

Thesis Title	Numerical Modeling of an Influenza Epidemic Model with Vaccination and Diffusion
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Abstract

This thesis studied the susceptible, vaccinated, exposed, infectious and recovered (*SVEIR*) influenza epidemic model. The model involving non-linear partial differential equation is studied. It is extended to incorporate diffusion in one-space dimension to enable the geographic spread of the disease in a population. Standard finite-difference and non-standard finite-difference method are constructed to obtain the numerical solutions of the proposed model. A parallel implementation procedure is also studied. The standard and non-standard finite difference methods are tested using two initial distributions and the proposed model is simulated by using the constructed method in order to investigate the rate of vaccination and vaccination efficiency.

Keywords : Non-Standard Finite-Difference Method / Parallel Implementation / Standard Finite-Difference Method / *SVEIR* Epidemic Model

หัวข้อวิทยานิพนธ์	การสร้างแบบจำลองเชิงตัวเลขของตัวแบบการแพร่ระบาดของโรคไข้หวัดใหญ่ที่มีการฉีดวัคซีนและมีการแพร่กระจาย
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บทคัดย่อ

วิทยานิพนธ์เล่มนี้ได้ศึกษาตัวแบบการแพร่ระบาดของโรคไข้หวัดใหญ่ชนิด กลุ่มเสี่ยงต่อการติดเชื้อ กลุ่มที่ได้รับวัคซีน กลุ่มฟักตัวของเชื้อ กลุ่มที่ติดเชื้อ และกลุ่มที่หายจากโรค (*SVEIR*) ในรูปแบบของระบบสมการเชิงอนุพันธ์ย่อยไม่เชิงเส้น โดยรวมการแพร่ระบาดของโรคในหนึ่งมิติที่พิจารณาการแพร่ระบาดทางภูมิศาสตร์ของโรคในกลุ่มประชากร วิธีผลต่างอันตะมาตรฐานและวิธีผลต่างอันตะไม่มาตรฐานได้สร้างขึ้นมาเพื่อศึกษาผลลัพธ์เชิงตัวเลขของตัวแบบ การคำนวณแบบขนานได้ถูกศึกษาทดสอบวิธีผลต่างอันตะมาตรฐาน โดยใช้สองเงื่อนไขเริ่มต้นเพื่อศึกษาอัตราการฉีดวัคซีนและประสิทธิภาพวัคซีน

คำสำคัญ : การคำนวณแบบขนาน / แบบจำลองการแพร่ระบาดของโรค *SVEIR* / วิธีผลต่างอันตะมาตรฐาน / วิธีผลต่างอันตะไม่มาตรฐาน

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LIST OF SYMBOLS

SYMBOL

β	Contact rate
β_E	Ability to cause infection by exposed individuals
β_I	Ability to cause infection by infectious individuals
β_v	Ability to cause infection by vaccination individuals
σ	Rate of latency
γ	Rate of clinically ill
δ	Rate of duration of immunity loss
μ	Natural mortality rate
r	Birth rate
K	Recovery rate of latents
α	Flu induced mortality rate
θ	Rate of susceptible
ϕ	Rate of vaccination
d_1	Diffusivity constants of susceptible population
d_2	Diffusivity constants of vaccinated population
d_3	Diffusivity constants of exposed population
d_4	Diffusivity constants of infectious population
d_5	Diffusivity constants of recovered population