THE ANALYSIS OF SHORT-TERM INTEREST RATE DYNAMICS IN THAILAND

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Abstract

This paper purposes the drift and diffusion function that can nest for all existing single-factor interest rate model. The drift functions can be classified to general model and linear drift model. The difference between these models is the non-linear term. This paper examines the effect on the significance of non-linear term based on three different data sets, inter-bank overnight rate, seven-day R/P rate, and one-month T-bill, and three different diffusion functions, affine diffusion function, CEV diffusion function, and combined form diffusion function by using log-likelihood ratio test .We find that the choice of the data sets and the choice of diffusion functions used do not affect the strength of the significance of the non-linearity when using Thai data.

Introduction

Interest rate modeling is created to satisfy the need to understand interest rate dynamic and to predict future movement of interest rates in order to price and to hedge interest rate product. The model provides a quantitative framework for explaining interest rate movements and valuing interest rate product.

The approach to model term structure of interest rate is to express interest rates in terms of one or more state variables, which follows continuous-time Markov processes. The model is thus defined by its state variables and by their process, which determines how they change through time.

In continuous-time for single factor term structure model there is only one state variable, which is the short or instantaneous rate of interest. So, the model is based on a short rate process. The short rate, r_t , is a key interest rate in all interest rate models, even though it cannot be observed directly. One alternative in practice is using the short term interest rate as the proxy of short rate because values of which can be observed directly from the market.

The interest rate model can be divided into two parts. The first part is deterministic, non-random, called the drift term, and the second part is stochastic term, also known as the volatility term.

The key feature of interest rate model is the mean reversion of interest rate process in the drift term. The mean reversion process is the critical property and it is the important difference between interest rates and stock prices. This property is that interest rate will be pulled back to the some long-run average level overtime. Hence, when interest rate is high, mean reversion causes the drift term to be a negative. And when interest rate is low, mean reversion causes the drift to be positive. Prior studies using different data generate confusing and conflicting conclusions about mean reversion process whether it is linear or non-linear. Bali and Wu (2005) use different interest rate data sets of U.S.A. to study linearity in drift term and find that the significance of non-linearity in the short-rate drift relies on the choice of data sets and on the specification of the diffusion function.

The nonlinear drift specification is critical because it can nest for mean reversion specification. If we find non-linear mean reversion of interest rate in drift term, it means that the interest rate model that applies linear drift specification such as affine model might be inappropriate. So, as proxies of short rate in interest rate model, the dynamics of interest rate series used are also important.

Therefore, this study focuses on studying dynamic of three different short term interest rate series of Thai's market, inter-bank overnight rate, seven-day R/P rate, and one-month T-Bill yield, because the periods of which, daily, weekly, and monthly are most often used as proxy of the short-rate.

Literature Reviews

There are many prior studies tending to explain the behavior of the short rate dynamic, but they generate confusing and sometime conflicting conclusion. Their studies are based on different data sets and different parametric and non parametric specification.

The studies that used non - parametric specification approach tend to infer non-linearity in drift term. Ait-Sahalia (1996) uses semi-non parametric approach and data set of daily seven day Eurodollar deposit rate and finds strong non-linearity in the drift term. Therefore, Ait-Sahalia rejects the models that assume linearity in drift term. Stanton (1997) employs data set of three-month Treasury-bill yield with non-parametric approach and also finds that the drift function of short rate contained important nonlinearity.

Non-parametric specification approach is argued by some studies. Chapman and Pearson (1998) apply non-parametric estimation to simulate sample paths of a square-root diffusion and find the point to weaken the use of non-parametric approach. They find that although the drift function is linear, this approach still generates nonlinear estimates for the drift function. Also, Pritsker (1998) studies the finite sample distribution of Ait sahalia’s non-parametric tests of continuous time model of the short-term rate and find that Ait-Sahalia’s test rejects true interest rate model too often. Then, Pritsker finds that the significance of nonlinearity in drift function declines when adjusted for the high persisitence of interest rate process.

The parametric specification approach is also used in many studies. Durham(2003) uses data set of three month treasury bill and find that the significance of nonlinearity in the drift function depends on the specification of the diffusion function. Chan(1992) uses generalized method of moment approach and rejects the commonly used square root diffusion model of Cox (1985), whose volatility is propertional to the square root of the level of interest rate. The result favours a model of which volatility is more sensitive to the current level of interest rate. The prior studies that applies parametric approach use the different data sets and already indicate that diffusion function has the effect on significance of non-linearity in the drift function, so Bali and Wu (2005) further study about short rate dynamic but rather using only one interest rate data set, they used three different interest rate data sets, which are different in maturity, to indicate the effect of maturity. They also study the effect on significance of non-linearity when using different diffusion function. As the result, they find that for the same diffusion specification, the significance of the non-linearity in the drift function declines as the maturity of the interest rate series increases and for the same data sets, the significance of the non-linearity in the drift function declines as the diffusion function changes from an affine function of short rate, to CEV function, and then to combined form.

There are the papers studying about dynamic of short rate using the data sets of interest rate of other countries instead of U.S.data. Fan and Lao (2004) use repo-rate in Chinese inter-bank market to compare the efficiency among different interest rate models and they find that generalized CKLS model with more flexible diffusion function is the best to model interest rate. Li (2000) uses Australian thirteen week

Treasury-note as data and applies GMM method to compare among different interest rate models. The result indicates that model with unrestricted diffusion function best fitted the historical data. Shi, Kagraoka, Tamura, and Ozaki (1999) use Japanese LIBOR rate to compare between linear drift model and the model that has CEV specification diffusion and exponential autoregressive drift. They find that linear drift model outperforms the exponential drift model in terms of capturing Japanese LIBOR rate.

Also, there are the studies that use Thai's data. Treepongkaruna and Gray (2006) study the characteristics of short-term interest rates in several countries including Thailand. For Thailand, they use inter-bank overnight rate as a proxy of short rate and they find the nonlinear mean reversion is jointly significant. In this paper, the result would not specify which interest rate serie is the best proxy for the instanetaneous interest rate but investigates the dynamic of different interest rate series instead. Nevertheless, the implication of this study is that one can use the result of this study for choosing the model that matches the dynamics of the particular interest rate serie.

Interest rate model

There are two approaches that the prior studies use in order to explain the interest rate dynamic, non-parametric and parametric approach. The choice of approach used probably causes the conflicting conclusion of the short rate dynamic. But, the non-parametric approach is argued about the efficiency to identify the interest rate dynamic. This study therefore uses the parametric approach of interest rate in single factor interest rate model. This study as well as study of Bali and Wu(2005) and study of Ait-Sahalia(1996) purposes the framework that encompasses most single-factor interest rate model with one diffusion specification.

Short rate model

In continuous time single-factor interest rate model, the interest rate is the only state variable and the interest rate dynamics follows time homogenous stochastic differential equation (SDE).

$$dr_t = \mu(r_t)dt + \sqrt{v(r_t)}dz \qquad (1)$$

where $\mu(r_t)$ and $v(r_t)$ are the drift and the diffusion function of the underlying variable, r, at time t. Both the drift and diffusion are liable to change overtime. The change of r in the small time interval between t and $t + \Delta t$, dt, is represented by dr_t. dz is the Standard Brownian motion.

According to time-homogenous diffusion process used in the study of Ait Sahalia (1996), the drift and diffusion specification for the empirical test of this paper are as follows.

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1} \tag{2}$$

$$v(r_t) = \beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta_3} \tag{3}$$

This drift and diffusion specification can nest all single-factor interest rate models as shown in table 1. This drift specification allows for departure from linearity in the drift. The diffusion function combines an affine specification with a CEV specification. Hence, this diffusion specification is sufficiently various for nesting for all diffusion specification.

Table 1
Nested cases of the single-factor diffusion model

Reference	**Drift function , $\mu(r_t)$**	**Diffusion function, $\sigma(r_t)$**
Vasicek (1977)	$\alpha_0 + \alpha_1 r_t$	β_0
Cox, Ingersoll, and Ross (1985) Brown and Dybvig (1986) Gibbons and Ramaswamy (1993) Pearson and Son (1994)	$\alpha_0 + \alpha_1 r_t$	$\beta_1 r_t$
Brennan and Schwartz (1979) Courtadon (1982)	$\alpha_0 + \alpha_1 r_t$	$\beta_2 r_t^{2}$
Chan et al. (1992) (β_3 >_ 1)	$\alpha_0 + \alpha_1 r_t$	$\beta_2 r_t^{\beta 3}$
General drift, CEV diffusion	$\alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1}$	$\beta_2 r_t^{\beta 3}$
General parametric model	$\alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1}$	$\beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta 3}$

Data and estimation

Another factor that probably leads to the conflicting conclusion is the data set used. The prior study used the different data sets that can be classified to four different maturities, daily, weekly, monthly, and quarterly. In the studies based on U.S. data, the most frequently used data set are the interest rates that have the shortest maturities, Fed funds rate, seven-day Eurodollar rate, and three month Treasury bill yield, representing the maturities of daily, weekly, and quarterly, respectively.

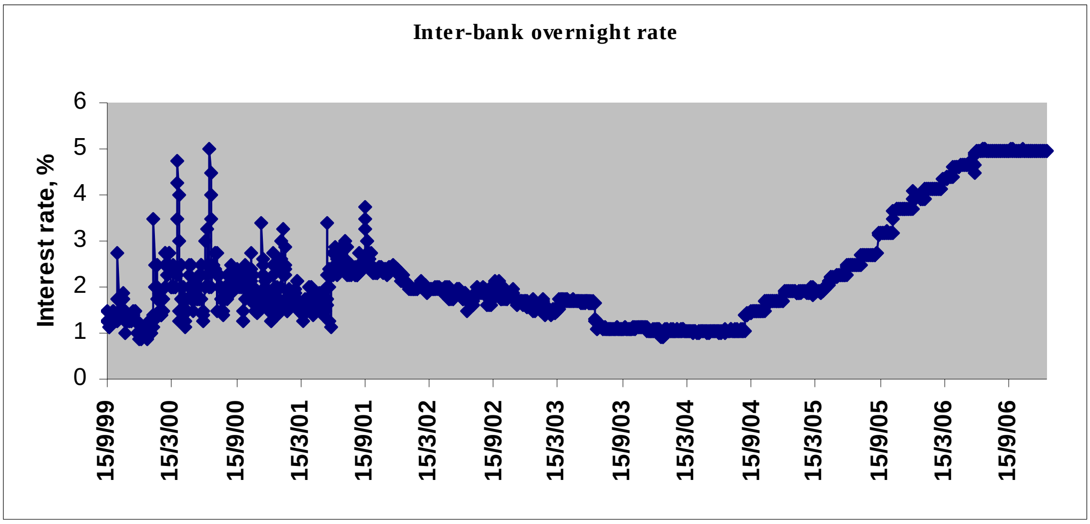
The studies based on data of the other countries as shown in table2 also use the data sets of interest rate whose maturity are as mentioned above. Also, to study the dynamic of short rate and to understand the effect of data set used, this paper uses the data sets of three shortest maturity interest rate, daily, weekly, and montly as proxy of short rate.

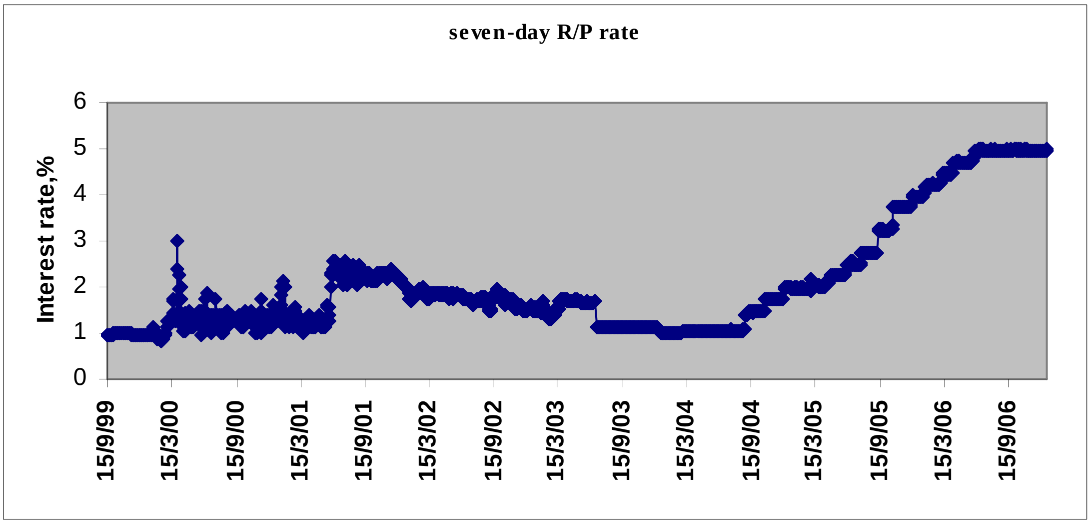
Table 2
Different data series used as proxies of short rate

Maturity	Studies	Data
Daily	Treepongkaruna and Gray(2006)	Thai Inter-bank overnight rate
Weekly	Ait Sahalia(1996)	Seven-day Eurodollar rate
	Fan and Lao(204)	Seven day Chinese repo- rate
Monthly	Shi, Kagraoka, Tamura, and Ozaki(1999)	One-month Japanese LIBOR rate
Quarterly	Stanton(1997)	Three-month U.S. Treasury bill
	Durham(2003)	Three-month U.S. Treasury bill
	Li(2000)	Thirteen week Australian Treasury-bill

The daily data of all three series are from Bank of Thailand’s website. All data sets start on 15 September 1999 and end on 29 December 2006, which includes 1,791 daily observations.

Figure 1
Interest-rate time series





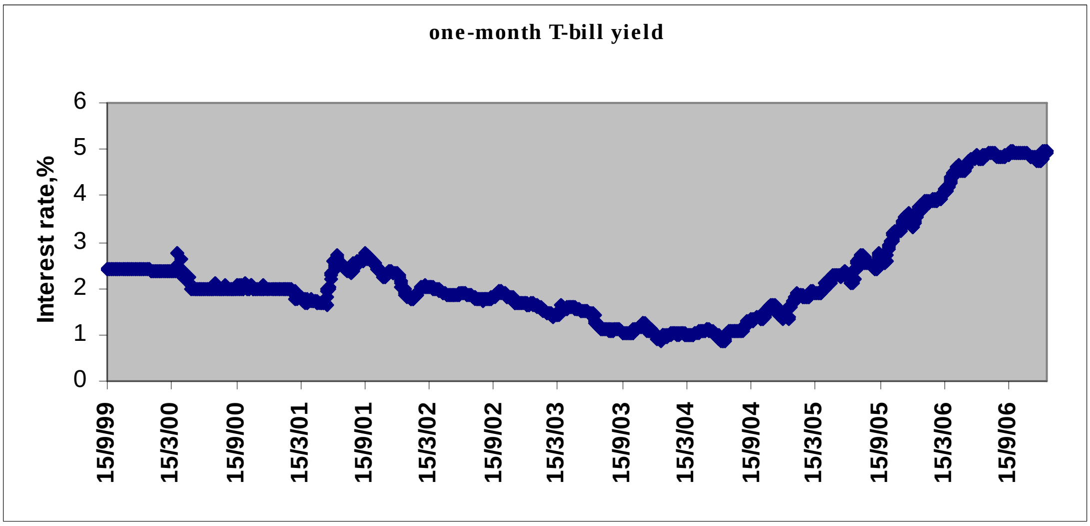


Figure 1 plots the time series of the three interest rate series. For each series, the data are quite flat for two-thirds of the period used. But after the last quarter of year 2004, the three interest rate series increase gradually, which is the result from increasing key indicator rate, fourteen-day R/P rate. The fourteen-day R/P rate is the pure jump process whose value is clearly related to the process of the money market. Therefore, the inter-bank overnight rate, seven-day R/P rate, and one-month T-bill are influenced by this key indicator rate increasing. Figure2 is the time series of the fourteen-day R/P rate. As can be observed, there are jumps in it at A and at the other times when the monetary authorities change key indicator rate. The data descriptions of all interest rate series are shown in table3.

Figure 2
The key indicator rate

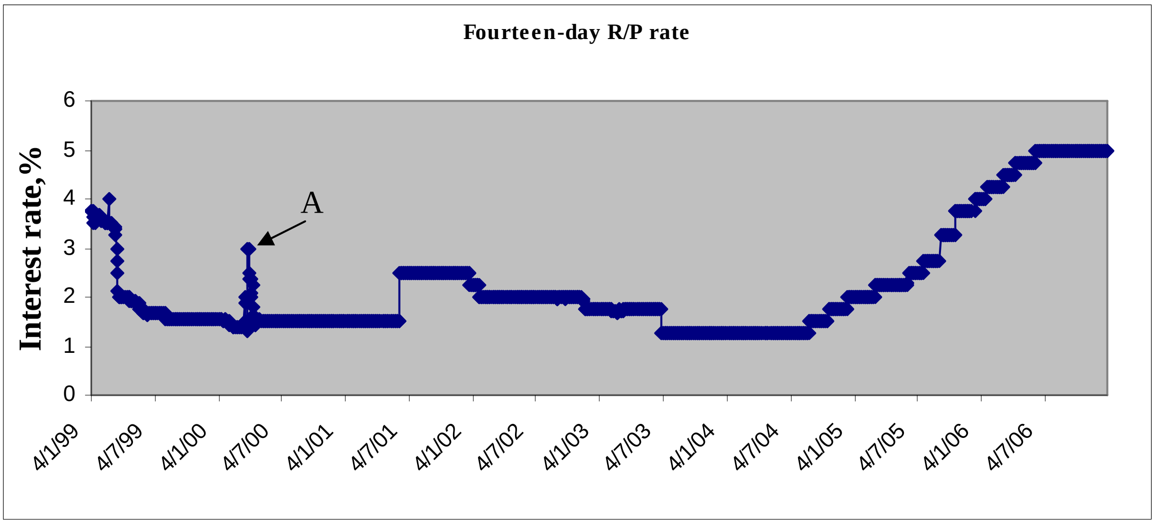


Table 3
Data Description

Interest rate	Mean	Variance	Max	Min
Inter-bank overnight rate	2.2428	1.3277	5.0000	1.32772
Seven-day R/P rate	2.0797	1.4825	5.0000	1.4825
0ne-month T-bill yield	2.2533	1.1750	4.9715	1.1750

The prior studies that use parametric approach to examine the dynamic of short rate mostly use either Generalized Method of Moment(GMM) or Maximum likelihood. This study as well as the study of Bali and Wu (2005) and the study of Durham (2003), uses the quasi-maximum likelihood because the number of sample used in this paper is relatively small. But, GMM requires a large-sample and its desirable properties are likely to be achieved only in very large sample. Moreover, GMM are asymptotically efficient in a large sample but are rarely efficient in finite sample.

SDE is in continuous-time setting but the data is in discrete-time setting. Following Euler approximation, the discrete time approximation of the continuous time model in Eq. (1) can be written as

$$\Delta r = \mu + \varepsilon \qquad (4)$$

where

$$\Delta r = r_t - r_{t-\Delta} \text{ and } \varepsilon_t = \sqrt{v(r_{t-\Delta})}Z$$

Stanton(1997) find that if applying monthly data or more frequent data, the error introduced by using approximation is extremely small, especially when compared with likely magnitude of estimation error. Hence, in this paper Δ denoting the length of the discrete interval is 1/365 for daily series.

From Eq. (8), the log-likelihood equation becomes

$$\ell\left(\{r_t\}_{t=2}^{T}\right) = -\frac{T-1}{2}\ln 2\pi - \frac{1}{2}\sum_{t=2}^{T}\left[\ln \nu\left(r_{t-\Delta}\right) + \frac{(r_t - r_{t-\Delta} - \mu\left(r_{t-\Delta}\right))^2}{\nu(r_{t-\Delta})\Delta}\right] \tag{5}$$

Where T is the number of observations.

This study uses log likelihood ratio test, LR to test the significance of non-linearity. The likelihood ratio test is $2\Delta\ell$, which is two times of the log likelihood difference between unrestricted model, general model, and restricted model, linear drift model. The specifications for the general model and linear drift model are as follows.

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1} \tag{6}$$

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t \tag{7}$$

The former is the unrestricted model and the latter is the restricted model. The likelihood ratio test is distributed as the Chi-square distribution with *df* equal to the number of restrictions imposed by the null hypothesis.

Empirical results

Table 4 - 6 contain the parameter estimates and the *t* – statistics in parenthesis for general model and linear drift with the combined diffusion function, the affine diffusion function, and the CEV diffusion function, respectively.

In each table, the results rank in order of the length of maturities from inter-bank overnight rate, seven-day R/P rate, and one-month T-bill. And, the parameter estimates of the general model, entitled general and a linear drift model, entitled affine, are reports in the first and the second column, respectively.

Table 4
Parameter estimates for combined diffusion function

Drift	**Inter-bank Overnight rate**		**Seven-day R/P Rate**		**One-month Treasury Yield**	
	General	**affine**	**General**	**affine**	**General**	**affine**
α_0	**0.0360**	**-0.0000**	**-0.0239**	**0.0143**	**0.0024**	**0.0038**
	(0.2443)	**(0.0080)**	**(0.3248)**	**(0.3920)**	**(0.2617)**	**(0.3604)**
α_1	**-0.8280**	**-0.0010**	**-0.0000**	**-0.0029**	**-0.0022**	**-0.0029**
	(0.2265)	**(0.0212)**	**(0.2654)**	**(0.9696)**	**(0.8301)**	**(0.2645)**
α_2	**-0.0001**	**-**	**0.1008**	**-**	**-0.0001**	**-**
	(0.0567)	**-**	**(0.4467)**	**-**	**(0.0392)**	**-**
α_3	**0.0002**	**-**	**0.0001**	**-**	**-0.0000**	**-**
	(0.3059)	**-**	**(0.5982)**	**-**	**(0.0690)**	**-**
β_0	**0.0000**	**0.3590**	**0.0003**	**0.3284**	**0.0005**	**-0.0270**
	(0.1786)	**(0.1515)**	**(1.1863)**	**(0.8090)**	**(0.0775)**	**(0.4036)**
β_1	**22.1187**	**0.0000**	**0.0000**	**0.0132**	**0.5349**	**0.0000**
	(1.4159)	**(0.0348)**	**(0.3497)**	**(0.3917)**	**(1.0476)**	**(0.2697)**
β_2	**128.2202**	**0.1136**	**0.1557**	**0.0504**	**0.0001**	**0.1123**
	(0.2462)	**(0.0860)**	**(3.0913)**	**(0.2112)**	**(0.21534)**	**(0.4676)**
β_3	**2.8105**	**0.0177**	**0.0000**	**-0.0000**	**0.00000**	**0.2709**
	(0.6177)	**(0.1916)**	**(0.2285)**	**(1.2384)**	**(0.05458)**	**(0.3869)**
Likelihood Value	**-1,646.2351**	**-1,646.2784**	**1,633.3717**	**-1,633.3794**	**-1,609.1786**	**-1,609.8845**

This table reports the maximum likelihood estimates of the short rate model and reports the absolute *t*-statistic in parenthesis.

$$dr_t = \mu(r_t)dt + \sqrt{v(r_t)}dz$$

with

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1} \quad \textbf{for General}$$

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t \quad \textbf{for Affine}$$

the diffusion function of the model is

$$v(r_t) = \beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta_3}$$

Table 5
Parameter estimates for affine diffusion function

Drift	Inter-bank Overnight rate		Seven-day R/P Rate		One-month Treasury Yield	
	General	affine	General	affine	General	affine
α_0	-0.0008	0.0310	-0.0332	0.0000	0.00912	0.0000
	(0.1525)	(0.2975)	(0.4191)	(0.2500)	(0.63131)	(0.0304)
α_1	-0.1244	-0.1288	-0.0093	-0.0526	0.00105	-0.0554
	(0.0261)	(0.2466)	(0.2897)	(0.3010)	-(0.07973)	(0.0955)
α_2	0.1249	-	0.1000	-	0.00389	-
	(0.0742)	-	(1.8887)	-	(0.11362)	-
α_3	0.0006	-	0.0006	-	-0.00008	-
	(0.0761)	-	(0.3372)	-	(013315)	-
β_0	0.0000	-0.0000	0.0970	0.0974	0.00001	0.0007
	(0.0355)	(0.0497)	(1.4217)	(1.5524)	(0.03860)	(0.0932)
β_1	21.5845	21.6434	0.0000	0.0117	0.57055	0.5302
	(1.4646)	(1.5859)	(0.0127)	(0.2222)	(1.39238)	(0.9207)
Likelihood Value	-1,646.3120	-1,646.3619	-1,633.3711	-1,633.3806	-1,609.9148	-1,609.9232

This table reports the maximum likelihood estimates of the short rate model and reports the absolute *t*-statistic in parenthesis.

$$dr_t = \mu(r_t)dt + \sqrt{v(r_t)}dz$$

with

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1} \quad \textbf{for General}$$

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t \quad \textbf{for Affine}$$

the diffusion function of the model is

$$v(r_t) = \beta_0 + \beta_1 r_t$$

Table 6
Parameter estimates for CEV diffusion function

Drift	Inter-bank Overnight rate		Seven-day R/P Rate		One-month Treasury Yield	
	General	affine	General	affine	General	affine
α_0	-0.0733	0.0300	0.0193	0.0238	0.0002	0.0179
	(0.7059)	(0.2095)	(0.4272)	(0.2299)	(0.1421)	(0.1589)
α_1	-0.6744	-0.5410	0.0037	-0.5515	0.3012	-1.0823
	(0.2235)	(0.0943)	(0.5157)	(0.8746)	(0.1995)	(0.2333)
α_2	-0.0009	-	-0.0859	-	-0.0065	-
	(0.0065)	-	(0.5015)	-	(0.1337)	-
α_3	0.0016	-	-0.0000	-	0.0000	-
	(1.0741)	-	(0.3091)	-	(0.0854)	-
β_2	1.2138	0.4690	0.1076	0.1142	0.7386	0.0627
	(1.2974)	(1.5501)	(1.3396)	(1.5117)	(0.1250)	(0.0929)
β_3	0.2601	0.0013	0.0260	0.0416	1.0643	0.4323
	(1.2355)	(0.1208)	(0.3304)	(0.4791)	(0.5235)	(0.1540)
Likelihood Value	-1,646.3030	-1,646.3766	-1,633.3872	-1,633.3885	-1,609.9184	-1,609.9419

This table reports the maximum likelihood estimates of the short rate model and reports the absolute *t*-statistic in parenthesis.

$$dr_t = \mu(r_t)dt + \sqrt{v(r_t)}dz$$

with

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1} \quad \text{for General}$$

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t \quad \text{for Affine}$$

the diffusion function of the model is

$$v(r_t) = \beta_2 r_t^{\beta_3}$$

For all tables, none of the mean reversion parameters are insignificantly different from zero, even the coefficients on the constant and linear terms. Some diffusion parameters are quite significant, indicating that the volatility merely depends on the level of the short rate for some choice of data sets and some diffusion function. Nevertheless, the results are consistent with Treepongkaruna and Gray (2006). When using specification of Ait Sahalia (1996) for short rate model, they find that none of parameter estimates is significant for interest rate series of many countries including Thailand. And Bali and Wu (2005) also find that the mean reversion parameters are insignificant when using U.S. three-month T-bill.

Nevertheless, the question remains, whether the choice for the data sets and diffusion function used affect the strength of the significance of the non-linearity. And whether there are the patterns for the significance of non-linearity as indicated by the prior study. Bali and Wu study find that the significance of non-linearity in mean reversion process decline when the conditional diffusion function changes from the affine function of short rate to the CEV function, and the combined form and when the maturity of interest rate series increase. To test the significance of non-linearity when using Thai data, we construct a log likelihood ratio test, LR, between the general model and linear drift model. In this case, the linear drift model is the restricted version of general model with $\alpha_2 = \alpha_3 = 0$ and *df* is equal to 2 and therefore the critical value is 5.99 at 95% confidence level.

Effect on mean reversion from choice of data sets and diffusion functions used

Table 7 contains eighteen log likelihood values of the unrestricted model and restricted model with the different diffusion functions, the multiplication of the three different interest rates with two drift specifications and three diffusion functions. The three diffusion specifications are as follows.

$$v(r_t) = \beta_0 + \beta_1 r_t \quad (8)$$

$$v(r_t) = \beta_2 r_t^{\beta_3} \quad (9)$$

$$v(r_t) = \beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta_3} \quad (10)$$

The diffusion specifications are the affine, the CEV, and the combined diffusion functions, respectively.

Table 7
Different data series used as proxies of short rate

Drift	**Inter bank Overnight rate**		**Seven-day R/P Rate**		**One-month Treasury Yield**	
	General	**affine**	**General**	**affine**	**General**	**affine**
Log likelihood Value						
Affine	-1,646.3120	-1,646.3619	-1,633.3718	-1,633.3806	-1609.9148	-1,609.9232
CEV	-1,646.3030	-1,646.3766	-1,633.3872	-1,633.3885	-1,609.9184	-1,609.9419
Combined	-1,646.2351	-1,646.2784	-1,633.3711	-1,633.3794	-1,609.1786	-1,609.8845

From table 7, we can construct the log likelihood ratio test values as shown in table 8. Relative to 5.99, none of the log likelihood ratio test is significantly different from zero, so we cannot reject the null hypothesis, the linear drift for inter-bank overnight rate, seven-day R/P rate, and one-month T-Bill yield.

Also, we can compare the significance of non-linearity specification when using different diffusion specifications. The result shows that whether we use any diffusion function, the log likelihood ratio tests are insignificantly different from zero.

Table 8
The log likelihood ratio test for the drift function

Drift LR = $2(\ell_{general} - \ell_{affine}) \sim \chi^2(2)$

	Interbank Overnight rate	**Seven-day R/P Rate**	**One-month Treasury Yield**
Affine	0.0998	0.0176	0.0168
CEV	0.1472	0.0026	0.0470
Combined	0.0866	0.0166	1.4118

But, the study of Bali and Wu favors combined diffusion specification. Hence, we compare the log likelihood ratio to test the significance of combined diffusion function against the affine and the CEV diffusion function for both general model and linear drift model.

In this case, the combined diffusion function is unrestricted model and the affine and the CEV diffusion functions are the restricted model. The null hypothesis that the diffusion is affine is $\beta_2 = \beta_3 = 0$. And the null hypothesis that the diffusion is CEV is $\beta_0 = \beta_1 = 0$. Hence, for both tests, df is equal to 2 and therefore the critical value is 5.99 at 95% confidence level.

Table 9
The log likelihood ratio test for the diffusion function

Variance LR = $2(\ell_{combined} - \ell_{affine/CEV}) \sim \chi^2(2)$

	Interbank Overnight rate		**Seven-day R/P Rate**		**One-month Treasury Yield**	
	General	**affine**	**General**	**affine**	**General**	**affine**
Affine	0.1538	0.1670	0.0014	0.0024	1.4724	0.0774
CEV	0.1358	0.1964	0.0322	0.0182	1.4796	0.1148

From table 9, none of log-likelihood ratios test is significant, therefore we cannot reject the null hypothesis for both the affine and the CEV diffusion functions.

Robustness test

In this section, we examine the robustness test of the models in terms of data used. As can be observed in the figure1, there are upward trend in the last part of the periods used. So, we use the data of the first 800 to eliminate the affect of the upward trend. But we also find that the parameter estimates are insignificant. The parameter estimates ae shown in appendix.

Conclusion

In this paper, we use the quasi-maximum likelihood for estimations. The parameter estimates are insignificant, which is consistent with the prior study using Thai data. Nevertheless, we examine whether there is the pattern of the strength of the significance of the non-linearity with log-likelihood ratio test. When using U.S. data Bali and Wu find the pattern of the strength of the significance of the significance of the nonlinearity, they find that the significance of non-linearity in mean reversion process decline when the conditional diffusion function changes from the affine function of short rate to the CEV function, and the combined form and when the maturity of interest rate series increase. We do not find the pattern of the strength of the significance of the non-linearity. With Thai data, we cannot find the pattern.

Appendix

Appendix1: Parameter estimates for combined diffusion function

Drift	Inter-bank Overnight rate		Seven-day R/P Rate		One-month Treasury Yield	
	General	**affine**	**General**	**affine**	**General**	**affine**
α_0	**0.0336**	**-0.0001**	**-0.0246**	**0.0122**	**-0.0182**	**-0.0027**
	(0.0349)	**(0.0691)**	**(0.1622)**	**(0.2219)**	**(0.2307)**	**(0.0246)**
α_1	**-0.8476**	**-0.1221**	**-0.0000**	**-0.0300**	**-0.0042**	**-0.0022**
	(0.0430)	**(0.1518)**	**(0.1480)**	**(0.8944)**	**(0.3751)**	**(0.0977)**
α_2	**-0.0001**	**-**	**0.1008**	**-**	**-0.0002**	**-**
	(0.0239)		**(0.0961)**		**(0.1671)**	
α_3	**0.0012**	**-**	**0.0001**	**-**	**0.0003**	**-**
	(0.0887)		**(0.2003)**		**(0.1105)**	
β_0	**0.0107**	**0.9236**	**0.0003**	**0.6708**	**0.0000**	**-0.0995**
	(0.1187)	**(1.0337)**	**(0.1662)**	**(0.6214)**	**(0.1050)**	**(0.0236)**
β_1	**25.4598**	**-0.0000**	**0.0000**	**0.0219**	**0.6511**	**0.1691**
	(0.0201)	**(0.1206)**	**(0.0527)**	**(0.1906)**	**(1.0359)**	**(0.0035)**
β_2	**103.1589**	**0.0297**	**0.2498**	**0.0961**	**0.0000**	**0.3936**
	(0.0964)	**(0.1021)**	**(1.2963)**	**(0.7950)**	**(0.0797)**	**(0.0657)**
β_3	**1.4184**	**0.0873**	**0.0283**	**-0.0000**	**0.0000**	**0.3294**
	(0.1058)	**(0.9922)**	**(0.1291)**	**(0.0917)**	**(0.0283)**	**(0.0398)**
Likelihood Value	**-736.1100**	**-736.2289**	**734.4480**	**-734.4758**	**-731.2860**	**-731.4462**

This table reports the maximum likelihood estimates of the short rate model and reports the absolute *t*-statistic in parenthesis.

$$dr_t = \mu(r_t)dt + \sqrt{v(r_t)}dz$$

with

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1}$$ **for General**

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t$$ **for Affine**

the diffusion function of the model is

$$v(r_t) = \beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta_3}$$

Appendix2: Parameter estimates for affine diffusion function

Drift	Inter-bank Overnight rate		Seven-day R/P Rate		One-month Treasury Yield	
	General	**affine**	**General**	**affine**	**General**	**affine**
α_0	**-0.0024**	**-0.0108**	**-0.1002**	**0.0020**	**-0.0068**	**-0.0023**
	(0.0805)	**(0.0980)**	**(0.1049)**	**(0.1279)**	**(0.1931)**	**(0.0576)**
α_1	**-0.0001**	**-0.0001**	**-0.0099**	**0.0319**	**-0.0007**	**-0.0744**
	(0.2526)	**(0.0878)**	**(0.1042)**	**(0.1034)**	**(0.0740)**	**(0.1168)**
α_2	**0.0007**	**-**	**0.1000**	**-**	**0.0382**	**-**
	(0.0904)		**(0.2180)**		**(0.1861)**	
α_3	**0.0016**	**-**	**0.0016**	**-**	**0.0000**	**-**
	(0.1534)		**(0.1511)**		**(0.0478)**	
β_0	**0.0002**	**0.9162**	**0.1215**	**0.1949**	**0.0000**	**0.0000**
	(0.0919)	**(0.6132)**	**(0.0776)**	**(1.0457)**	**(0.0000)**	**(0.0890)**
β_1	**46.8114**	**3.7864**	**4.6653**	**0.0639**	**0.6540**	**0.6580**
	(1.0214)	**(0.0801)**	**(0.0577)**	**(0.1789)**	**(1.0402)**	**(1.0236)**
Likelihood Value	**-736.1209**	**-736.2326**	**-734.4500**	**-734.4794**	**-731.5289**	**-731.5295**

This table reports the maximum likelihood estimates of the short rate model and reports the absolute *t*-statistic in parenthesis.

$$dr_t = \mu(r_t)dt + \sqrt{v(r_t)}dz$$

with

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1} \quad \textbf{for General}$$

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t \quad \textbf{for Affine}$$

the diffusion function of the model is

$$v(r_t) = \beta_0 + \beta_1 r_t$$

Appendix3: Parameter estimates for CEV diffusion function

Drift	Inter-bank Overnight rate		Seven-day R/P Rate		One-month Treasury Yield	
	General	**affine**	**General**	**affine**	**General**	**affine**
α_0	**-0.694**	**0.0010**	**-0.1677**	**0.0000**	**-0.0000**	**0.0039**
	(0.1460)	**(0.1879)**	**(0.1769)**	**(0.0408)**	**(0.0973)**	**(0.0957)**
α_1	**-0.0007**	**-0.5120**	**-0.0397**	**-0.0009**	**-0.0007**	**-0.3411**
	(0.0368)	**(0.0436)**	**(0.1307)**	**(0.0471)**	**(0.0772)**	**(0.1000)**
α_2	**-0.0009**	**-**	**-0.2691**	**-**	**0.1244**	**-**
	(0.0146)		**(0.1304)**		**(0.2003)**	
α_3	**0.0129**	**-**	**0.0025**	**-**	**0.0000**	**-**
	(0.1539)		**(0.2051)**		**(0.0746)**	
β_2	**0.9568**	**1.0000**	**12.5575**	**988.8111**	**988.3932**	**994.0901**
	(0.9547)	**(0.9590)**	**(0.0800)**	**(0.1003)**	**(0.0716)**	**(0.0557)**
β_3	**-0.0019**	**0.0010**	**0.9912**	**2.0052**	**2.9130**	**2.9147**
	(0.0484)	**(0.1023)**	**(0.3378)**	**(0.8656)**	**(0.8035)**	**(0.6268)**
Likelihood Value	**-736.2026**	**-736.2462**	**-734.4562**	**-734.5584**	**-731.4272**	**-731.4552**

This table reports the maximum likelihood estimates of the short rate model and reports the absolute *t*-statistic in parenthesis.

$$dr_t = \mu(r_t)dt + \sqrt{v(r_t)}dz$$

with

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1} \quad \textbf{for General}$$

$$\mu(r_t) = \alpha_0 + \alpha_1 r_t \quad \textbf{for Affine}$$

the diffusion function of the model is

$$v(r_t) = \beta_2 r_t^{\beta_3}$$

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