LITERATURE REVIEWS

The nanofluid behavior has been observed consistently by different researchers at different organizations and with different nanofluids. They found that thermal conductivity, viscosity and the convective heat-transfer coefficient increase relative to the base fluid.

1. Thermal conductivity of nanofluids

The earliest observations of thermal conductivity enhancement in liquid dispersions of submicronic solid particles nanoparticles were reported in 1993 by Masuda *et al*. (Masuda *et al*., 1993) However, the term nanofluid was first proposed by Choi about a decade ago to denote engineered colloids composed of nanoparticles dispersed in a base fluid (Eastman *et al*., 2004).

1.1 Experimental thermal conductivity of nanofluids

Masuda *et al*. (1993) measured the thermal conductivity of Al_2O_3/water nanofluid. The sizes of nanoparticles averaged 13 nm in diameter. They found that the thermal conductivity of the nanofluid depends mainly on the volume fraction.

In 1996 Eastman *et al*. (1996) also measured the thermal conductivity of Al_2O_3/water nanofluid but with sizes of nanoparticles of 33 nm in diameter. They also found that the thermal conductivity of the nanofluid depends mainly on the volume fraction. Similarly, Eastman *et al*.(1997) tested the thermal conductivity of CuO/water of diameter 33 nm reporting also that thermal conductivity it depend mainly on volume fraction. Pak and Cho (1999) using water base fluid with Al_2O_3 and TiO_2 nanoparticles (diameter 13 nm and 27nm, respectively) reported a similar dependence. Lee *et al*. (1999) experimented using the following mixtures: water/ Al_2O_3 (38.4nm), water/CuO (23.6nm), ethylene glycol/Al_2O_3 (38.4nm) and ethylene glycol/CuO (23.6nm). Their conclusions accorded with those reported by the researches listed above. Eastman *et al*. (2001) reported similar conclusions using ethylene glycol/CuO (35nm). Later, Xie et al. (2002) using ethylene glycol/Al_2O_3 reported similarly.

However, when Das *et al*. (2003) did similar experimental work using water/ Al_2O_3 (38.4nm), water/CuO (28.6nm) they also tested for thermal conductivity dependence on temperature. They found that while thermal conductivity was strongly dependent on volume fraction, it was also dependent on temperature. Finally, Chon *et al*. (2005) using water/Al_2O_3 and varying the nanoparticles sizes (11, 47 and 150 nm) reported that thermal conductivity was dependent on nanoparticle size as well as volume fraction and temperature. The data referred to above can be found in Appendix A.

1.2 Thermal conductivity models of nanofluids

Previously, the enhancement in the thermal conductivity of the nanofluids was assumed to depend mainly on the volume fraction (Masuda *et al*., 1993; Eastman *et al*., 1997; Pak and Cho, 1998; Lee *et al*., 1999; Eastman et al., 2001; Xie *et al*., 2002; Kwak and Kim, 2005). A thermal conductivity model that was based on the above assumption (e.g. Maxwell's model (1873) and Hamilton and Crosser's model (1962)) was called a stationary model. However, later works showed that the increase in temperature (Das *et al*., 2003; Chon *et al*., 2005; Li and Peterson, 2006) also enhanced the thermal conductivity of nanofluids. A dynamic model was therefore developed due to the Brownian motion of nanoparticles, based on the stationary model with the inclusion of the thermal dispersion effect (Khannafer *et al*., 2003; Koo and Kleinstreuer, 2004). Although, the temperature-dependent effect could be predicted by the dynamic model, the volume-fraction-dependent effect was not satisfactorily predicted.

Recently, the thermal conductivity of the interfacial layer of the nanoparticles has been shown to have an effect on the thermal conductivity of nanofluids (Patal *et al*., 2003; Wang *et al*., 2003; Yu and Choi, 2003; Yu and Choi, 2004; Xue *et al*., 2004; Leong *et al*., 2006). Leong *et al*. (2006) has proposed a stationary model that includes the interfacial layer effect. Their model has been shown to have higher accuracy in predicting the effect of volume fraction than the other stationary models. Nevertheless, the Leong et al.'s model has not been able to predict the temperature-dependent effect due to its stationary nature.

1.2.1 Stationary models

In 1873, the Maxwell (MG) model was developed to determine the effective thermal conductivity of liquid-solid suspensions for different volumetric loadings of spherical particles. It is given by

$$\frac{k_{eff}}{k_f} = \frac{(1-\phi)\left(k_p + 2k_f\right) + 3\phi k_p}{(1-\phi)\left(k_p + 2k_f\right) + 3\phi k_f} \tag{1}$$

where k_{eff} is the effective thermal conductivity of liquid with particle suspension, k_f is the thermal conductivity of host medium (based fluid), k_p is the thermal conductivity of particles and ϕ is the volume fraction of particles.

The basis of effective thermal conductivity for a two-component mixture is as follows

$$k_{eff} = \frac{k_p\phi_p\left(\frac{dT}{dx}\right)_p + k_f\phi_f\left(\frac{dT}{dx}\right)_f}{\phi_p\left(\frac{dT}{dx}\right)_p + \phi_f\left(\frac{dT}{dx}\right)_f} \tag{2}$$

Seventy years later, in 1962, Hamilton and Crosser developed an elaborate model for the effective thermal conductivity of two-component mixtures as a function of the conductivity of the pure materials, the composition of the mixture, and the shape of the disperse particles. For non-spherical particles, in which the ratio of conductivity of two phases is larger than 100, The Hamilton and Crosser model of thermal conductivity can be determined as follows

$$\frac{k_{eff}}{k_f} = \frac{k_p + (n-1)k_f - (n-1)\phi(k_f - k_p)}{k_p + (n-1)k_f + \phi(k_f - k_p)} \tag{3}$$

where n is the empirical shape factor given by $n = 3/\psi$ and ψ is the sphericity defined as the ratio of the surface area of a sphere with a volume equal to that of the particle, to the surface area of the particle. For spherical particles, the Hamilton and Crosser (H-C) model reduces to the Maxwell-Garnett model.

In 1977, Wasp introduced an alternative expression for calculating the effective thermal conductivity of solid-liquid mixtures. It is given by

$$\frac{k_{eff}}{k_f} = \frac{k_p + 2k_f - 2\phi\left(k_f - k_p\right)}{k_p + 2k_f - \phi\left(k_f - k_p\right)} \tag{4}$$

where ϕ is the volume fraction of the particles and defined as $\phi = \frac{\forall_p}{\forall_f + \forall_p} = m\frac{\pi}{6}d_p^3$ and m is the number of the particles per unit volume and d_p is the average diameter of the particle. Comparison between the Hamilton and Crosser model and the Wasp model reveals that Wasp model is a special case of the Hamilton and Crosser model with the sphericity 1.0.

Xuan and Li (2000) applied the Hamilton and Crosser model to a water-alumina nanoparticles suspension to obtain a rough estimation of the thermal conductivity of nanofluids. The data show that the model results for ψ = 0.7 were close to their experimental data. However, this model was not suitable for predicting the thermal conductivity of CuO nanofluids (Trisakri and Wongwises, 2005).

A number of research groups have proposed that one of the major mechanisms behind this thermal conductivity enhancement is the layering of liquid molecules at the solid-particle surface which is commonly referred to as the interfacial layer or nanolayer in the case of nanofluids. This solid-like layer acts as a thermal bridge between a solid nanoparticle and the base liquid and so is an important mechanism to enhancing the thermal conductivity. Liquid molecules are known to form a layered structure at solid surfaces whose atomic structures are more ordered than those of the bulk liquid. Since this layer is in an ordered solid-like state, it would have a thermal conductivity greater than the bulk liquid and smaller than the solid particle (Leong et al, 2006).

Wang *et al*. (2003) established a fractal model for predicting the effective thermal conductivity of liquid with nanoparticle inclusion that considers the

effect of particle size and surface adsorption. The thermal conductivity of nanoparticles will be replaced by the thermal conductivity of nanoparticle clusters into MG model. The enhancement in effective thermal conductivity is given by

$$\frac{k_{eff}}{k_f} = \frac{(1-\phi)+3\phi\int_0^{\infty}\frac{k_{cp}(r)n(r)}{k_{cp}(r)+2k_f}dr}{(1-\phi)+3\phi\int_0^{\infty}\frac{k_{cp}n(r)}{k_{cp}(r)+2k_f}dr} \tag{5}$$

where $k_{cp}(r)$ is the thermal conductivity of nanoparticle clusters predicted by Bruggeman model and $n(r)$ is the radius distribution function. The proposed fractal model predicts well the trend for variation of the effective thermal conductivity with low volume fraction of nanoparticles for 50 nm CuO particles suspended in deionized water when ϕ = 0.5% (Wang et al., 2003).

Tinga *et al*. (1973) derived a model for the complex dielectric constant of a multiphase mixture (air–water–cellulose) with confocal ellipsoidal shell (water). Their model for the three-phase spherical inclusions can be simplified for thermal conductivity (k) instead of dielectric constant (ε) by considering cellulose, water, and air as solid particle (p), interfacial layer (lr), and host medium (f), respectively. The model can be expressed as

$$\frac{k_{eff}}{k_f} = 1+\frac{3\phi_p\left[\left(\beta^3-1\right)\left(2k_{lr}+k_p\right)\left(k_{lr}-k_f\right)-\left(k_{lr}-k_p\right)\left(2k_{lr}+k_f\right)\right]}{\left(2k_f+k_{lr}\right)\left(2k_f+k_p\right)-\left(\frac{2}{\beta^3-1}\right)\left(k_{lr}-k_f\right)\left(k_{lr}-k_p\right)-3\phi_p k_{lr}\left(k_p-k_f\right)} \tag{6}$$

where $\beta = 1+h/a$, h is the shell thickness and a is the solid particle radius.

Yu and Choi (2003) modified the Maxwell model for the effective thermal conductivity of solid/liquid suspensions to include the effect of this ordered nanolayer. The modified the Maxwell model is given by

$$\frac{k_{eff}}{k_f} = \frac{k_{pe} + 2k_f + 2\left(k_{pe} - k_f\right)\left(1+\gamma\right)^3 \phi}{k_{pe} + 2k_f + 2\left(k_{pe} - k_f\right)\left(1+\gamma\right)^3 \phi} \tag{7}$$

where $\gamma = h/a$ is the ratio of the nanolayer thickness to the original particle radius. k_{pe} is the equivalent thermal conductivity of the equivalent particles. For large particles ($a >> h, \gamma \to 0$), the nanolayer impact is small and the modified Maxwell model reduces to the original Maxwell model. However, this renovated Maxwell model is limited to suspensions with spherical particles.

Yu and Choi (2004) also extend the Hamilton–Crosser model for suspensions of nonspherical particles to include the effect of a solid/liquid interface. The model is defined by

$$\frac{k_{eff} - k_f}{k_e + \left(n-1\right)k_f} = \frac{f_e}{3} \sum_{e=a,b,c} \frac{k_{pe} - k_f}{k_{pe} + (n-1)k_f} \tag{8}$$

where *n* is the empirical shape factor, k_{pe} is the equivalent thermal conductivity and f_e and the equivalent volume concentration. The renovated Hamilton–Crosser model correctly predicts the magnitude of the thermal conductivity of nanotube-in-oil nanofluids and shows the important role that the solid/liquid interfacial layers play in the enhanced thermal conductivity of nanofluids. However, this model is not able to predict the nonlinear behavior of the nanofluid thermal conductivities

Xue *et al*. (2004) proposed a formula for calculating the effective thermal conductivity of nanofluids which is based on Maxwell theory and average polarization theory that includes the interfacial shell effect. It is given as

$$9\left(1-\frac{\nu}{\lambda}\right)\frac{k_{eff} - k_f}{2k_{eff} + k_f} + \frac{\nu}{\lambda}\left[\left(\frac{k_{eff} - k_{c,x}}{k_{eff} + \beta_{2,x}\left(k_{c,x} - k_{eff}\right)}\right) + \left(4\frac{k_{eff} - k_{c,y}}{2k_{eff} + \left(1-\beta_{2,x}\right)\left(k_{c,y} - k_{eff}\right)}\right)\right] \tag{9}$$

where $k_{c,x}$ and $k_{c,y}$ are the thermal conductivity components of the complex elliptical

particle along the x and y axes, respectively and ν/λ is the volume fraction of nanoparticle with interfacial shell. In validating Xue's model, the thermal conductivity of interfacial shell and an interfacial shell thickness of 3 nm were chosen to match his model results with experimental data of Yu and Choi (2004) for carbon nanotube/oil nonofluid and Xie et al. (2004) for alumina/water nanofluid. However, the depolarization factor in his model was estimated (Trisaksri and Wongwises, 2005).

Another new model for the effective thermal conductivity including interfacial layer was proposed by Leong et al. (2006).

$$k_{eff} = \frac{\left(k_p - k_{lr}\right)\phi k_{lr}\left[2\beta_1^3 - \beta^3 + 1\right] + \left(k_p + 2k_{lr}\right)\beta_1^3\left[\phi\beta^3\left(k_{lr} - k_f\right) + k_f\right]}{\beta_1^3\left(k_p + 2k_{lr}\right) - \left(k_p - k_{lr}\right)\phi\left[\beta_1^3 + \beta^3 - 1\right]} \tag{10}$$

where $\beta = 1+\gamma$, $\beta_1 = 1+\frac{\gamma}{2}$ and $\gamma = \frac{h}{a}$ is ratio of interfacial layer thickness and particle radius. The Leong et al model is in good agreement with most of the experimental results obtained from their study and the literature when the layer thickness is chosen as 1 nm and its thermal conductivity is assumed to be as 2 to 3 times that of base fluid thermal conductivity. Therefore, there is a need to determine the interfacial layer thickness and its thermal conductivity in order to develop a model for the effective thermal conductivity of nanofluids.

1.2.2 Dynamic models

Khanafer et al.'s model (2003) proposes the effective thermal conductivity of the nanofluid takes the following form:

$$k_{eff} = k_{Static} + k_V \tag{11}$$

where the static part k_{Static} of the dynamic model derives from Wasp's stationary model as follows

$$\frac{k_{eff}}{k_f} = \frac{k_p + 2k_f - 2\phi\left(k_f - k_p\right)}{k_p + 2k_f - \phi\left(k_f - k_p\right)} \tag{12}$$

Therefore, the dynamic part k_V which is the enhancement in the thermal conductivity due to the thermal dispersion is given as

$$k_v = C\left(\rho c\right)_{nf} \left|\overline{V}\right| \phi d_p \tag{13}$$

where $\left|\overline{V}\right| = \sqrt{u^2 + v^2 + w^2}$ and C is an unknown constant which should be determined from experimental data.

Jang *et al*. (2004) added the fundamental role of the dynamic nanoparticle was in predicting thermal conductivity that took into account energy transport in the nanofluids. It is given as

$$k_{eff} = k_f\left(1-\phi\right) + k_p\phi + 3C\frac{d_f}{d_p}k_f \,\mathrm{Re}_{d_p}^2 \,\mathrm{Pr}\,\phi \tag{14}$$

where Re_{d_p} is the Reynolds number defined by $\mathrm{Re}_{d_p} = \left(\overline{C}_{RM} d_p\right)\big/\nu$, C is a proportional constant, $\overline{C}_{RM}$ is the random motion velocity of nanoparticles, ν is the dynamic viscosity of base fluid and Pr is Prandtl number.

Koo and Kleinstreuer's model (2004) proposes the effective thermal conductivity of the dynamic model can be written as

$$k_{eff} = k_{static} + k_V \tag{15}$$

where the static part k_{static} of the dynamic model of Koo and Kleinstreuer (2004) is from Maxwell's stationary model as follows:

$$\frac{k_{eff}}{k_f} = \frac{(1-\phi)\left(k_p + 2k_f\right) + 3\phi k_p}{(1-\phi)\left(k_p + 2k_f\right) + 3\phi k_f} \tag{16}$$

The dynamic part k_V which is from the effect of the particle movement is given by Koo and Kleinstreuer (2004) as follows:

$$k_v = p\rho c\left|\bar{V}\right|\phi d_p f\left(T,\phi\right) \tag{17}$$

where $\left|\bar{V}\right| = \sqrt{\frac{18k_bT}{\pi\rho_d d_p^3}}$ from (Probstein, 2003), p is the probability for a particle to travel along any direction and $f\left(T,\phi\right)$ is determined by matching experimental data.

Chon *et al*. (2005) report an experimental correlation following the previously proposed conjecture from the theoretical work on Jang and Choi (2004), it is experimentally validated that the Brownian motion of nanoparticles constitutes a key mechanism of the thermal conductivity enhancement with increasing temperature and decreasing nanoparticle sizes.

$$\frac{k_{eff}}{k_f} = 1 + 64.7 \cdot f^{0.7460}\left(\frac{d_f}{d_p}\right)^{0.3690}\left(\frac{k_p}{k_f}\right)^{0.7476} \times \mathrm{Pr}^{0.09955}\,\mathrm{Re}^{1.2321} \tag{18}$$

Under specified f and d_f, Assuming constant k_p and ρ_f

$$\frac{k_{eff}}{k_f} = 1 + const \cdot \left(\frac{\mathrm{Pr}\left(T\right)^{0.9955} T^{1.2321}}{d_p^{0.369} k_f\left(T\right)^{0.7476} \mu^2\left(T\right)}\right) \tag{19}$$

2. Experimental convective transport in nanofluids

Convective heat transfer enhancements were reported by Pak and Cho (1998) for turbulent flow of alumina/water and titana/water nanofluid in round tubes that Nusselt numbers up to 30% higher than predicted by pure fluid. The correlation Nusselt numbers for turbulent flow is given as

$$\mathrm{Nu}_{nf} = 0.021 \cdot \mathrm{Re}_{nf}^{0.8} \mathrm{Pr}_{nf}^{0.5} \tag{20}$$

Xuan and Li (2003) also reported Nusselt numbers up to 30% higher than predicted for turbulent flow of copper/water nanofluids. The correlation Nusselt numbers with the experimental data is given as

For turbulent flow

$$\mathrm{Nu}_{nf} = 0.0059 \cdot \left[1 + 7.6286 \cdot \phi^{0.6886} \left(\mathrm{Re}_{nf} \mathrm{Pr}_{nf} \frac{d_p}{D} \right)^{0.001} \right] \mathrm{Re}_{nf}^{0.9238} \mathrm{Pr}_{nf}^{0.4} \tag{21}$$

For laminar flow

$$\mathrm{Nu}_{nf} = 0.4328 \cdot \left[1 + 11.285 \cdot \phi^{0.754} \left(\mathrm{Re}_{nf} \mathrm{Pr}_{nf} \frac{d_p}{D} \right)^{0.218} \right] \mathrm{Re}_{nf}^{0.333} \mathrm{Pr}_{nf}^{0.4} \tag{22}$$