

ห้องสมุดงานวิจัย สำนักงานคณะกรรมการการวิจัยแห่งชาติ



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OPISTHORCHIS VIVERRINI INFECTIONS IN HAMSTER AND
GERBIL: MORPHOLOGY, BIOLOGY AND PATHOLOGY

MISS ORASA WONKCHALEE

A THESIS FOR THE DEGREE OF MASTER OF SCIENCE
KHON KAEN UNIVERSITY

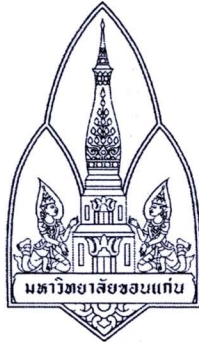
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**AN ACCURACY COMPARISON OF SOLUTIONS BETWEEN BOUNDARY
ELEMENT METHOD AND MESHLESS METHOD
FOR LAPLACE EQUATION**



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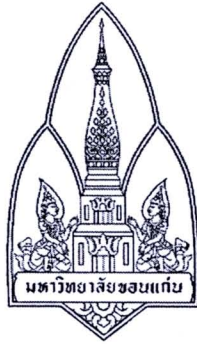
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**AN ACCURACY COMPARISON OF SOLUTIONS BETWEEN BOUNDARY
ELEMENT METHOD AND MESHLESS METHOD
FOR LAPLACE EQUATION**

MISS NISSAYA CHUATHONG

**A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE
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วัตถุประสงค์หลักของการศึกษาค้นคว้าครั้งนี้เพื่อศึกษาและเปรียบเทียบความสามารถของวิธีเชิงตัวเลขสองวิธีคือ
วิธีสมาชิกตามขอบและวิธีเมฆเลส ซึ่งนำมาใช้แก้ปัญหามสมการลาปลาซในสองมิติ โดยมีรูปแบบ โดเมนและ
เงื่อนไขที่แตกต่างกันและได้นำโปรแกรมแมทแลบมาช่วยในการคำนวณ เมื่อนำผลลัพธ์ที่ได้จากวิธีเชิงตัวเลขทั้ง
สองมาตรวจสอบกับผลลัพธ์เชิงวิเคราะห์พบว่าผลลัพธ์ที่ได้นี้ยังคงอยู่ในเกณฑ์ดี อย่างไรก็ตามเป็นที่น่าสนใจที่
จะเห็นว่าการเปรียบเทียบผลเชิงตัวเลขกับผลเฉลยเชิงวิเคราะห์สำหรับโดเมนและเงื่อนไขที่แตกต่าง วิธีเมฆเลสจะ
มีความเหนือกว่าวิธีสมาชิกตามขอบเล็กน้อยในแง่ของความแม่นยำของผลเฉลย ซึ่งแสดงให้เห็นศักยภาพในเชิง
บวกว่าเมื่อมีการแก้ปัญหามีความซับซ้อนมากขึ้น วิธีเมฆเลสจะสามารถจัดการกับปัญหาเหล่านี้ได้ดีกว่าวิธี
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Thesis Advisor: Dr. Wattana Toutip

ABSTRACT

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The main purpose of this study is to investigate and compare the capability of two different numerical techniques; the boundary element method and the meshless method, in solving problems described by the Laplace equation in two dimensions. Domains with boundary conditions were chosen for the task and the MATLAB computer program was utilized for the calculation process. The results obtained from both numerical techniques were validated against the exact ones and are found to remain in a satisfactory agreement. Nevertheless, it is interesting to see that comparing the numerical results with the analytical solution for all domains with boundary conditions, the meshless method has revealed a slight but noticeable superior aspect to the boundary element method in terms of the accuracy. This indicates its positive potential when dealing with more complex problems and is well worth further investigation.

**Goodness Portion to the Present Thesis is Dedicated
for my Parents and Entire Teaching Staff**

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