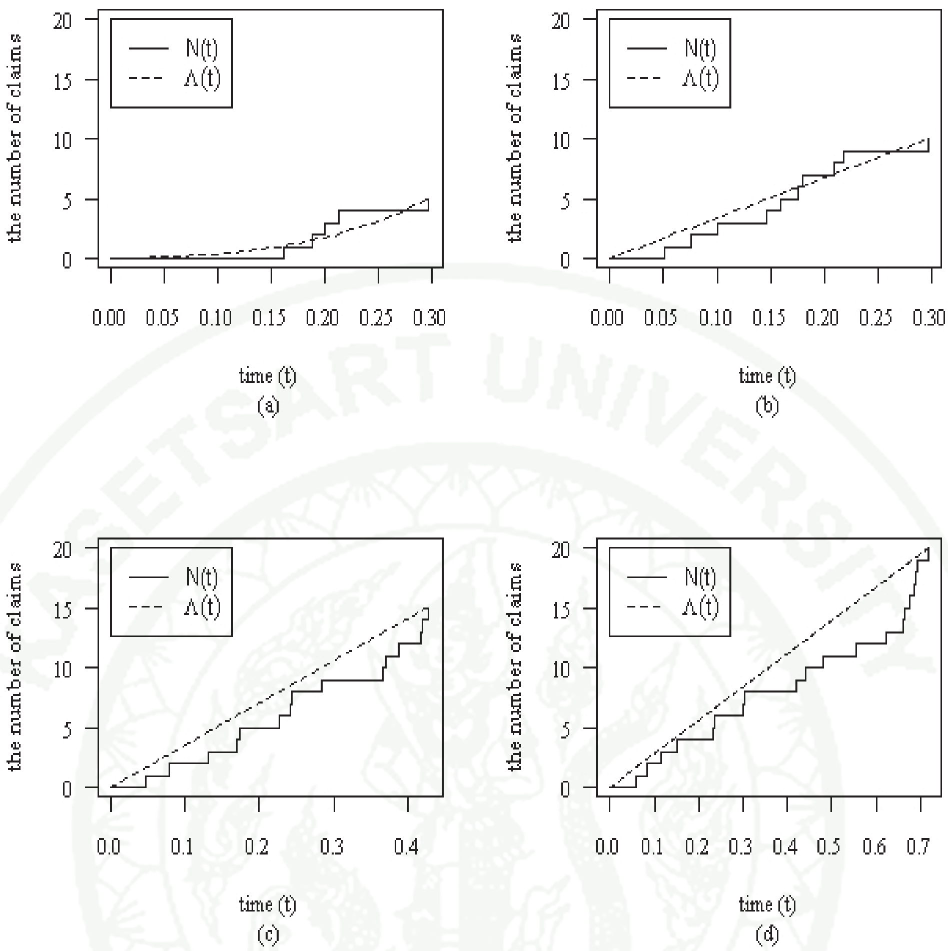
1.1 The estimation of claim counts based on the NHPP with a bell-shaped intensity



Appendix Figure A1 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 0.1, \sigma = 0.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

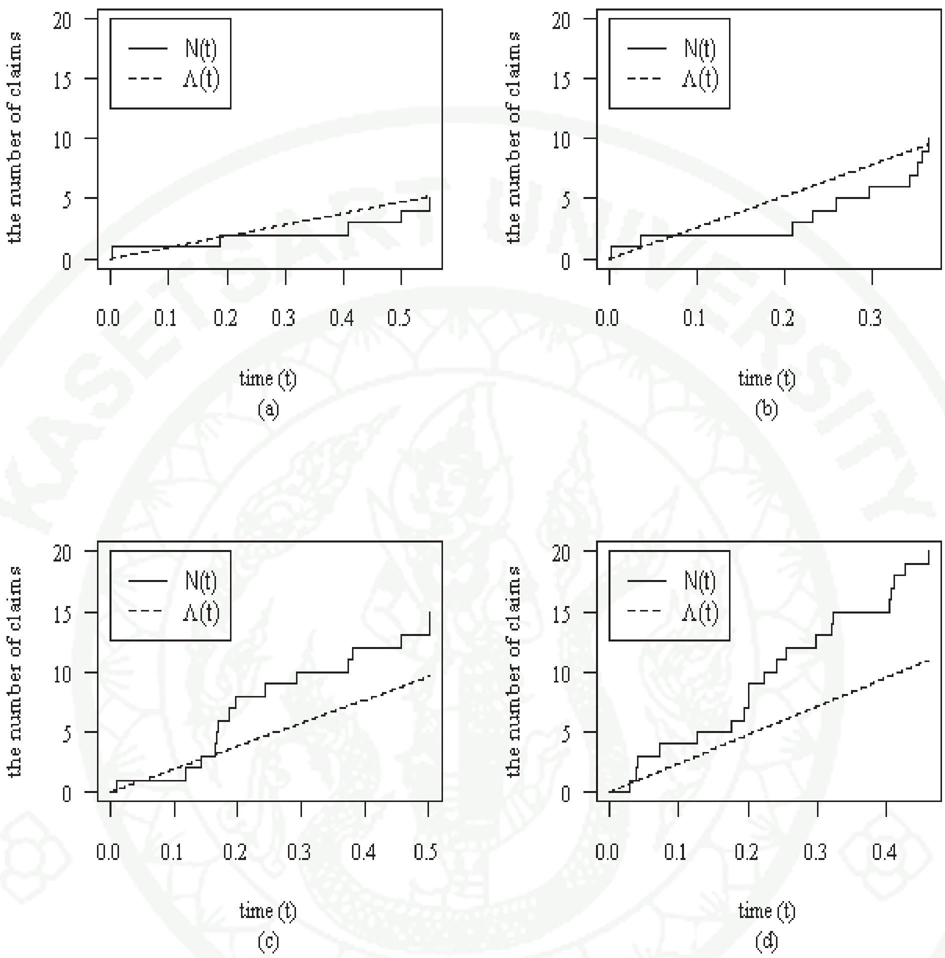


Appendix Figure A2 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 5, \sigma = 0.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

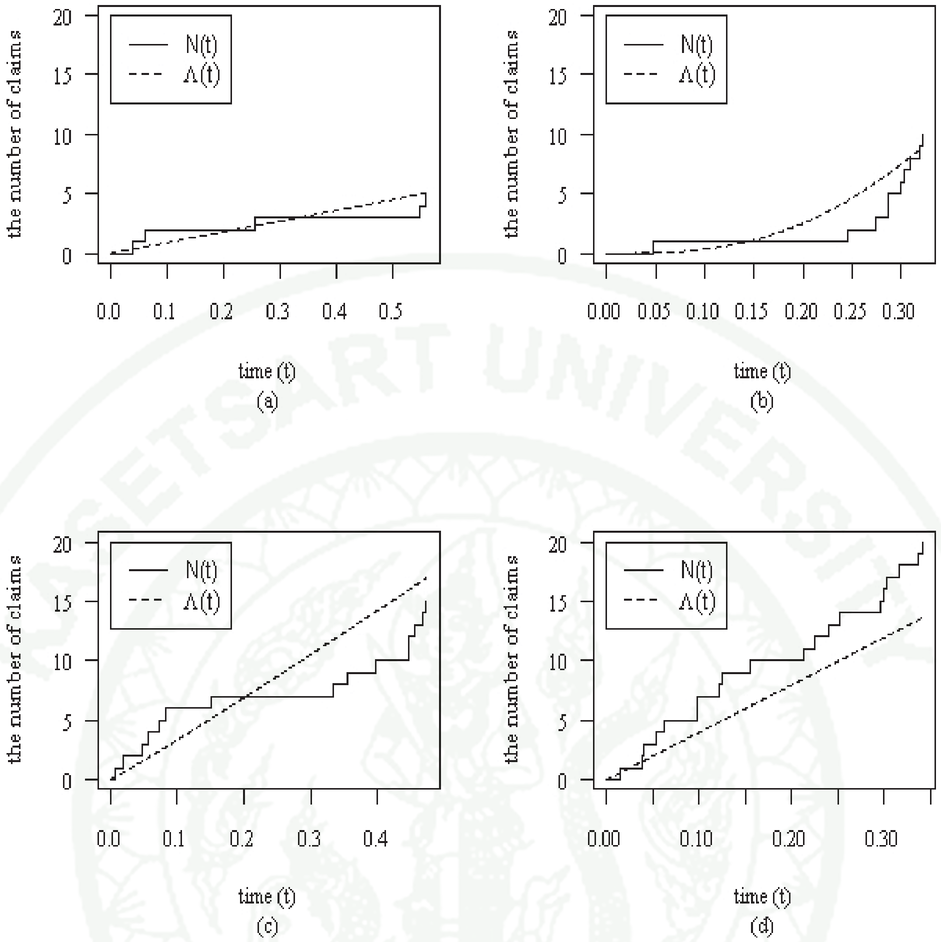


Appendix Figure A3 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 10, \sigma = 0.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

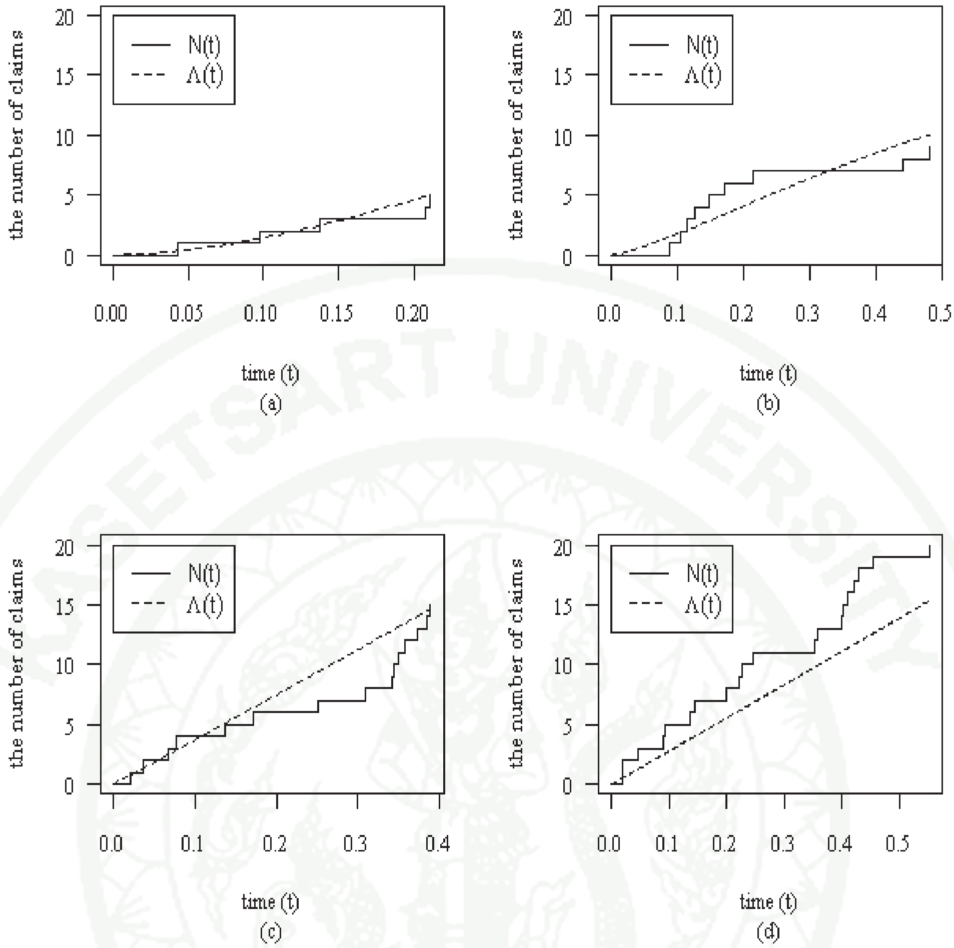
1.2 The estimation of claim counts based on the NHPP with a beta-shaped intensity



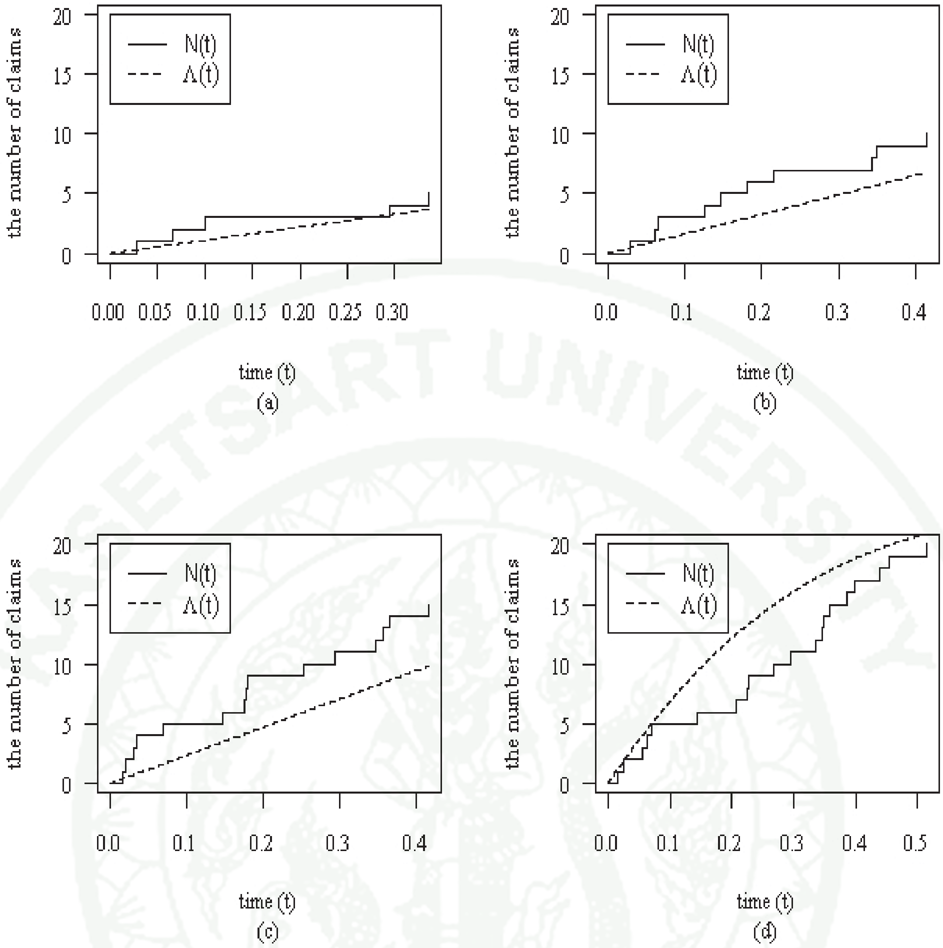
Appendix Figure A4 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 0.1, p = q = 2$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity



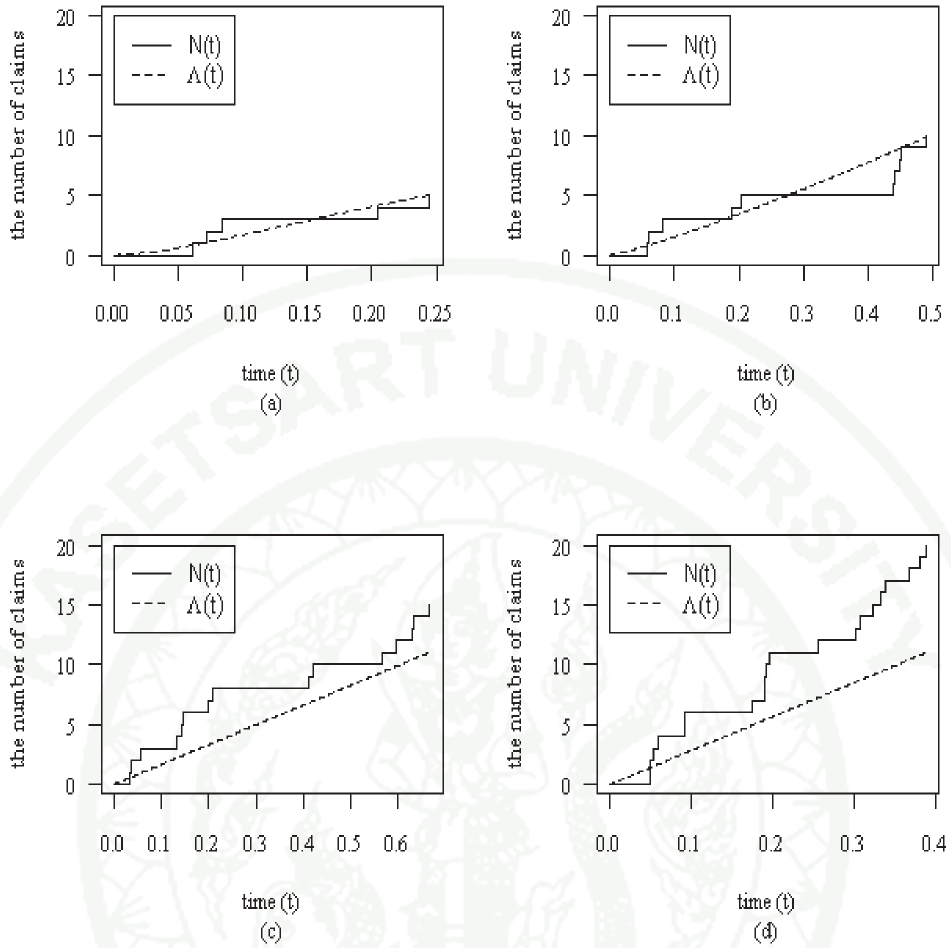
Appendix Figure A5 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 5, p = q = 2$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity



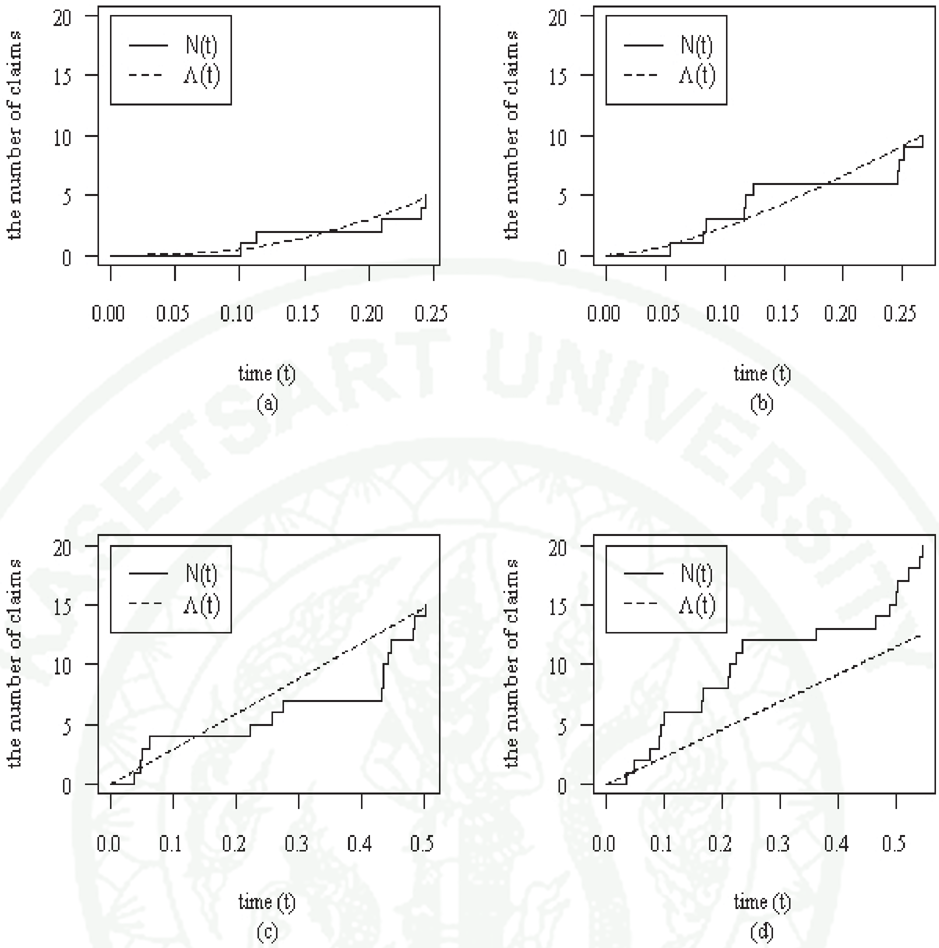
Appendix Figure A6 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 10, p = q = 2$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity



Appendix Figure A7 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 0.1, p = q = 3$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity



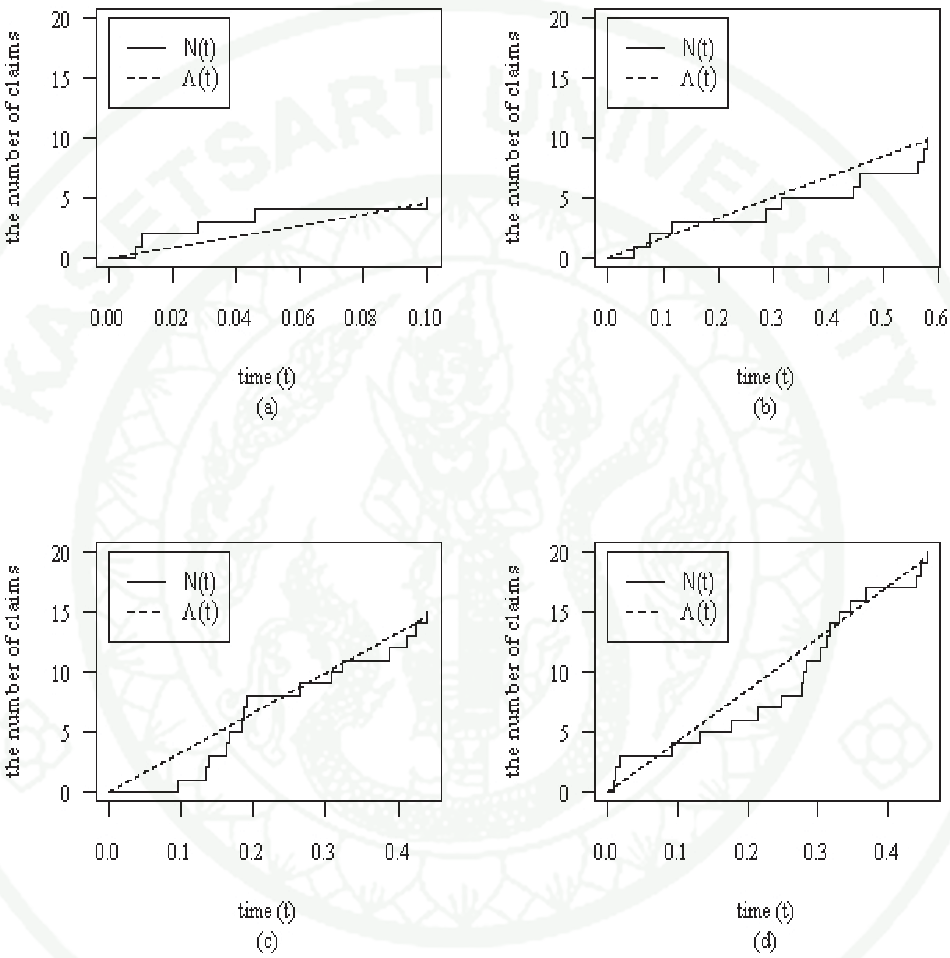
Appendix Figure A8 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 5, p = q = 3$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity



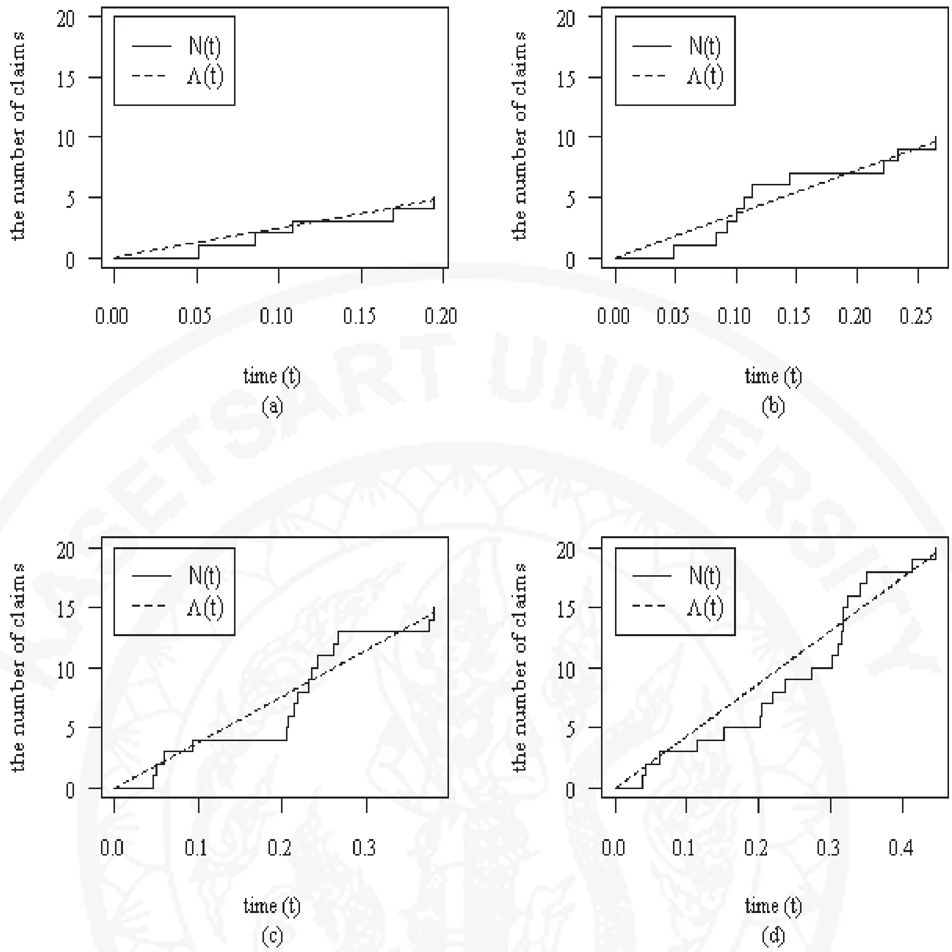
Appendix Figure A9 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 10, p = q = 3$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the MLE for estimating the model parameters of claim intensity

2. The BE for Estimating the Model Parameters of Claim Intensity

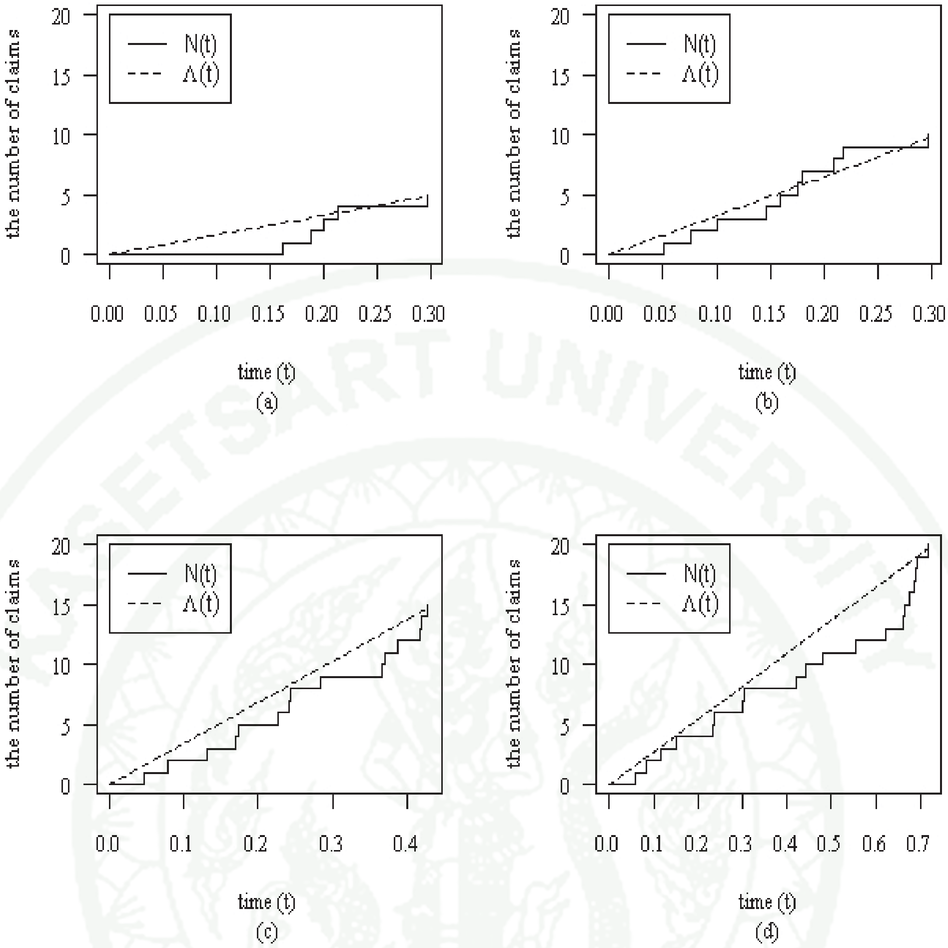
2.1 The estimation of claim counts based on the NHPP with a bell-shaped intensity



Appendix Figure A10 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 0.1, \sigma = 0.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

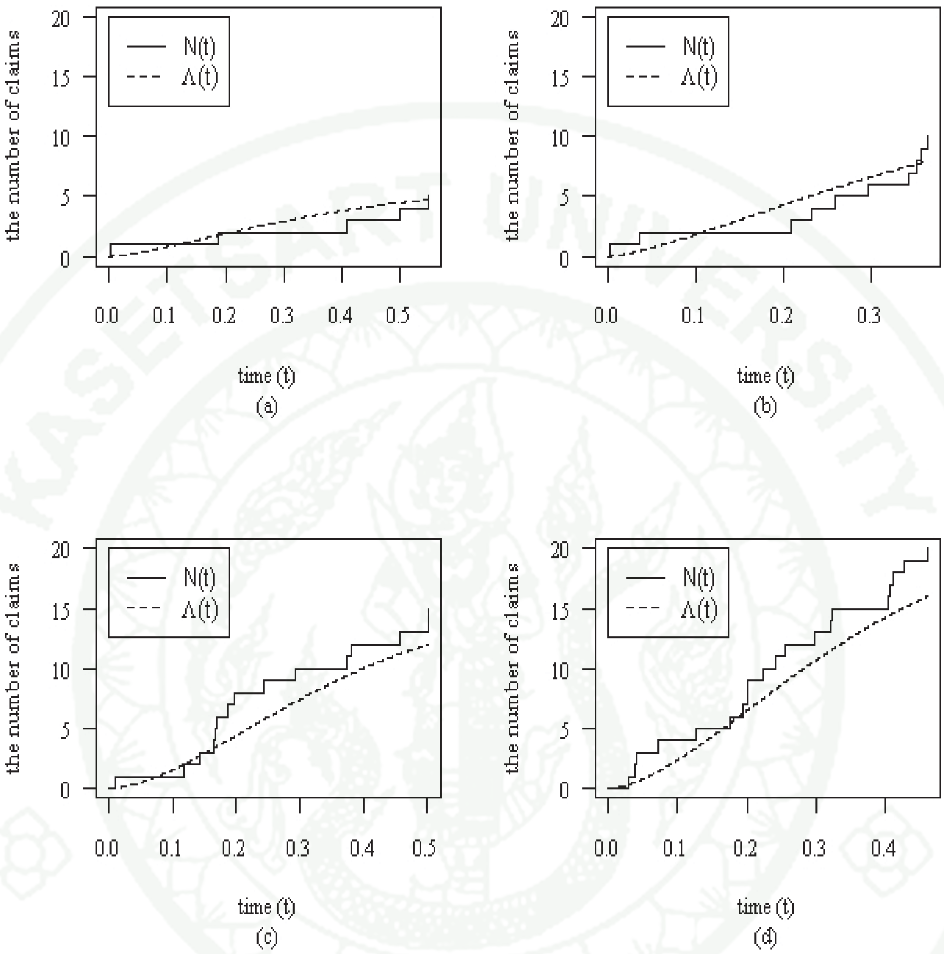


Appendix Figure A11 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 5, \sigma = 0.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

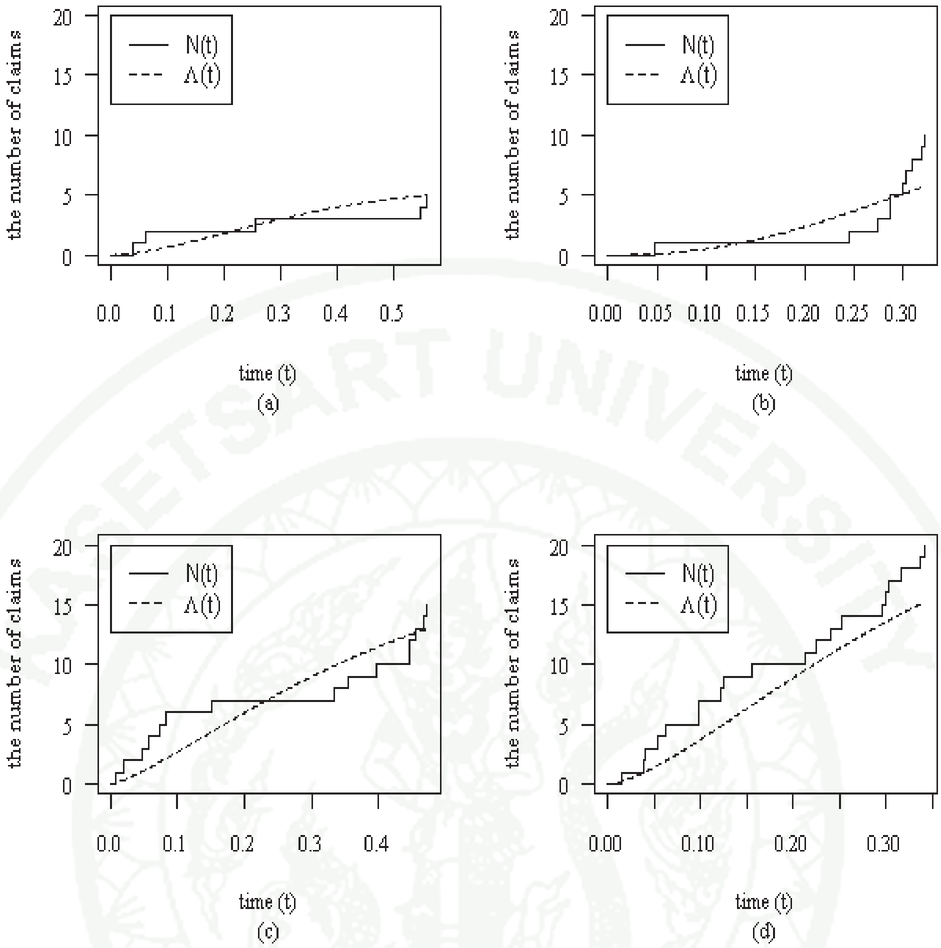


Appendix Figure A12 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of bell-shaped intensity $\lambda^* = 10, \sigma = 0.25$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

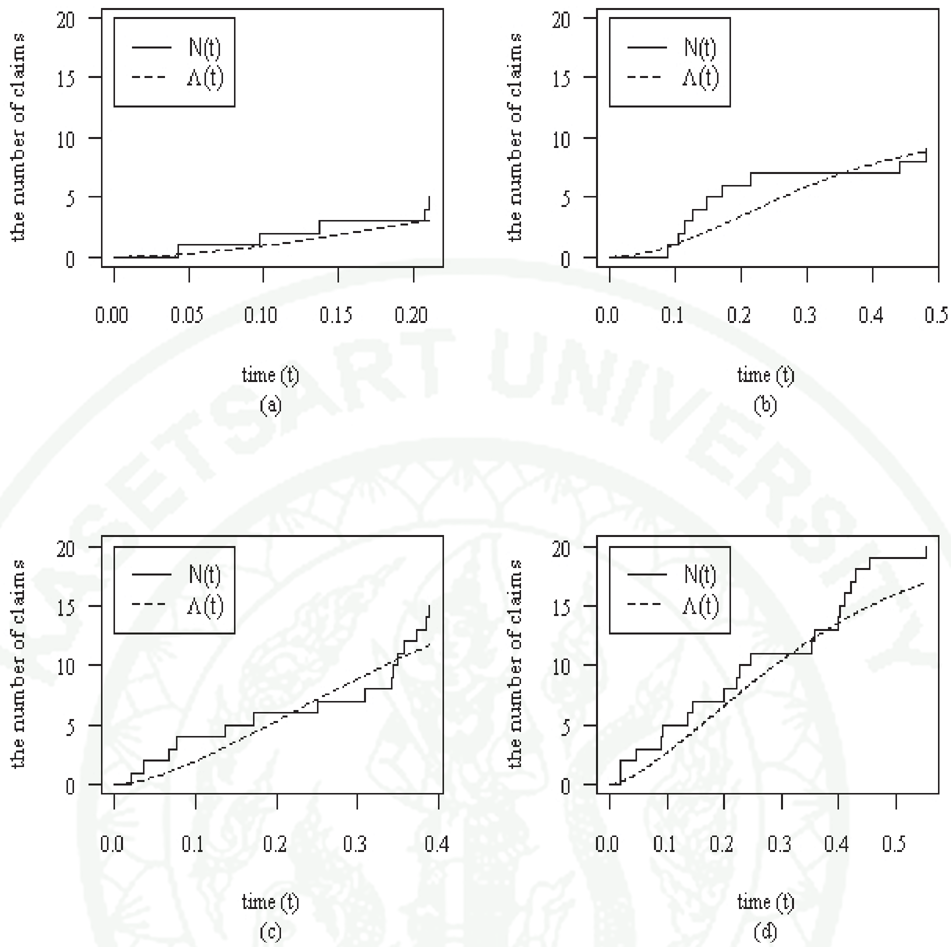
2.2 The estimation of claim counts based on the NHPP with a beta-shaped intensity



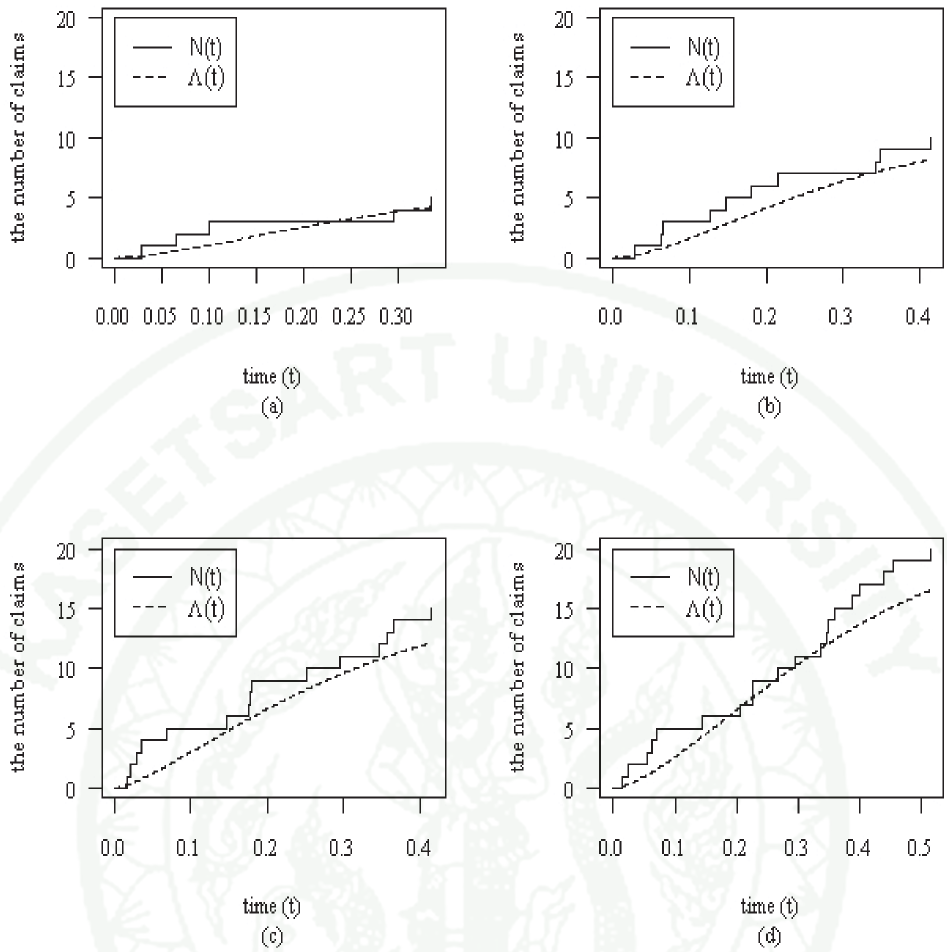
Appendix Figure A13 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 0.1, p = q = 2$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity



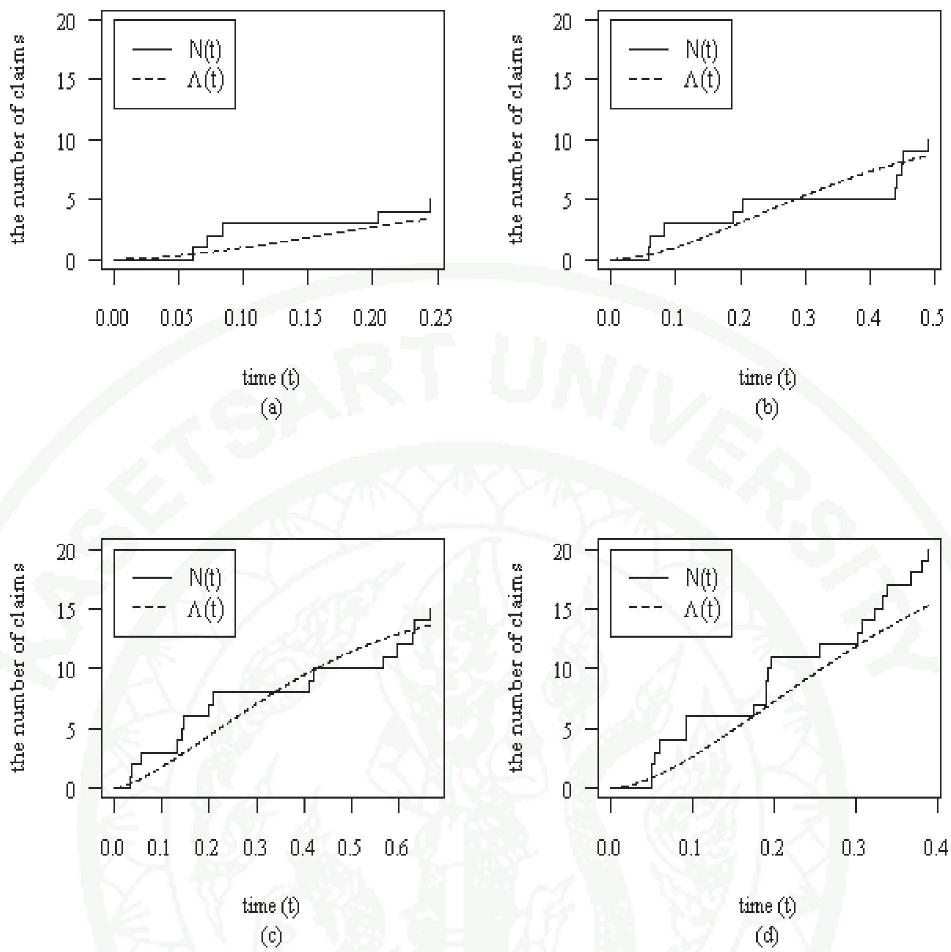
Appendix Figure A14 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 5, p = q = 2$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity



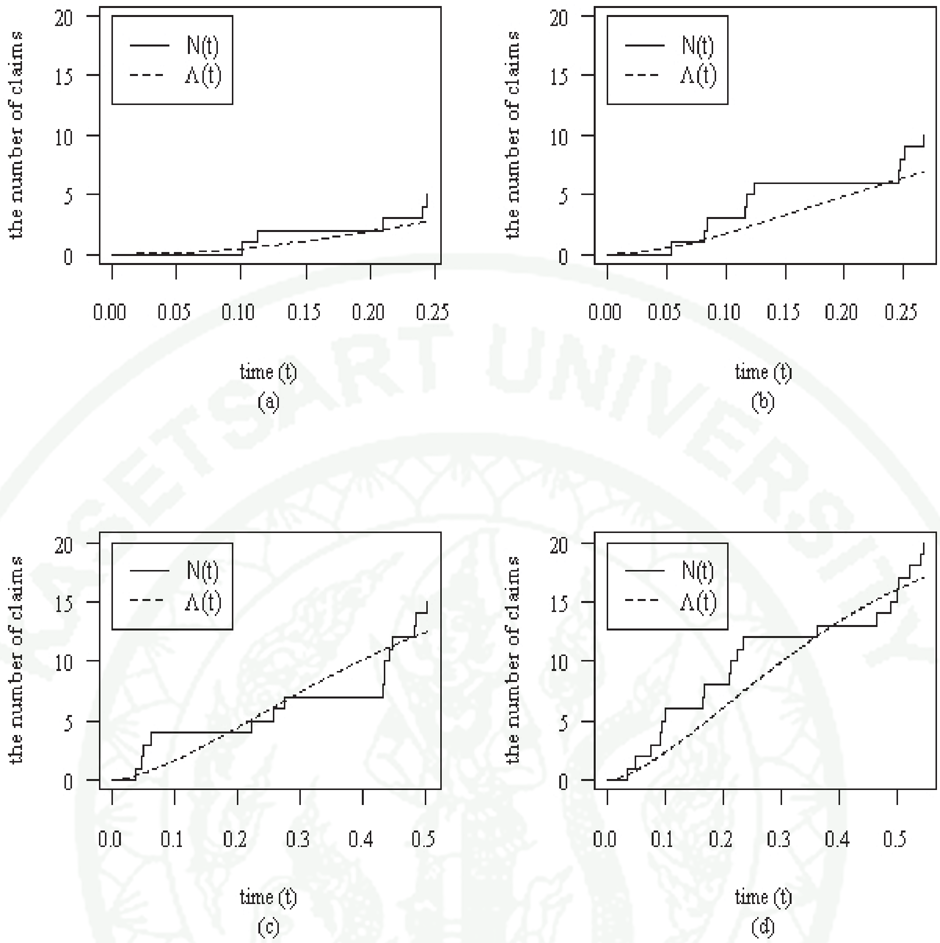
Appendix Figure A15 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 10, p = q = 2$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity



Appendix Figure A16 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 0.1, p = q = 3$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity



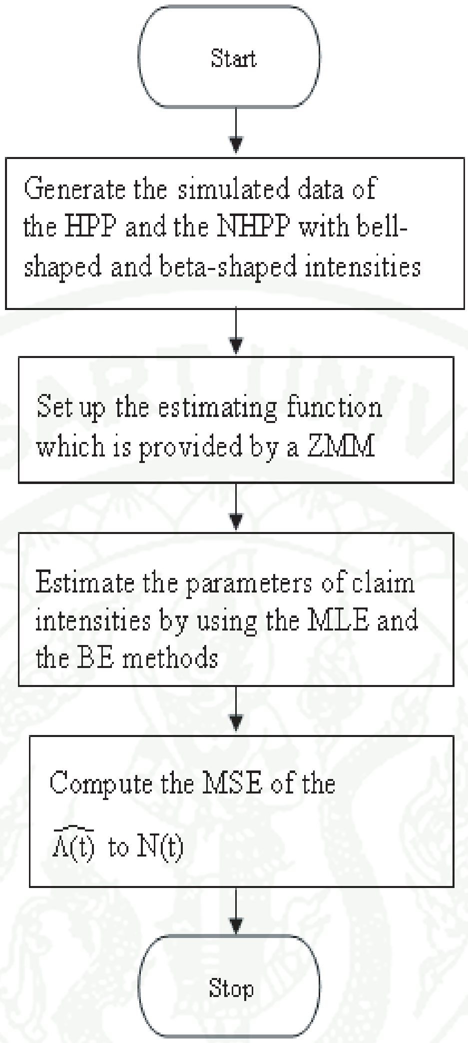
Appendix Figure A17 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 5, p = q = 3$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity



Appendix Figure A18 The behavior of $\Lambda(t)$ relating to a specified $N(t)$ based on the NHPP with the parameters of beta-shaped intensity $\lambda^* = 10, p = q = 3$ for some samples of (a) 5 claims, (b) 10 claims, (c) 15 claims and (d) 20 claims, using the BE for estimating the model parameters of claim intensity

Appendix B

Methodology flow chart and source code



Appendix Figure B1 Methodology flow chart

3. R Code of HPP with a Constant Claim Intensity Rate λ (lambda)

3.1 Generating the simulated data

```
n                  # the number of the observed times of claim occurrences
n.plus1<-n+1
T<-rep(0,n.plus1)  # arrival times of claim occurrences
W<-rep(0,n.plus1)  # inter-arrival times of claim occurrences
       for(i in 2:n.plus1){
       W[i]<-rexp(1,lambda)
       T[i]<-T[i-1]+W[i]
        }
ArrT<-T
interW<-W
```

3.2 Estimating the parameters of claim intensity by using the MLE and the BE methods

```
   mle.lamda<-1/mean(interW[2:n.plus1])              # the parameter estimate
                                                     # of claim intensity
   BE.lamda<-(n+alpha.prior)*(beta.prior/(1+(beta.prior
                 *sum(interW[2:n.plus1]))))          # the parameter estimate
                                                     # of claim intensity
```

4. R code of NHPP with a bell-shaped intensity λ^{*}, σ (lambda.star,sigma)

4.1 Generating the simulated data

```
n                      # the number of the observed times of claim occurrences
n.plus1<-n+1
T<-rep(0,n.plus1)     # arrival times of claim occurrences of HPP
                       # with claim intensity rate=1
W<-rep(0,n.plus1)     # inter-arrival times of claim occurrences  of HPP
                       # with claim intensity rate=1

T.NPP<-rep(0,n.plus1)
term1.T.NPP<-0
term2.T.NPP.pnorm1<-0
term2.T.NPP.pnorm1.1<-0
term2.T.NPP.pnorm2<-0
term2.T.NPP.pnorm2.1<-0
term2.T.NPP.qnorm<-0
lambda.pop<-0
term2.T.NPP.pnorm1<-(-1)/(2*sigma)
term2.T.NPP.pnorm1.1<-pnorm(term2.T.NPP.pnorm1,0,1)
term2.T.NPP.pnorm2<-1/(2*sigma)
term2.T.NPP.pnorm2.1<-pnorm(term2.T.NPP.pnorm2,0,1)
lambda.pop<-lambda.star/(term2.T.NPP.pnorm2.1-term2.T.NPP.pnorm1.1)

   for(i in 2:n.plus1){
    W[i]<-rexp(1,1)
    T[i]<-T[i-1]+W[i]
    term1.T.NPP<-T[i]/lambda.star
term2.T.NPP.qnorm<-((T[i]-lambda.star*as.integer(term1.T.NPP)))/lambda.pop)
                    +term2.T.NPP.pnorm1.1
T.NPP[i]<-as.integer(term1.T.NPP)+(sigma*qnorm(term2.T.NPP.qnorm,0,1))+0.5
```

```
    }

ArrT.NPPbell<-T.NPP     # arrival times of claim occurrences of NHPP
                        # with a bell-shaped intensity

Wbell<-rep(0,n.plus1)
        for(ii in 2:n.plus1){
          Wbell[ii]<-ArrT.NPPbell[ii]-ArrT.NPPbell[ii-1]
        }

interWbell<-Wbell           # inter-arrival times of claim occurrences of NHPP
                            # with a bell-shaped intensity
```

4.2 Estimating the model parameters of claim intensity by using the MLE and the BE methods

```
#######################
#    MLE Method       #
#######################
dif.sq.ArrT.NPPbell<-rep(0,n)
 ssq.ArrT.NPPbell<-0
dif.sq.ArrT.NPPbell<-(ArrT.NPPbell[2:n.plus1]-0.5)^2
ssq.ArrT.NPPbell<-sum(dif.sq.ArrT.NPPbell)
####################################
ml<-function(lamda.star.est,sigma.est){
    term1<-(-n)*log(lamda.star.est)
term2<-n*log(pnorm((1/(2*sigma.est)),0,1)-pnorm(((-1)/(2*sigma.est)),0,1))
    term3<-n*log(sigma.est)
    term4<-(n/2)*log(2*pi)
    term5<-(1/(2*(sigma.est^2)))*ssq.ArrT.NPPbell
    term7<-lamda.star.est*(pnorm(((ArrT.NPPbell.adj[samp2,n.plus1]-0.5)
      /sigma.est),0,1)-pnorm(((-1)/(2*sigma.est)),0,1))
```

```
    /(pnorm((1/(2*sigma.est)),0,1)-pnorm(((-1)/(2*sigma.est)),0,1))
logl<-term1+term2+term3+term4+term5+term7
     }

library("stats4")
lamda.star.pop1<-lamda.star  # define initial value in function of mle()
sigma.pop1<-sigma            # define initial value in function of mle()
est2<-rep(0,2)
est2<- mle(minuslogl=ml,start=list(lamda.star.est=lamda.star.pop1,
          sigma.est=sigma.pop1))
  ml.lamda.star1<-coef(est2)[1]          # the parameter estimate  of
                                         # the model of claim intensity
  ml.sigma1<-coef(est2)[2]               # the parameter estimate of
                                         # the model of claim intensity


########################
#      BE Method       #
########################
interWbellC<-interWbell[2:n.plus1]
ArrT.NPPbellCB<-ArrT.NPPbell[2:n.plus1]
sigma.init<-ml.sigma1     # define initial value in function of BEbell()

BEbell<-function(interWbellC,ArrT.NPPbellCB,n,a1.lamdPrior,l1.lamdPrior,
              a1.sigmaPrior,l1.sigmaPrior,lamda.star,sigma.init,n1.iter)
{
library("R2WinBUGS")
coef<-rep(0,3)
lamda.star.hat<-0
sigma.hat<-0
sigma.sq.hat<-0
wi<-interWbellC
ti<-ArrT.NPPbellCB
```

```
sigma.sq.init<-sigma.init^2
data<-list(w=wi, t=ti,L=n, a.lamdPrior=a1.lamdPrior, l.lamdPrior=l1.lamdPrior,
          a.sigmaPrior=a1.sigmaPrior, l.sigmaPrior=l1.sigmaPrior)
inits<-function(){list(Baylamda.star=lamda.star,Baysigma.sq=sigma.sq.init)}
Baybell<-bugs(data,inits,model.file="d:/NHPPbell.txt",parameters=
            c("Baylamda.star","Baysigma","Baysigma.sq"),n.iter=n1.iter,
            n.thin=1,n.chains=1,bugs.directory="c:/Program Files/WinBUGS14/")
lamda.star.hat<-Baybell$mean$Baylamda.star[1]
sigma.hat<-Baybell$mean$Baysigma[1]
sigma.sq.hat<-Baybell$mean$Baysigma.sq[1]
coef[1]<-lamda.star.hat
coef[2]<-sigma.hat
coef[3]<-sigma.sq.hat
coef
}

bay<-BEbell(interWbellC,ArrT.NPPbellCB,n,a1.lamdPrior,l1.lamdPrior,
           a1.sigmaPrior, l1.sigmaPrior,lamda.star,sigma.init,n1.iter)
BE.lamda.star1<-bay[1]          # the parameter estimate of
                                # the model of claim intensity
BE.sigma1<-bay[2]               # the parameter estimate of
                                # the model of claim intensity


######  BUGS code for the NHPP with a bell-shaped intensity ######
######  is contained in the file NHPPbell.txt                ######
model   {
         for (i in 1 : L) {
         w[i] ~ dexp(lamdaBell[i])
 lamdaBell[i]<-(Baylamda.star/(phi(0.5/sqrt(Baysigma.sq))-phi(-0.5
             /sqrt(Baysigma.sq))))*exp((-0.5/Baysigma.sq)
             *pow((t[i]-0.5),2))/sqrt(Baysigma.sq*2*3.141592654)
         }
```

```
    Baylamda.star ~ dgamma(a.lamdPrior,l.lamdPrior)
    Baysigma.sq ~ dgamma(a.sigmaPrior,l.sigmaPrior)
    Baysigma<-sqrt(Baysigma.sq)
  }
```

5. R code of NHPP with a beta-shaped intensity λ^*, p, q (lambda.star, p,q)

5.1 Generating the simulated data

```
n              # the number of the observed times of claim occurrence
n.plus1<-n+1
m1=0
m2=1
D=m2-m1
T<-rep(0,n.plus1)            # arrival times of claim occurrences of HPP
                                 # with claim intensity rate=1
W<-rep(0,n.plus1)            # inter-arrival times of claim occurrences of HPP
                                 # with claim intensity rate=1
T.NPP<-rep(0,n.plus1)
comp.alp.pop<-0
t.star.pop<-0
alp1.star.pop<-0
bet.fun.pop<-0
comp.gen.data<-0
comp.alp.pop<-(p-1)/(p+q-2)
t.star.pop<-m1+D*comp.alp.pop
alp1.star.pop<-(((t.star.pop-m1)/D)^(p-1))*((1-((t.star.pop-m1)/D))^(q-1))
bet.fun.pop<-beta(p,q)
comp.gen.data<-alp1.star.pop/(lamda.star*D*bet.fun.pop)

   for(i in 2:n.plus1){
     term1.T.NPP<-0
```

```
        term2.T.NPP<-0
        term.T.NPP.qbeta<-0
        W[i]<-rexp(1,1)
        T[i]<-T[i-1]+W[i]
        term1.T.NPP<-T[i]*comp.gen.data
        term2.T.NPP<-as.integer(term1.T.NPP)
        term.T.NPP.qbeta<-term1.T.NPP-term2.T.NPP
        T.NPP[i]<-D*qbeta(term.T.NPP.qbeta,p,q)+m1+term2.T.NPP
      }

ArrT.NPPbet<-T.NPP  # arrival times of claim occurrences of NHPP
                                # with a beta-shaped intensity

Wbet<-rep(0,n.plus1)
        for(ii in 2:n.plus1){
             Wbet[ii]<-ArrT.NPPbet[ii]-ArrT.NPPbet[ii-1]
        }
interWbet<-Wbet        # inter-arrival times of claim occurrences of NHPP
                             # with a beta-shaped intensity
```

5.2 Estimating the parameters of the model of claim intensity by using the MLE and the BE methods

```
#######################
#    MLE Method       #
#######################
dif.ArrT.NPPbet<-ArrT.NPPbet[2:n.plus1]-as.integer(ArrT.NPPbet[2:n.plus1])-m1
       for (h in 1:n){
           if (dif.ArrT.NPPbet[h]==0){
          dif1.ArrT.NPPbet[h]<-0.0001}
           else
          dif1.ArrT.NPPbet[h]<- dif.ArrT.NPPbet[h]
```

```
        }
          dif2.ArrT.NPPbet<-1-(dif1.ArrT.NPPbet/D)
        log.dif1.ArrT.NPPbet<-log(log(exp(dif1.ArrT.NPPbet)))
        log.dif2.ArrT.NPPbet<-log(log(exp(dif2.ArrT.NPPbet)))
        slog1.ArrT.NPPbet<-sum(log.dif1.ArrT.NPPbet)
        slog2.ArrT.NPPbet<-sum(log.dif2.ArrT.NPPbet)

logl<-0
ml<-function(lamda.star.est,p1.est,q1.est){
      term1<-(-n)*log(log(exp(lamda.star.est)))
      comp.alp.est<-(p1.est-1)/(p1.est+q1.est-2)
      t.star.est<-m1+D*comp.alp.est
      alp1.star.est<-(((t.star.est-m1)/D)^(p1.est-1))
                    *((1-((t.star.est-m1)/D))^(q1.est-1))
      term2<-n*log(log(exp(alp1.star.est)))
      term3<--1*(p1.est-1)*(slog1.ArrT.NPPbet-n*log(D))
      term4<--1*(q1.est-1)*slog2.ArrT.NPPbet
      termin.probBet<-(ArrT.NPPbet[n.plus1]-as.integer(ArrT.NPPbet[n.plus1])
                      -m1)/D
      term5<-(lamda.star.est/alp1.star.est)*D*beta(p1.est,q1.est)
             *(as.integer(ArrT.NPPbet[n.plus1])+pbeta(termin.probBet,
             p1.est,q1.est))
      logl<-term1+term2+term3+term4+term5
        }

library("stats4")
lamda.star.pop1<-lamda.star # define initial value in function of mle()
p1.pop1<-p                 # define initial value in function of mle()
q1.pop1<-q                 # define initial value in function of mle()
est3<-rep(0,3)
est3<-mle(minuslogl=ml,start=list(lamda.star.est=lamda.star.pop1,
          p1.est=p1.pop1,q1.est=q1.pop1))
 ml.lamda.star1<-coef(est3)[1]  # the parameter estimate of
```

```
                                # the model of claim intensity
 ml.p1<-coef(est3)[2]           # the parameter estimate of
                                # the model of  claim intensity
 ml.q1<-coef(est3)[3]             # the parameter estimate of
                                # the model of claim intensity
#######################
#      BE Method       #
#######################

interWbetC<-interWbet[2:n.plus1]
ArrT.NPPbetCB<-ArrT.NPPbet[2:n.plus1]
lamda.star.init<-lamda.star             # define initial value in
                          # function of BEbell()
p1.init<-p                # define initial value in function of BEbell()
q1.init<-q                # define initial value in function of BEbell()

BEbet<-function(interWbetC,ArrT.NPPbetCB,n, a1.lamdPrior, l1.lamdPrior,
                a1.p1Prior,l1.p1Prior, a1.q1Prior,l1.q1Prior,lamda.star.init,
                p1.init,q1.init,n1.iter)
{
library("R2WinBUGS")
coef<-rep(0,3)
lamda.star.hat<-0
p1.hat<-0
q1.hat<-0
wi<-c()
ti<-c()
wi<-interWbetC
ti<-ArrT.NPPbetCB
data<-list(w=wi, t=ti,L=n, m22=m2,m11=m1,a.lamdPrior=a1.lamdPrior,
           l.lamdPrior=l1.lamdPrior, a.p1Prior=a1.p1Prior, l.p1Prior
           =l1.p1Prior,a.q1Prior=a1.q1Prior,l.q1Prior=l1.q1Prior)
inits<-function(){list(lamda.star=lamda.star.init,p1=p1.init,q1=q1.init)}
```

```
Baybet<-bugs(data,inits,model.file="d:/NPPBeta.txt",parameters
             =c("lamda.star","p1","q1"),n.iter=n1.iter,n.thin=1,n.chains=1,
             bugs.directory="c:/Program Files/WinBUGS14/")

lamda.star.hat<-Baybet$mean$lamda.star[1]
p1.hat<-Baybet$mean$p1[1]
q1.hat<-Baybet$mean$q1[1]
coef[1]<-lamda.star.hat
coef[2]<-p1.hat
coef[3]<-q1.hat
coef
}

bay<-BEbet(interWbetC,ArrT.NPPbetCB,n,a1.lamdPrior, l1.lamdPrior, a1.p1Prior,
           l1.p1Prior,a1.q1Prior,l1.q1Prior,lamda.star.init,p1.init,q1.init,
           n1.iter)
BE.lamda.star1<-bay[1]         # the parameter estimate of
                      # the model of claim intensity
BE.p1<-bay[2]                    # the parameter estimate of
                      # the model of claim intensity
BE.q1<-bay[3]                    # the parameter estimate of
                      # the model of claim intensity

######  BUGS code for the NHPP with a beta-shaped intensity ######
######  is contained in the file NHPPbeta.txt               ######

model{
 for(i in 1:L){
    lamdaBeta[i]<-exp(ll[i])
    w[i]~dexp(lamdaBeta[i])
    ll[i]<-log(lamda.star)-(p1-1)*(log(p1-1)-log(p1+q1-2))-(q1-1)*(log(q1-1)
              -log(p1+q1-2))+(p1-1)*(log(t[i]-m11)-log(m22-m11))+(q1-1)
```

```
                *(log(m22-t[i])-log(m22-m11))
              }
lamda.star~dgamma(a.lamdPrior,l.lamdPrior)
p1~dgamma(a.p1Prior,l.p1Prior)I(1,)
q1~dgamma(a.q1Prior,l.q1Prior)I(1,)
}
```

CURRICULUM VITAE

NAME : Ms. Uraiwan Jaroengeratikun

BIRTH DATE : July 11, 1970

BIRTH PLACE : Srisaket, Thailand

EDUCATION :

YEAR	INSTITUTE	DEGREE/DIPLOMA
1993	Khonkaen Univ.	B.Sc (Statistics)
1997	Chulalongkorn Univ.	M.Sc (Insurance)

POSITION : Lecturer of Department of Applied Statistics

WORK PLACE : Faculty of Applied Science, King Mongkut's University of Technology North Bangkok

PUBLICATIONS :

1. **U. Jaroengeratikun**, W. Bodhisuwan, A. Thongteeraparp. A Statistical Analysis of Intensities Estimation on the Modeling of Non-Life Insurance Claim Counting Process. *Applied Mathematics*, 2012, 3(1): 100-106.

2. **U. Jaroengeratikun**, W. Bodhisuwan, A. Thongteeraparp. A Bayesian Inference of Non-Life Insurance Based on Claim Counting Process with Periodic Claim Intensity. *Open Journal of Statistics*, 2012, 2(2): 177-183.