CHAPTER II

TWO-PHASE FLOW

In this chapter, flow patterns inside tube will be described for vertical upward flows. Next, the different heat transfer regions in two-phase flow will be presented. Finally, the balance equations for two-phase flow will be discussed.

2.1 Flow patterns in vertical tubes [21]

For co-current upflow of gas and liquid in a vertical tube, the liquid and gas phases distribute themselves into several recognizable flow structures. These are referred to as flow patterns and they are depicted in Fig. 2.1 and can be described as follows:

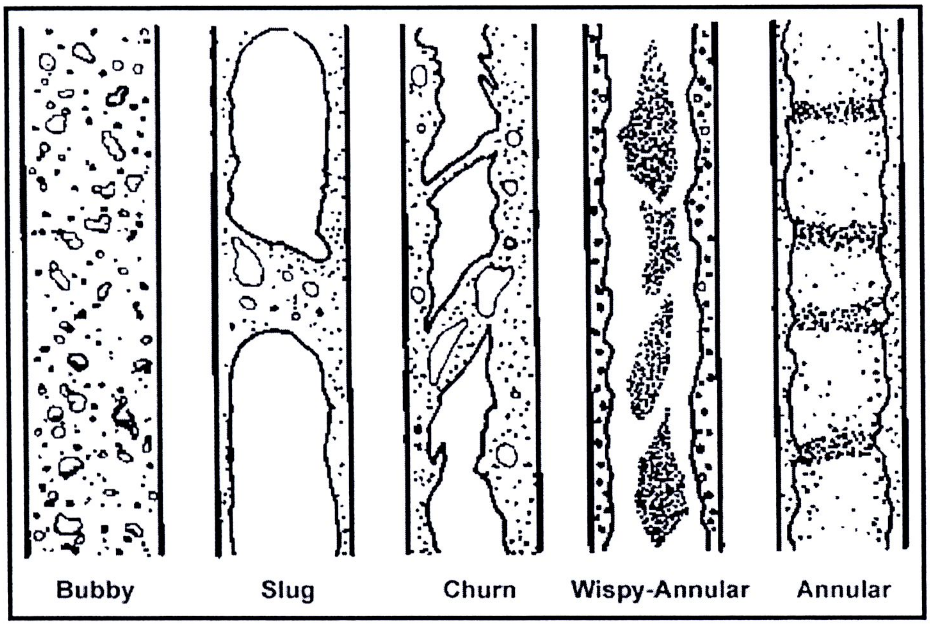

Fig. 2.1 Two-phase flow patterns in vertical upflow [21].

- **Bubbly flow.** Numerous bubbles are observable as the gas is dispersed in the form of discrete bubbles in the continuous liquid phase. The bubbles may vary widely in size and shape but they are typically nearly spherical and are much smaller than the diameter of the tube itself.

- **Slug flow.** With increasing gas void fraction, the proximity of the bubbles is very close such that bubbles collide and coalesce to form larger bubbles, which are similar in dimension to the tube diameter. These bubbles have a characteristic shape similar to a bullet with a hemispherical nose with a blunt tail end. They are commonly referred to as Taylor bubbles after the instability of that name. Taylor bubbles are separated from one another by slugs of liquid, which may include small bubbles. Taylor bubbles are surrounded by a thin liquid film between them and the tube wall, which may flow downward due to the force of gravity, even though the net flow of fluid is upward.

- **Churn flow.** Increasing the velocity of the flow, the structure of the flow becomes unstable with the fluid traveling up and down in an oscillatory fashion but with a net upward flow. The instability is the result of relative parity of the gravity and shear forces acting in opposing directions on the thin film of liquid of Taylor bubbles. This flow pattern is in fact an intermediate regime between the slug flow and annular flow regimes. In small diameter tubes, churn flow may not develop at all and the flow passes directly from slug flow to annular flow. Churn flow is typically a flow regime to be avoided in two-phase transfer lines, such as those from a reboiler back to a distillation column or in refrigerant piping networks, because the mass of the slugs may have a destructive consequence on the piping system.

- **Annular flow.** Once the interfacial shear of the high velocity gas on the liquid film becomes dominant over gravity, the liquid is expelled from the center of the tube and flows as a thin film on the wall (forming an annular ring of liquid) while the gas flows as a continuous phase up the center of the tube. The interface is disturbed by high frequency waves and ripples. In addition, liquid may be entrained in the gas core as small droplets, so much so that the fraction of liquid entrained may become similar to that in the film. This flow regime is particularly stable and is the desired flow pattern for two-phase pipe flows.

- **Wispy annular flow.** When the flow rate is further increased, the entrained droplets may form transient coherent structures as clouds or wisps of liquid in the central vapor core.

- **Mist flow.** At very high gas flow rates, the annular film is thinned by the shear of the gas core on the interface until it becomes unstable and is destroyed, such that all the liquid in entrained as droplets in the continuous gas phase, analogous to the inverse of the bubbly flow regime. Impinging liquid droplets intermittently wet the tube wall locally. The droplets in the mist are often too small to be seen without special lighting and/or magnification.

2.2 The different heat transfer regions in two-phase flow [22]

We shall now consider subcooled liquid fed into the bottom of a vertical evaporator tube, which is uniformly heated along its entire length. The heat flux $\dot{q}$ is assumed to be low and the tube should be long enough such that the liquid can be completely evaporated. Fig. 2.2 shows, on the left, alongside the various heat exchange regions that have already been explained, the profiles of the liquid and wall temperatures.

As long as the wall temperature stays below that required for the formation of vapor bubbles, heat will be transferred by single-phase, forced flow. If the wall is adequately superheated, vapor bubbles can form even though the core liquid is still subcooled. This is a region of subcooled boiling. In this area, the wall temperature is virtually constant and lies a few Kelvin above the saturation temperature. The transition to nucleate boiling is, by definition, at the point where the liquid reaches the saturation temperature at its centre, and with that the thermodynamic quality is $x_{th}^{*} = 0$. In reality, as Fig. 2.2 indicates, the liquid at the core is still subcooled due to the radial temperature profile, whilst at the same time vapor bubbles form at the wall, so that the mean enthalpy is the same as that of the saturated liquid.

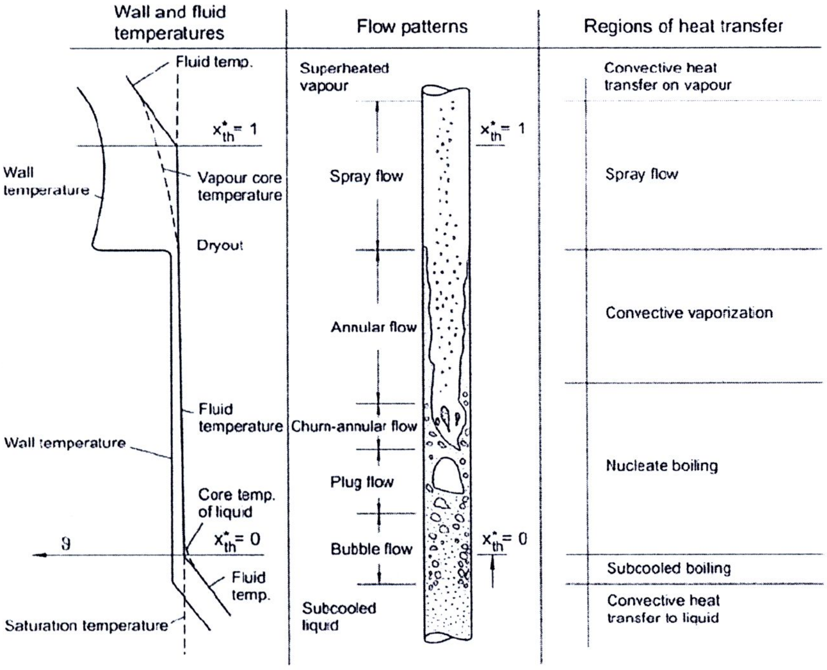

Fig. 2.2 Wall and liquid temperatures, flow pattern and the associated heat transfer regions, heated tube [22].

In the nucleate boiling region heat transfer is chiefly determined by the formation of vapor bubbles and only to a small extent by convection. This region encompasses the bubble, plug, churn and a part of the annular flow regimes. The vapor content constantly increases downstream, and at sufficiently high vapor content the churn flow converts into an annular flow, with a liquid film at the wall and vapor, with liquid droplets, in the core. The entire nucleate boiling region is characterized by the formation of vapor bubbles at the wall. However in annular flow, the liquid film downstream is so thin and its resistance to heat transfer is so low, that the liquid close to the wall is no longer sufficiently superheated, and the formation of bubbles at the wall is suppressed. Heat is conducted principally by the liquid that is evaporating at its surface. Heat is transferred by "convective evaporation".

As soon as the liquid film at the wall is completely evaporated, the temperature of a wall being heated with constant heat flux rises. This transition is known as dryout. The spray flow region is entered, followed by a region where all the liquid droplets being carried along by the vapor are completely evaporated, in which heat is transferred by convection to the vapor.

2.3 One-Dimensional Two-fluid Model [23]

The two-fluid model treats each phase separately in terms of two sets of conservation equations that govern the balance of mass, momentum and energy with proper averaging methods. Some simplifying assumptions are adopted to reach their most commonly used form. These simplifying assumption are discussed below.

1. The average of products is approximated by the products of averages.

It is quite clear that $\langle ab \rangle \neq \langle a \rangle \langle b \rangle$, since the two variables may have non-uniform distributions on the cross sections; assuming $\langle ab \rangle \approx \langle a \rangle \langle b \rangle$ is a useful but rough approximation; however, to make it better, we should know the local distributions of a and b, just the information we have lost in space averaging. We will also suppress the explicit indication of space averages by $\langle \circ \rangle$.

2. The two phasic pressures are assumed to be equal.

This assumption appears a reasonable one, considering that in 1D duct generally the difference between phasic pressures is very small. However, the assumption $p_l = p_v = p$ is less trivial that it seems; in fact:

- Such a choice implies that pressure perturbations are transferred instantaneously from one phase to the other, which is obviously not true;

- If we would also assume that the interfacial pressure is equal to the common value p this would lead to the impossibility to simulate the transport of void fraction, e.g., in the case of stratified flow, where superficial waves propagate due to pressure difference in the two phases; to overcome this problem, pressure at the interface is sometimes assumed different from the common value of phasic pressures;

- Assuming $p_l = p_v = p$ has also consequence on the mathematical character of balance equations that, also because of that, tend to lose the hyperbolic well-posed character it would be desirable, showing in some cases a partially-elliptic behavior; we will come back on this aspect later on.

3. The interface is assumed as an immaterial surface.

The jump conditions are written assuming that then interface cannot accumulate mass or momentum or energy. This appears obvious, but there are detailed aspects related to surface tension that should be considered. We will neglect them in the present treatment.

4. Axial diffusion terms are neglected.

Also this choice might appear obvious, since conductivity and viscosity of water (largely the most important fluid in our applications) are small enough that even a little velocity in the fluid will make advection transport so overwhelming with respect to diffusion transport to immediately justify this assumption. However, this choice eliminates any axial diffusion, changing the mathematical character of the equations. Anyway, numerical discretization will make spurious axial diffusion terms, even much larger than the physical ones, to appear again: nothing to be really desired...

5. Phasic velocities and pressures at the interface are assumed equal.

For the continuity of the velocity field trough the interface, it is:

$$w_{z,li} = w_{z,vi} = w_{z,i}$$

On the other hand, assuming

$$\langle p_l \rangle_i = \langle p_v \rangle_i = p_i$$

Requires neglecting the effect of surface tension on stresses at the interface, something we normally do not really need to account for.

With these assumptions we have:

MASS BALANCE EQUATIONS:

$$\frac{\partial}{\partial t} A\alpha_k \rho_k + \frac{\partial}{\partial z} A\alpha_k \rho_k w_k = A\Gamma_k \qquad (k = l, v)$$

(rate of change + advection = mass transfer rate)

In this equation, the only constitutive term is the mass transfer rate, Γ_k. The related jump condition is $\Gamma_v = -\Gamma_l$

MOMENTUM BALANCE EQUATIONS:

$$\frac{\partial}{\partial t} A\alpha_k \rho_k w_k + \frac{\partial}{\partial z} A\alpha_k \rho_k w_k^2 + A\alpha_k \frac{\partial p}{\partial z}$$

$$+(p-p_i)\frac{\partial}{\partial z} A\alpha_k = A\alpha_k \rho_k g_z + A\Gamma_k w_i - AF_{k,vm}^{'} \qquad (k = l, v)$$

$$-AF_{k,i} - AF_{k,w} + C_{w,k}(p_w - p_i)\frac{\partial A}{\partial z}$$

(rate of change + advection + phasic pressure gradient

+ interfacial pressure term = body force + mass transfer term - virtual mass term

- interfacial friction - wall friction + wall pressure term)

In this equation, constitutive laws are required for specifying:

- The mass transfer rate $\Gamma_k = \left[\frac{kg}{m^3 s}\right]$: this is related to heat transfer at the interface as discussed below;

- The interfacial velocity $w_i = \left[\frac{m}{s}\right]$: it is generally assumed $w_i = \lambda w_v + (1-\lambda) w_l \ (0 \le \lambda \le 1)$;

- The virtual mass term $F_{k,vm}^{'} = \left[\frac{force}{volume}\right] = \left[\frac{kg}{m^2 s^2}\right]$: it is due to the local variation of interfacial pressure due to accelerating phases (the $\Delta p_{ki}^{'}$ term) and it appears as a "virtual" (or "added") mass sticking to one phase when it accelerates with respect to the other in space or time; the classical formulation adopted to account for it is:

$$F_{k,vm} = \pm C_{vm} \alpha_l \alpha_v \rho_m \left[\frac{D_v w_v}{Dt} - \frac{D_l w_l}{Dt}\right] \qquad (+ for\ vapor, - for\ liquid)$$

where C_{vm} is a flow regime dependent virtual mass coefficient (e.g., taking the values 0.5 and 0 respectively for bubbly flow and stratified flow), ρ_m is a mixture density and the Lagrangian derivatives appearing above are defined as:

$$\frac{D_k \circ}{Dt} = \frac{\partial \circ}{\partial t} + w_k \frac{\partial \circ}{\partial z}$$

- The interfacial friction term $F_{k,i} = \left[\frac{force}{volume}\right] = \left[\frac{kg}{m^2 s^2}\right]$: this generally depends on the square of the relative velocity $w_r = w_v - w_l$ and it is strongly flow regime dependent, also through the interfacial area per unit volume a_i;

- The wall friction term $F_{k,w} = \left[\frac{force}{volume}\right] = \left[\frac{kg}{m^2 s^2}\right]$: it generally depends on the square of the phasic velocity w_k and is also flow regime dependent, also according to the fraction of the k-th phase assumed to be present at the wall;

- The pressure at the interface p_i: this term is of basic importance for two reasons:

1. Its neglect may give rise to an ill-posed problem;
2. As said, it is important to simulate gravity waves in stratified horizontal flow

- The pressure at the wall p_w: it may be assumed equal or different from the value of the common phasic pressure p, also depending on the flow regime;

- The fraction of presence of the k-th phase at the wall, $C_{w,k}$ also flow regime dependent; it must be $C_{w,l} + C_{w,v} = 1$. The jump condition in this case is, obviously enough, $F_{l,i} = -F_{v,i}$

A simple rule to check if the jump condition for an equation is correct is adding up term by term the equations for the two phases and imposing that the summation of the terms that should not appear in a mixture equation is zero. In the present case, it is:

$$\begin{aligned}
&\frac{\partial}{\partial t} A(\alpha_l \rho_l w_l + \alpha_v \rho_v w_v) + \frac{\partial}{\partial z} A(\alpha_l \rho_l w_l^2 + \alpha_l \rho_l w_l^2) \\
&+ A(\underbrace{\alpha_l + \alpha_v}_{=1}) \frac{\partial p}{\partial z} + (p - p_i) \frac{\partial}{\partial z} A(\underbrace{\alpha_l + \alpha_v}_{=1}) \\
&= A(\alpha_l \rho_l + \alpha_v \rho_v) g_z + A(\underbrace{\Gamma_l + \Gamma_v}_{=0}) w_i - A(\underbrace{F'_{l,vm} + F'_{v,vm}}_{=0}) \\
&- A(\underbrace{F_{l,i} + F_{v,i}}_{=0}) - A(\underbrace{F_{v,w} + F_{v,w}}_{overall\ wall\ friction}) + (\underbrace{C_{w,l} + C_{w,v}}_{=1})(p_w - p_i) \frac{\partial A}{\partial z}
\end{aligned}$$

And then:

$$\begin{aligned}
&\underbrace{\frac{\partial}{\partial t} A(\alpha_l \rho_l w_l + \alpha_v \rho_v w_v)}_{\substack{mixture\ rate\ of\ change\ of\ momentum \\ per\ unit\ length}} + \underbrace{\frac{\partial}{\partial z} A(\alpha_l \rho_l w_l^2 + \alpha_l \rho_l w_l^2)}_{\substack{mixture\ advection\ of\ momentum \\ per\ unit\ length}} + \underbrace{A \frac{\partial p}{\partial z}}_{Total\ pressure\ term} \\
&= \underbrace{A(\alpha_l \rho_l + \alpha_v \rho_v) g_z}_{\substack{mixture\ body\ force \\ per\ unit\ length}} - A \underbrace{(F_{l,w} + F_{v,w})}_{mixture\ wall\ friction} \underbrace{- (p - p_i) \frac{\partial}{\partial z} A + (p_w - p_i) \frac{\partial A}{\partial z}}_{=0, if\ p_w = p}
\end{aligned}$$

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ENERGY BALANCE EQUATIONS:

$$\frac{\partial}{\partial t}A\alpha_k\rho_k\left(u_k+\frac{w_k^2}{2}\right)+\frac{\partial}{\partial z}A\alpha_k\rho_k\left(h_k+\frac{w_k^2}{2}\right)w_k$$

$$=-p_i\frac{\partial}{\partial t}(A\alpha_k)+A\alpha_k\rho_k g_z w_k+A\Gamma_k\left(h_k+\frac{w_i^2}{2}\right) \qquad (k=l,v)$$

$$+AQ_{k,i}+AQ_{k,w}+AF_{k,i}w_i+AF_{k,vm}w_i$$

(rate of change + advection = interfacial pressure term + body force term

+ mass transfer term + interfacial heat transfer + wall heat transfer

+ interfacial friction term + virtual mass term)

The additional constitutive terms involved in these equations are:

- The phasic heat transfer rate at the interface $Q_{k,i}=\left[\frac{W}{m^3}\right]$: it is generally evaluated on the basis of interfacial heat transfer coefficients, $H_{k,i}=[W/(m^3K)]$, also accounting for the interfacial area per unit volume, multiplied by the difference between the interfacial and the phasic temperatures, $T_i(=T_{sat}(p)$ in the absence of noncondensable gases) and T_k respectively:

$$Q_{k,i}=H_{k,i}(T_i-T_k)$$

- The phasic heat transfer rate at the wall $Q_{k,w}=\left[\frac{W}{m^3}\right]$: it is generally evaluated on the basis of wall heat transfer coefficients, $H_{k,w}=[W/(m^3K)]$, also accounting for the phasic heat transfer area with the wall, multiplied by the difference between the wall and the phasic temperatures, T_w and T_k respectively:

$$Q_{k,w}=H_{k,w}(T_w-T_k)$$

In this case, we will find the appropriate jump condition by adding the phasic energy balance equations and discussing the obtained equation in front of the expected form of the mixture energy balance equation. It is:

$$\frac{\partial}{\partial t}A\left[\alpha_l\rho_l\left(u_l+\frac{w_l^2}{2}\right)+\alpha_v\rho_v\left(u_v+\frac{w_v^2}{2}\right)\right]$$
$$+\frac{\partial}{\partial z}A\left[\alpha_l\rho_l\left(h_l+\frac{w_l^2}{2}\right)w_l+\alpha_v\rho_v\left(h_v+\frac{w_v^2}{2}\right)w_v\right]$$
$$=\underbrace{-p_i\frac{\partial}{\partial t}A\underbrace{(\alpha_l+\alpha_v)}_{=1}}_{=0,\ \mathit{for\ a\ rigid\ pipe}}+A(\alpha_l\rho_l g_z w_l+\alpha_v\rho_v g_z w_v)$$
$$+A\left[\Gamma_l\left(h_l+\frac{w_i^2}{2}\right)+\Gamma_v\left(h_v+\frac{w_i^2}{2}\right)\right]+A(Q_{l,i}+Q_{v,i})$$
$$+A(Q_{l,w}+Q_{v,w})+A\underbrace{(F_{l,i}+F_{v,i})}_{=0}w_i+A\underbrace{(F_{l,vm}+F_{v,vm})}_{=0}w_i$$

And then, considering that $\Gamma_l=-\Gamma_v$, it is:

$$\underbrace{\frac{\partial}{\partial t}A\left[\alpha_l\rho_l\left(u_l+\frac{w_l^2}{2}\right)+\alpha_v\rho_v\left(u_v+\frac{w_v^2}{2}\right)\right]}_{\mathit{mixture\ rate\ of\ change\ of\ energy\ per\ unit\ length}}$$
$$\underbrace{+\frac{\partial}{\partial z}A\left[\alpha_l\rho_l\left(h_l+\frac{w_l^2}{2}\right)w_l+\alpha_v\rho_v\left(h_v+\frac{w_v^2}{2}\right)w_v\right]}_{\mathit{mixture\ advection\ of\ energy\ per\ unit\ length}}$$
$$=\underbrace{Ag_z(\alpha_l\rho_l w_l+\alpha_v\rho_v w_v)}_{\substack{\mathit{power\ of\ the\ mixture\ body\ force}\\ \mathit{per\ unit\ length}}}+\underbrace{A(Q_{l,w}+Q_{v,w})}_{\substack{\mathit{heat\ transfer\ between\ the\ mixture}\\ \mathit{and\ the\ wall\ per\ unit\ length}}}$$
$$\underbrace{+A\Gamma_v(h_v-h_l)+A(Q_{l,i}+Q_{v,i})}_{\substack{=0,\ \mathit{if\ the\ mixture\ energy\ equation}\\ \mathit{must\ be\ satisfied}}}$$

Therefore, the jump condition for energy is: $\Gamma_v(h_v-h_l)+(Q_{l,i}+Q_{v,i})=0$.

This jump condition needs a bit more of consideration.

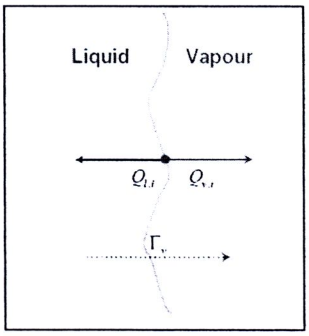

Fig. 2.3 The phasic heat transfer rate at the interface [23]

The relationship can be rewritten as:

$$\Gamma_v = -\frac{Q_{l,i} + Q_{v,i}}{h_v - h_l}$$

Showing that at the interface heat and mass transfer are intimately linked with each other

In particular:

- If the interface releases heat to the phases, i.e., the summation of the convection interfacial heat transfer rates is positive, $Q_{l,i} + Q_{v,i} > 0$, then it is $\Gamma_v < 0$, i.e., condensation take places; the heat released is equal to the latent heat released by the condensation process, $-\Gamma_v (h_v - h_l)$;

- If the interface receives heat from the phases, i.e., the summation of the convection interfacial heat transfer rates is negative, $Q_{l,i} + Q_{v,i} < 0$, then it is $\Gamma_v > 0$, i.e., boiling or evaporation takes places; the heat received by the interface is equal to the latent heat needed by the vaporization process, $\Gamma_v (h_v - h_l)$.

This view of the simultaneous heat and mass transfer processes can be applied to both "bulk" and "wall" mass transfer processes:

- Bulk mass transfer refers to homogeneous boiling or condensation, where the interface is made of bubble or droplet surfaces or even the liquid surface in stratified or annular flow conditions;

- Wall mass transfer refers to heterogeneous processes due to local non-equilibrium, as subcooled boiling or film wise condensation, where the interface is located close to the wall in a region where steep temperature gradients occur.

Apportioning the mass transfer in bulk and wall contribution is one of the tasks pursued in codes for evaluating heat and mass transfer, as the two conditions refer to different phenomena