CHAPTER V

LEAF SPRING MODELING AND VERIFICATION

The leaf spring model was created to represent its characteristics and properties. A good precise model must be able to exhibit overall phenomena obtained from real experiment. This research aims to investigate the relationship between spring parameters towards vehicle ride comfort when design parameters are selected. The objective is to build a leaf spring model that is general and reliable enough to predict ride value. The parameters of the model related to physical properties of the leaf spring model can be used for design purpose.

In the investigation, the characteristics were observed mainly on simulation. For this study, a leaf spring model was proposed, regarding the effect of nonlinearities both from contact friction and from the other assembled component at installation. The modified Dahl's model and the three link equivalent model were combined to represent a nonlinear leaf spring system. The model was then verified by a leaf spring test rig that was designed and constructed to perform deflection measurement under static loading condition. The parameters of the leaf spring model were adjusted by successive comparison of the results to the real measured data.

5.1. Leaf Spring Modeling

There are various types and shapes of leaf springs available in market although the term of leaf spring usually referred to a circular shape of flat metal. The basic common design and the one that used for studying and modeling in this research is a symmetrical semi-elliptic with equal eyes at both ends as shown in Fig.5-1. This type of leaf spring is generally used with the Hotchkiss suspensions which are commonly used with general commercial trucks.

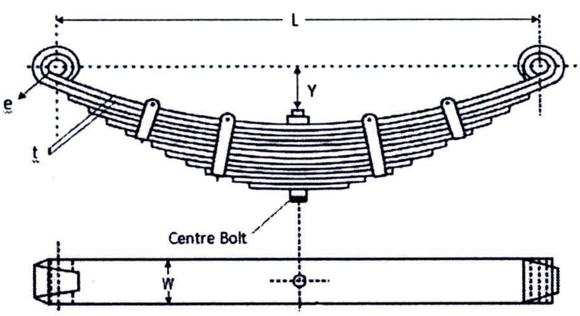

Fig.5-1 Symmetrical multi-leaf spring with design parameters

5.1.1. Existing Leaf Spring Models

There are a lot of theories and methods, both in computational and graphical ways to capture the essence of leaf spring property. The modern computer program such as ADAMS and finite element can be applied to analyze and solve nonlinear problem of leaf spring model, including optimization. One classical simplified method is the linear beam deflection theory which is subjected to small deflections and the other ones mentioned in the leaf spring manual are the center link extension method, the two-point deflection method which are not discussed in details for this paper. Both modern computer programs and classical geometrical techniques have both advantages and disadvantages, so that the users should account for any limitations and assumptions when each method is applied.

5.1.1.1. Classical Beam Theory

This is a commonly used approach to model the leaf spring as a beam and to use the simple beam theory to predict the stiffness of the spring. The basic assumption is to consider the leaf spring as a cantilever beam with variable cross-sectional area.
The Society of Automotive Engineers adopted the beam theory to calculate the equivalent stiffness of the leaf spring. Design formulae and correction factors to account for different leaf spring designs and geometries are available. A comprehensive presentation of different configurations of the leaf spring and the equations used to

calculate the stress induced in the leaf spring can be found in the Spring Design Manual [1].

5.1.1.2. Finite-Element Method

The finite-element method provides a practical approach to analyze frictional contact problems. In this approach, the Lagrange multiplier method or penalty method can be used to treat the geometrical contact. The elastoplastic constitutive laws are adopted in order to overcome the non-differentiability of the friction law.

5.1.1.3. Multibody Kinematic

The method is used to model stiff leaf springs that experience small elastic deformation is based on the finite-element floating frame of reference formulation. Using the assumption that there is no relative rigid body displacement between the leaves of the spring, the leaf spring can be modeled as one flexible body in the floating frame of reference formulation. The leaves are discretized using the finite-element method. The gross motion of the leaves is described by the displacement of the spring (body) coordinate system. The deformation of the leaves with respect to the body coordinate system is described using the finite-element nodal coordinates. The leaves of the spring at some sections can experience intermittent contacts and friction due to the relative displacements.

5.1.1.4. The Three Link Equivalent Model

This method was proposed in 1944 by Maurice Olley and other member of SAE Spring Committee [15]. The model represents the leaf spring as three bar linkages, connected to one another with torsion springs so that each member can move in angle and in plane, relative to each other. This method has been used to model and to study the characteristic of leaf spring in suspension system such as the work of Ekici [16].

In simulation, the shackle linkage is also attached to the original model to allow more degree of freedom for leaf spring displacement when subjected to the external applied force.

5.1.2. Friction Model and Hysteresis Effects

The hysteresis effect is the very important characteristic of leaf springs which arises from interleaf friction and stick-slip phenomena. In the proposed model, the modified first-order differential equation Dahl model is used to describe hysteresis relationship between applied force and spring deflections. This modified version (also known as "the Restoring force model") was proposed by Al Majid [17]. The details and its formulation can be found in Ref.[15]. For this presented work, the generalized mathematical equations of the modified model which was used to describe the stick-slip phenomena in mechanical system (for example, see Ref.[18]) are presented below,

$$\frac{dF}{dt} = \beta \frac{du}{dt}(h - F \operatorname{sgn} \frac{du}{dt})^{\mu} \quad 5\text{-}1$$

$$h = \frac{1}{2}[(h_u + h_l)\operatorname{sgn}\frac{du}{dt} + (h_u - h_l)] \quad 5\text{-}2$$

Where

$$h_u(u) = au + b \quad 5\text{-}3$$

$$h_l(u) = du + e \quad 5\text{-}4$$

In Eq.(5-1), time derivative of force is related to velocity of displacement u and the other remaining term which depends on the boundary of hysteresis curve and direction of velocity. The terms h_u and h_l represents the upper and lower boundaries of the curve, respectively. Parameters μ , β, a, b, d, and e are related to the shape and characteristic of the enveloped curve so that these parameters can be adjusted to give desirable shape of hysteresis loop, compared with the experimental results.

5.1.3. Description of the Leaf Spring Model

A model was constructed with the three link equivalent model. For the case of leaf spring studied here, a shackle linkage is also attached to allow leaf spring motion when subjected to external load.

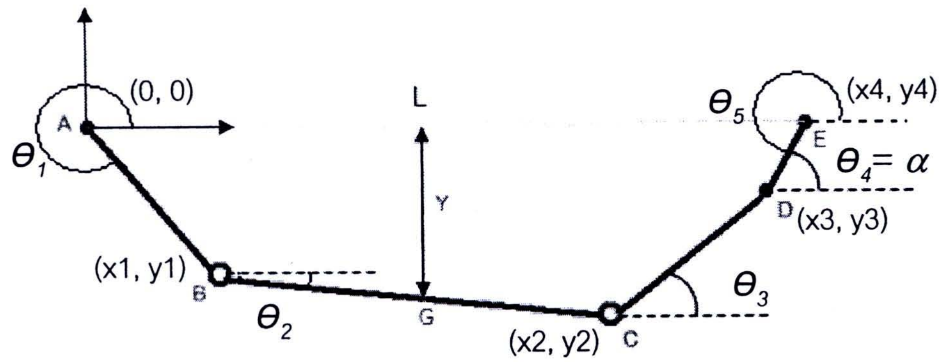

Fig.5-2 Five link mechanism of a leaf spring model

In simulation, the initial model parameters must be prescribed before running the program. The geometric parameters in the five link mechanism, AE, AB, BC, CD, DE, Y, and shackle angle (α) can be measured and calculated from the tested leaf spring at initial static equilibrium state with zero load. The relative motion between linkages was derived from leaf spring geometry as presented below,

$$AB = 0.75L \tag{5-5}$$

according to the deflection theory which are recommended by the Spring Design Manual [1] and CD is set to be equal in length with AB.

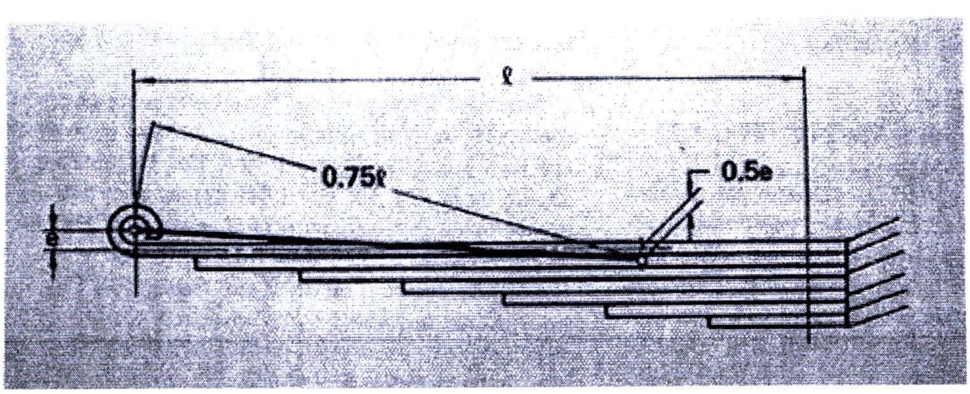

Fig.5-3 Equivalent linkage of the upturned eye cantilever spring [1]

The length of AE, DE, and Y, were measured from a real leaf spring at installation which are 119, 10 and 15 cm, respectively.

From Fig.5-2, the motion of each link is related to the other adjacent member. If a point on one link is considered, it moves in X-Y plane both in vertical and horizontal direction. The two adjacent links are moving in angle with each other to produce angular displacement θ. The coordinates (x0, y0), (x1, y1), (x2, y2), (x3, y3), and (x4, y4) represent the position of the joints, referred to the reference frame. The initial parameters θ_1 θ_5 and θ_4 are set to be 340, 180, and 70 degree, respectively.

$$y1 = AB\sin\theta_1 \qquad \text{5-6(a)}$$

$$x1 = AB\cos\theta_1 \qquad \text{5-6(b)}$$

$$y2 = 2y - y1 \qquad \text{5-6(c)}$$

$$x3 = AE - DE\cos\theta_4 \qquad \text{5-6(d)}$$

$$y3 = -DE\sin\theta_4 \qquad \text{5-6(e)}$$

$$\theta_3 = \sin^{-1}[(y3 - y2)/CD] \qquad \text{5-6(f)}$$

$$x2 = x3 - CD\cos\theta_3 \qquad \text{5-6(g)}$$

$$\theta_2 = \tan^{-1}[(y2 - y1)\ /\ (x2 - x1)] \qquad \text{5-6(h)}$$

$$BC = (x2 - x1)/\cos\theta_2 \qquad \text{5-6(i)}$$

5.1.4. The Full Leaf Spring Model Construction

Full leaf spring system was accomplished by a combination of a five link mechanism model and Dahl's hysteresis model. As illustrated in Fig.5-2. Hysteresis effect was added to the joints between linkages (points B,C). The completed model is shown in Fig.5-4 and the sub system of Dahl's model at two joints is shown in Fig.5-5

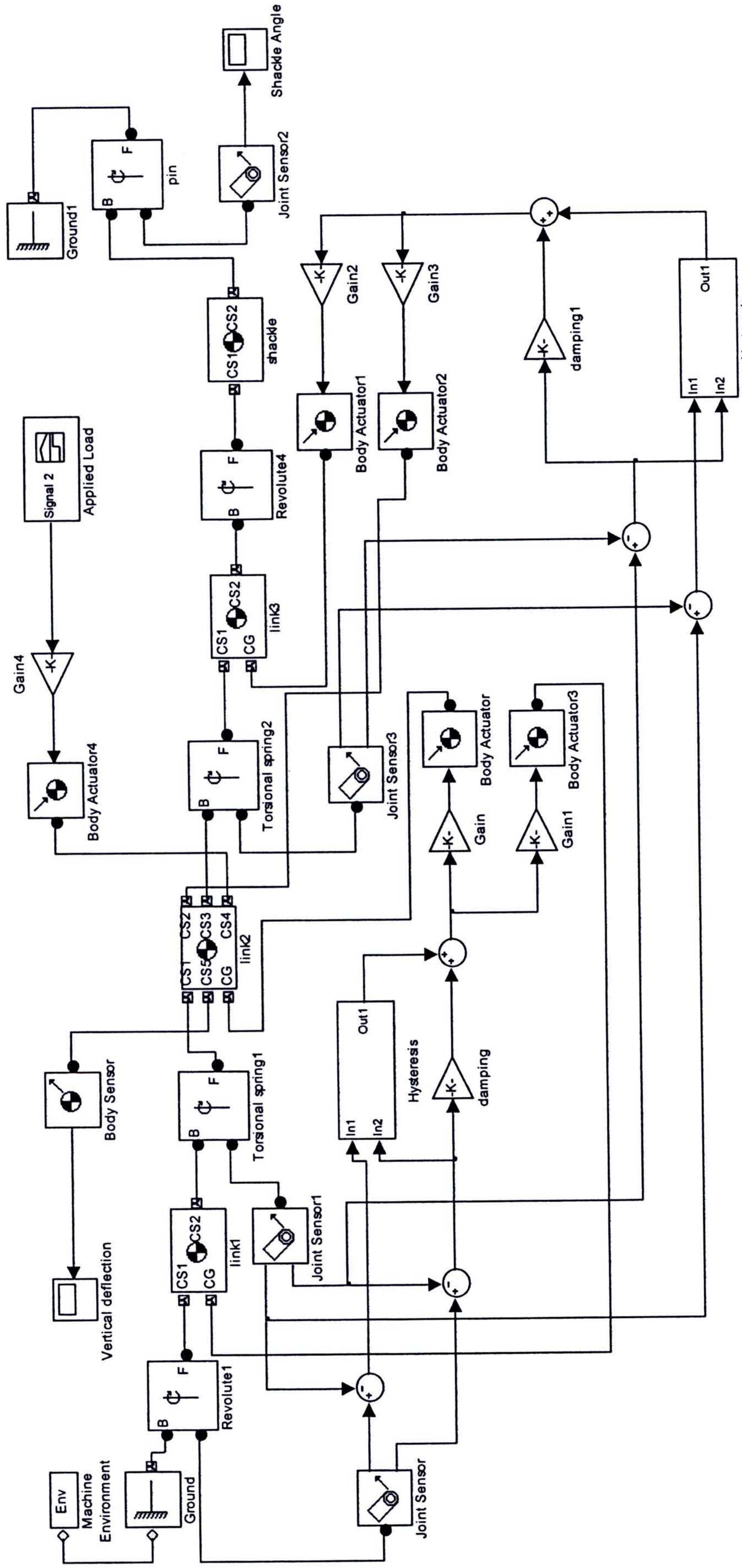

Fig.5-4 Full leaf spring model, regarding hysteresis effects

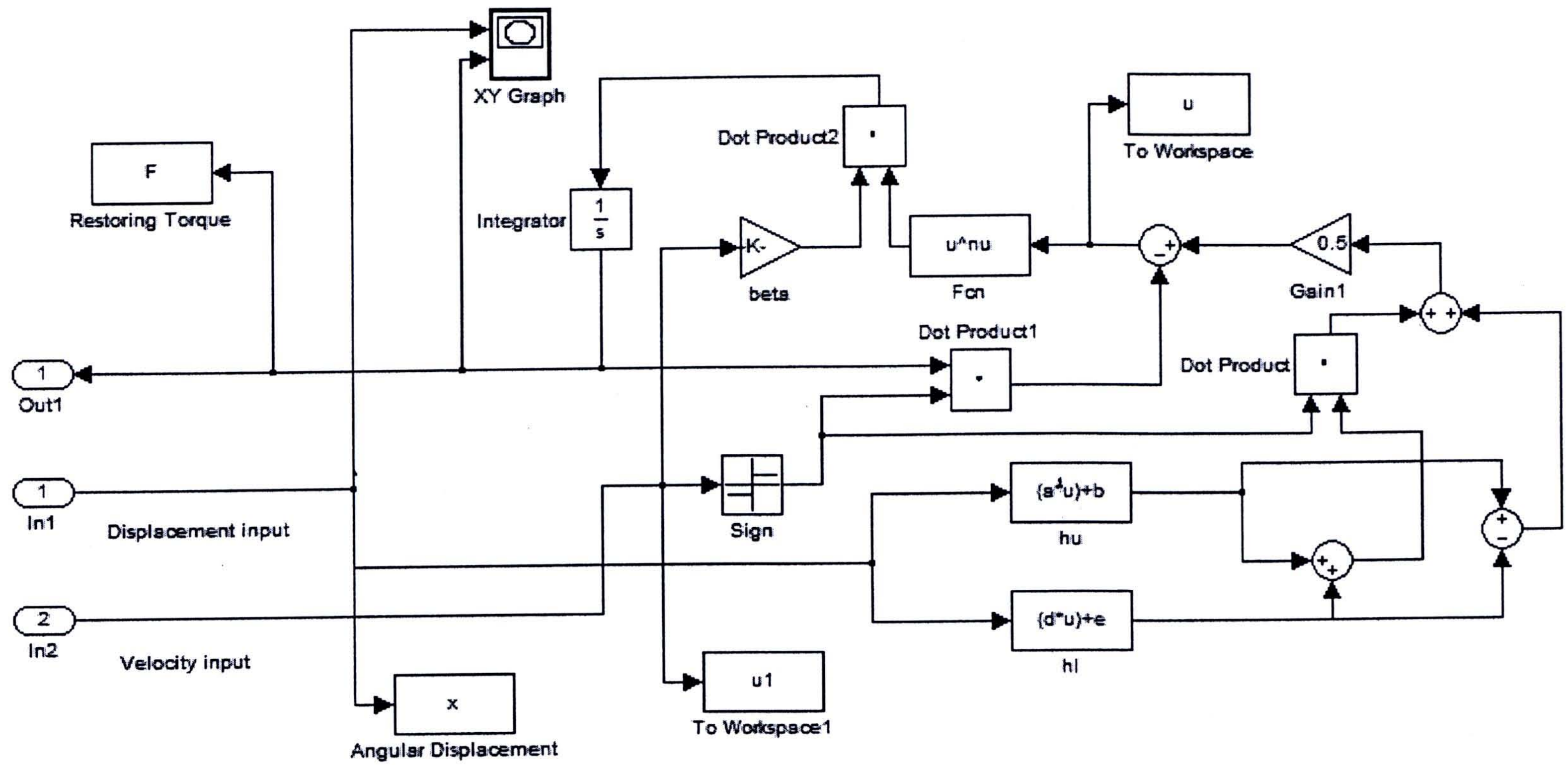

Fig.5-5 Hysteresis component of a leaf spring model

5.2. Verification of the Leaf Spring Model

In order to verify the leaf spring model, some experiments were carried out to compare the results with those of the simulation's. In this work, a leaf spring test rig was designed and constructed by a group of senior students at Chulalongkorn University.

5.2.1. Design and Construction of the Leaf Spring Test Rig

The leaf spring test rig is composed of three main components which are the frame, the load generation and measurement unit, and the deflection measurement unit. Details for each component and their functions is given individually as follows,

5.2.1.1. Test Rig Frame

The test rig frame was designed to withstand the maximum applied load at least 1,000 Kg force as shown in Fig.5-6.

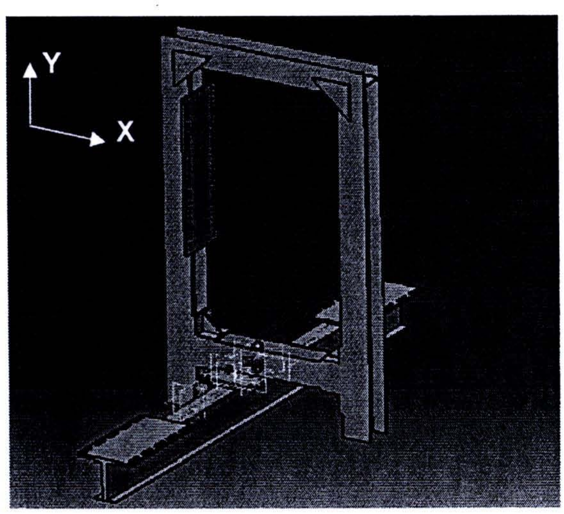

Fig.5-6 Final design of the test rig frame

This main structure can fit with various sizes of leaf springs and provides suitable space for additional attachments. In the test, leaf spring was installed downturned with one end fixed and the other end shackled so that the applied load from an actuator is in -Y direction.

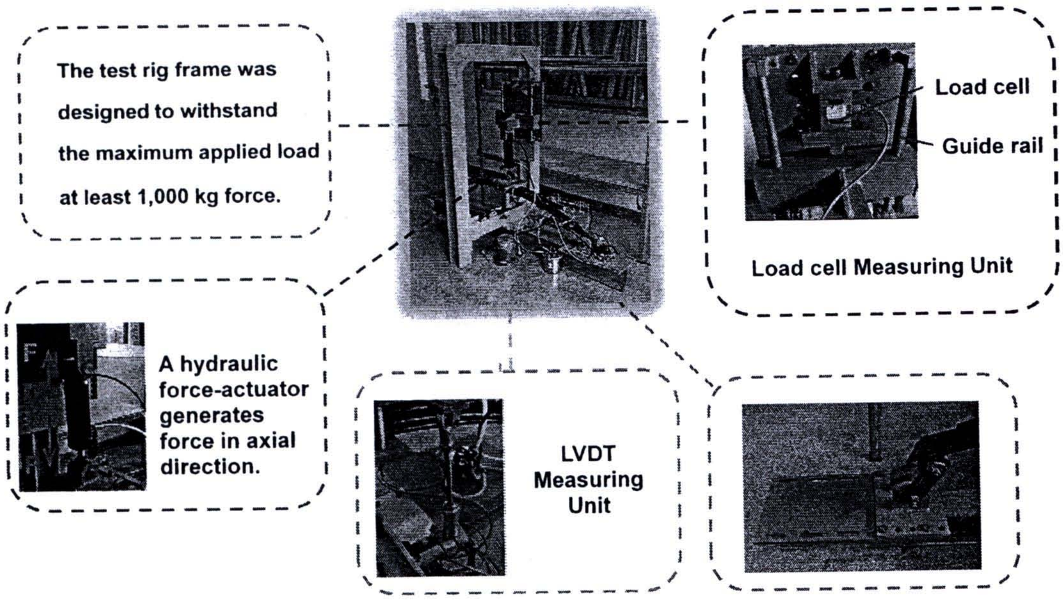

Fig.5-7 Configuration of the leaf spring test rig

5.2.1.2. Load Measuring Unit

In the experiment, the applied force was increased in step by a hydraulic actuator and was measured by a load cell. The load cell was placed on top of a guided slider to ensure that only axial load is applied to it.

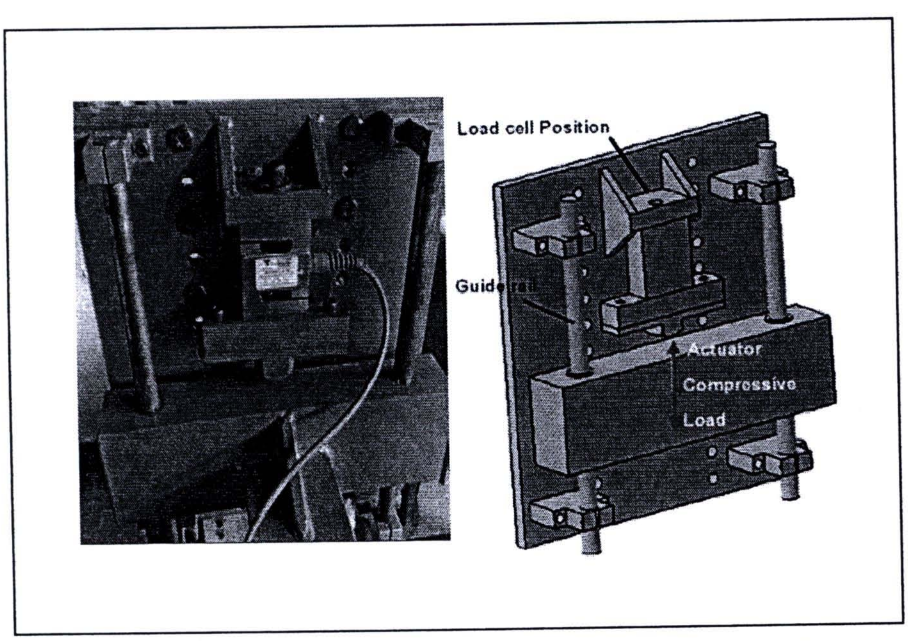

Fig.5-8 The guide base for load cell and force-actuator attachment

5.2.1.3. Deflection Measuring Unit

In the experiment, only leaf spring displacement in vertical direction was considered, hence an LVDT sensor was attached perpendicular to leaf spring with one side of LVDT connected to leaf spring's lock and the other side attached to the base of the test rig frame. For this design, the housing of LVDT can be fitted with different lengths of leaf spring and various types of sensor.

Fig.5-9 The LVDT measuring unit

5.2.2. Verification Testing

Fig.5-10 shows the two main components of the proposed leaf spring model which are the three link equivalent model that describes the mechanism in term of geometrical relation and the hysteresis Dahl's model.

When external force is applied in vertical direction, leaf spring starts to move from its original position. Each linkage moves in angle/Θ (rad) with each other at the revolute joint so that the angular displacement is produced. At each joint, the input angular displacement generated the output restoring torque by hysteresis relationship as introduced in Eqs.(5-1) - (5-4). This restoring torque represents the resistance in spring which resists movement from the initial state. The vertical displacements of leaf spring at CG point in Y-axis at steady state position were recorded for every increasing step of force. The relationship between the applied force (kg) and the corresponding displacement (cm) could be found.

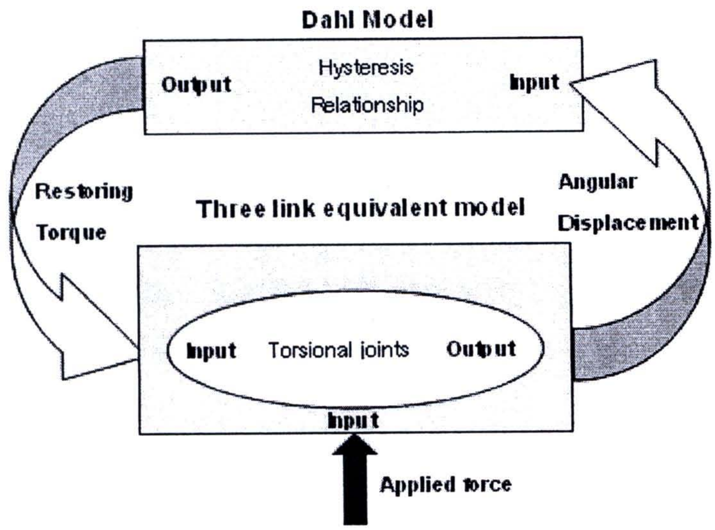

Fig.5-10 Five link mechanism of a leaf spring model

For solving the first order differential equations and the leaf spring movement, the model was implemented in SimMechanics, MATLAB software. The Force-displacement relation can be simulated and animation can be displayed graphically as shown in Fig.5-11. As the force was applied, the corresponding deflections started to form the hysteresis loop, according to different directions of loading.

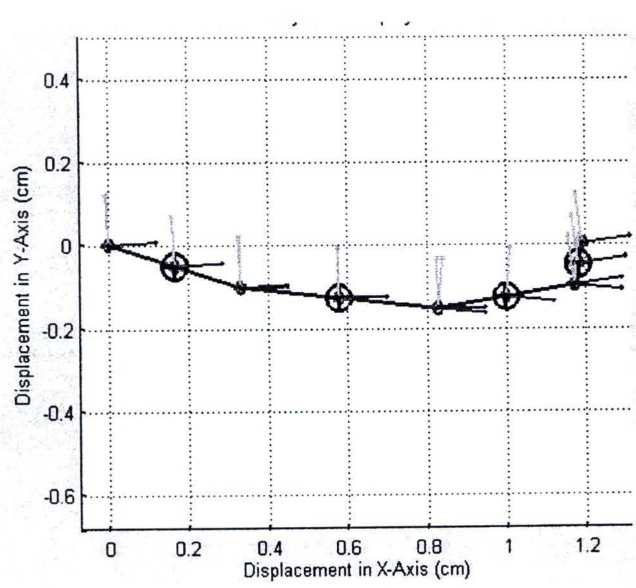

Fig.5-11 The animated results of simulated model

The experiment was carried out to measure the leaf spring vertical deflections when series of loads were applied. The spring was loaded with the load interval of 20 kg from zero to the prescribed maximum deflection and back to zero. The amount of force was initially generated from a hydraulic pump which was controlled via human-interfaced control panel by LabVIEW software. All values of the applied force were received via a load cell while the deflections of leaf spring were captured by an LVDT sensor. The results show a non-linear relationship between the applied loads and the leaf spring deflection for both directions of loading in form of a hysteresis loop shown in Fig.5-12.

	Load (kg)	Deflection (cm)
Load	29.9310	0.0034
	63.4771	0.5855
	86.7014	1.3098
	117.6670	2.1493
	149.9230	3.2116
	186.6950	4.8539
Unload	176.2410	4.7701
	115.3845	3.7293
	72.1125	2.6469
	54.6821	1.9905
	21.4910	0.7700
	-1.5429	0

Table 5-1 Experimental result of leaf spring deflection subjected to vertical applied load.

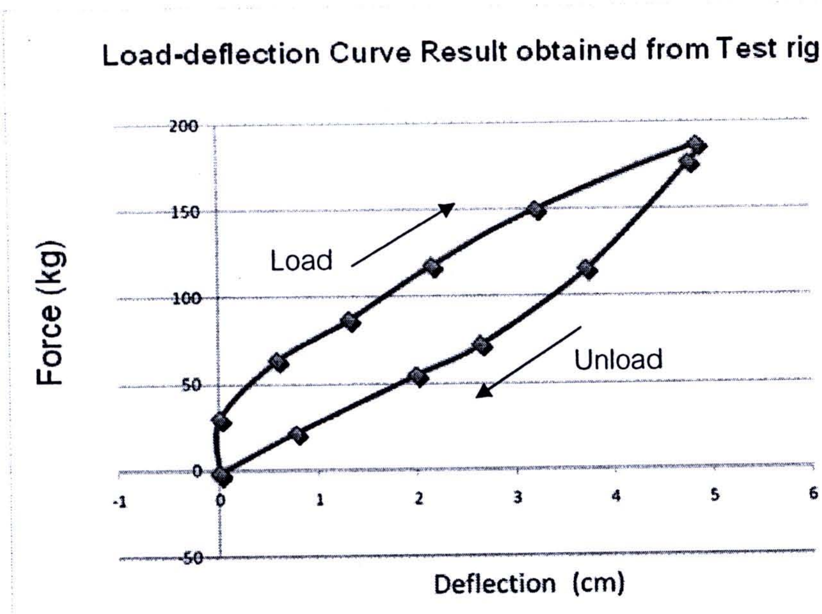

Fig.5-12 The load-deflection curve result obtained from test rig experiment.

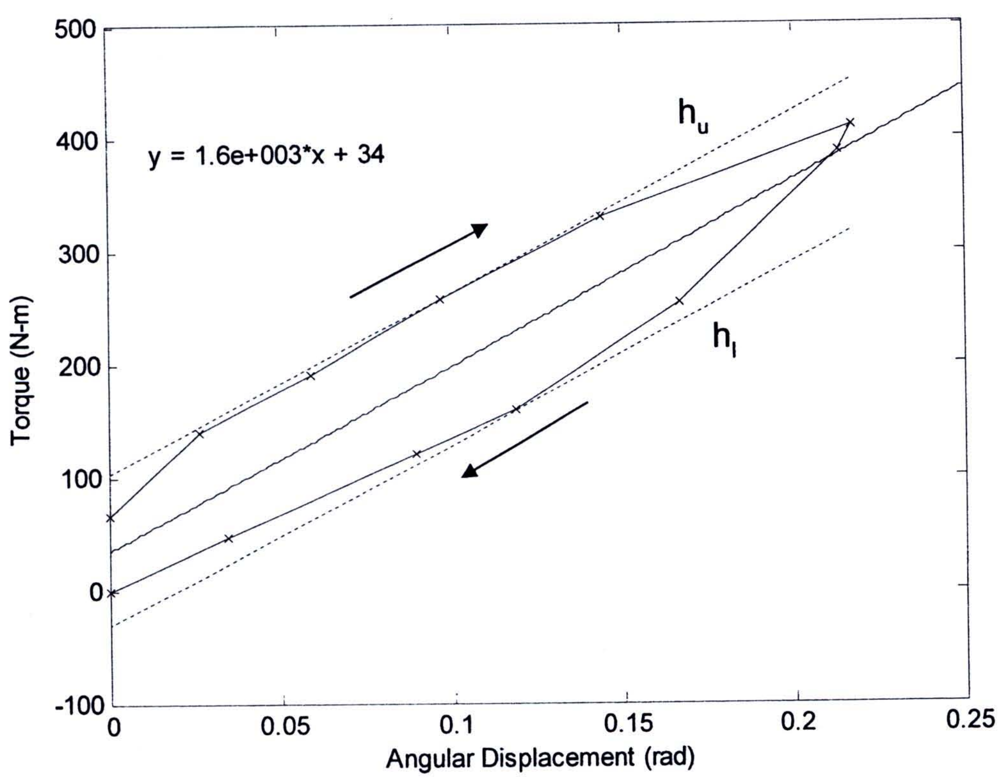

Fig.5-13 The load-deflection curve of torque and angular displacement

5.2.3. Parameters Identification

Parameters identification was made by comparison of the simulation and experimental result. The average regression line or nominal stiffness could be examined from the load-deflection curve obtained from the experiment. (a linear solid line in Fig.5-13). The slope of the line should reflect the average stiffness or value of *a* and *d* in Eqs.(5-3) and (5-4) which are the slope of the two linear curves, i.e., the upper boundary and the lower boundary, respectively.

A linear leaf spring model without friction/hysteresis component was use to calibrate a slope of a nominal stiffness (a linear solid line in Fig.5-13) which is located parallel to both upper and lower envelopes curves, h_u and h_l. Once the approximated nominal stiffness is known, the values of *a* and *d* are found which are the slope of the other two curves. These parameters were used in the hysteresis/Dahl's model. In this case, *a* and *b* are assumed to be equal so that the remaining unknown parameters are β, μ, *b*, and *e*. These parameters were adjusted until the best fit between the loop shapes was satisfied.

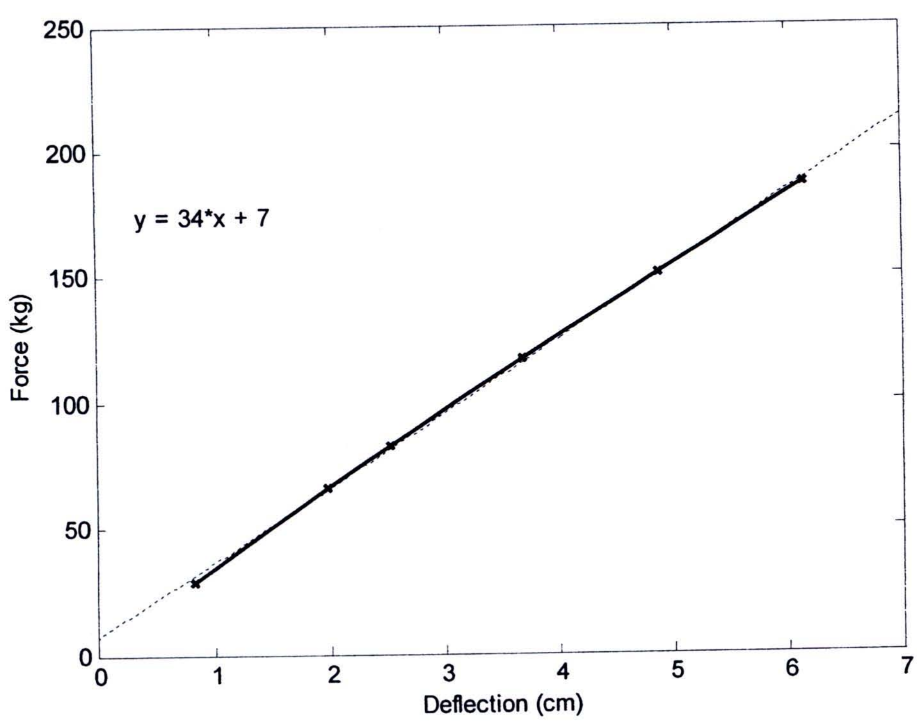

Fig.5-14 Load-deflection curve result of linear leaf spring model

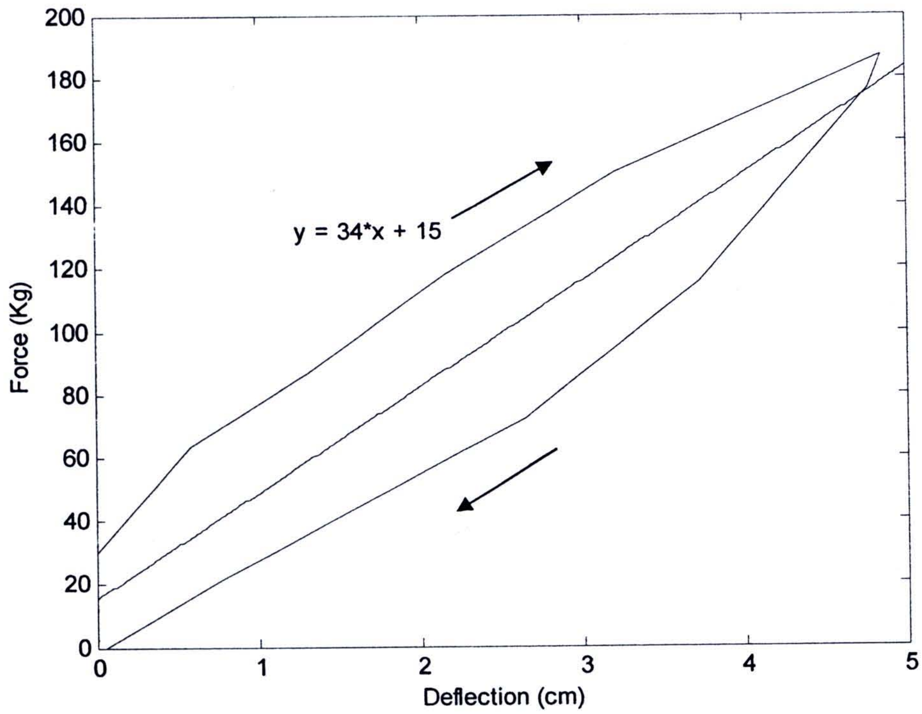

Fig.5-15 Load-deflection curve of force (kg) and displacement (cm)

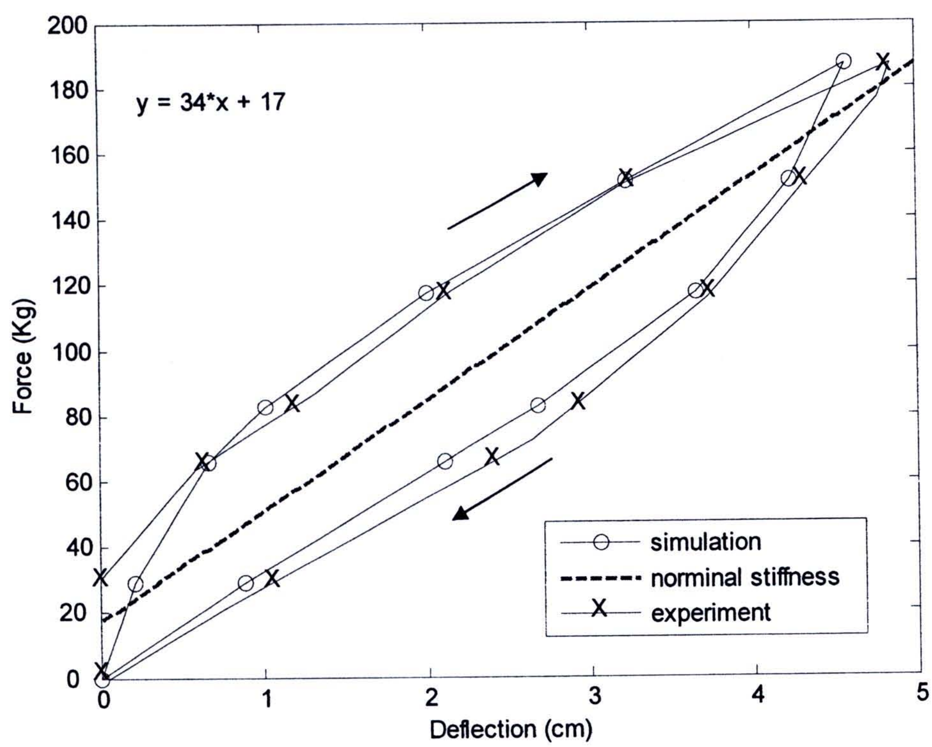

Fig.5-16 Comparison between load-deflection curve results from experiment and simulation

The final values obtained are below.

$a = 3.3354 \times 10^4$ N/m, $b = 4.9907 \times 10^2$ N,

$d = 3.3354 \times 10^4$ N/m, $e = -2.0653 \times 10^2$ N,

$\beta = 100$, and $\mu = 1$

The shapes of curves are not exactly the same, but the model can establish most of the characteristics in steady state. Major difference appears at both ends of the loops as the model does not capture well in the transient state transition, however the result shows some agreements between the test rig experiment and the simulating model which is able to capture the overall characteristics of leaf spring. Further analysis should be carefully made again for these areas. However, the recent work was lack of sufficient data in the transition state which might be due to the limitation and ability of the leaf spring test rig that can allow for static testing only. Apart from this, some obstacles still exist to limit the performance of the test rig such as misalignment between the hydraulic actuator and the force sensor must be eliminated in order to provide better accuracy. The next step of this research is to identify parameters that can be used in leaf spring design and to find relationship between those parameters and vehicle ride comfort.