CHAPTER I

INTRODUCTION

Partial differential equations (PDEs) are important in the mathematics and many fields concerning with PDEs such as science and engineering. Some PDEs with geometric shapes are not difficult to solve but there are some complicated ones with complex shapes, are difficult or even unobtainable to find the analytical solution. Therefore, approximation of the solutions has been important and helped in finding a solution, called numerical methods, although the methods were not interested due to their capabilities and also limitations of the calculation engine in the past.

In recent decades, computers have been developed rapidly. At present, numerical methods are no less important than analytical methods. The main numerical methods used for finding the numerical solutions of PDEs in science and engineering are finite difference method (FDM), finite element method (FEM) and boundary element method (BEM). FDM is easy to understand and has been popular for a long time. It can be applied to any regular shapes. FEM, however, is better than FDM in terms of capabilities. Although, it is more difficult to understand but its advantages are able to solve the problems of complex geometry domains. To deal with the problem, nevertheless, it requires elements within the domain which the generation more difficult and time consuming for higher dimension. In order to avoid the element generation process within domain, BEM is used because it requires only elements on the boundary domain to deal with the problem. This method reduces the unknown variables making it easy to solve. But it still requires elements as FDM and FEM. Recently, alternative methods that do not need element generation have gained increasing interest. These methods are known collectively as meshless methods or meshfree methods. It should be noted, however, that meshless methods do not required a mesh, but they still require a specification of node distribution within the domain.

1.1 Boundary element method (BEM)

BEM is attractive and important computational technique for solving problems in applied science and engineering. BEM is used to reduce the dimension of the problems being solved. The boundary of the domain will be discretized into boundary elements. The main process is to construct a system of algebraic equations to obtain the value of functions and normal derivatives at the boundary points and then use these values to compute function values at internal points in the domain. One of the most interesting features of the BEM is the much smaller system of equations and the considerable reduction of the data required to run a program. Moreover, the numerical accuracy of the method is generally greater than that of the FEM [11].

1.2 Motivation of boundary element method

During the last few decades, the BEM was known as boundary integral equation method (BIEM) or boundary integral method. It was used to solve the problems of mathematical physics and it had been popular during the period from about 1960 through 1975 [15]. The BIEM was used with the work of Jaswon [40], Symm [41], and Cruise and Rizzo [39]. The BEM was first used by Brebbia [10] at the second conference at Southampton, UK [15]. At present, BEM method is well known in engineering and science and it is developed very rapidly due to the capability of computer.

Toutip [8] described how to solve the linear and nonlinear problems in two dimensions by using linear boundary element method. Kaennakham [37] described how to solve Berger's equation in two dimensions by using linear boundary element method and MATLAB was utilized for computing. Chanthawara [38] described how to solve Biharmonic equation on irregular domains by using dual reciprocity boundary element method with MATLAB. Lucha [18] described how to find Laplace equation by using quadratic boundary element method with MATLAB.

Using a scattered set of points instead of using elements, the complex mesh generation process can be alleviated.

1.3 Meshless method

Mesh generation of a complicated geometry is always time consuming in dealing with problems by employing numerical methods, e.g., FDM, FEM and BEM. These methods cannot handle such problems at all, or are limited to very special (regular) situations. In the last decade, researchers have paid attention to the meshless methods without employing the concept of elements. Meshless methods are often better suited to cope with changes in the geometry of the domain of interest than classical discretization techniques such as FDM, FEM or BEM [12]. Another obvious advantage of meshless discretizations is their independence from a mesh, using a set of nodes scattered within the problem domain as well as a set of nodes scattered on the boundaries of the domain to represent (not discretize) the problem domain and its boundaries [17]. These sets of scattered nodes do not form a mesh, which means that no information on the relationship between the nodes is required, at least for field variable interpolation.

Mesh generation is still the most time consuming part of any mesh-based numerical simulation. Since meshless discretization techniques are based on only a set of nodes scattered, these costs of mesh generation are eliminated [12, 17].

1.4 Motivation of meshless method

During the last two decades, meshless methods have been developed and effectively applied to solve many problems in science and engineering [12, 14, 19, 20]. Several meshless methods have also been reported in the literature, for example, the domain-based methods including the element-free Galerkin method [1], the reproducing kernel method [2], and boundary-based methods including the boundary node method [3], the meshless local Petrov-Galerkin approach [4], the local boundary integral equation method [6] and the radial basis function (RBF) approach [7], which is focused for this thesis.

Meshless methods based on radial basis function, which depends only on the distance between one point to any other points, and this distance is Euclidean norm. In [23], Kansa was the first to demonstrate methods that utilized RBF in deal with multivariate data for scattered data interpolation [22] and PDEs solution finding. Kansa introduced an unsymmetrical form of an RBF method in an attempt to improve accuracy and conditioning. This is also known as the RBF collocation method [25]. Thus, Kansa's RBF-based methods have become very popular in the field of science and engineering. Li [26] demonstrated the comparison between RBF method and the finite element method in terms of accuracy and efficiency, and concluded that the accuracy of the RBF method was more superior than FEM. Sharan [27] described using the popular multiquadric RBF to solve elliptic PDEs. In a similar work, Sarler [28] formulated a solution method for diffusion problems based on RBF. In a more general work, Wendlend [25] integrated the theory of Galerkin methods with radial basis functions. More recently, Divo and Kassa [26, 27] described using RBF to model convective viscous flows and heat transfer problems. Chinchapatnam [28] computed incompressible viscous flows by using a localized RBF method. Radial basis methods for compressible flows are much less common and Shu [29] has recently proposed. Fasshauer [13] described a method related to scattered Hermite interpolation for which the solution of elliptic partial differential equations by collocation method and compared the method of [23] with their method. Besides, mehless methods theory is still compared with that of FEM and FDM. However, development of meshless methods theory has increased and it is investigated to obtain the advantage of these procedures in solving the problems.

1.5 Objectives of this study

The purpose of this thesis is to investigate a capability of two numerical techniques; BEM and meshless method. Both methods are described in solving two dimensions Laplace equation. Results of two methods are compared with analytical solution in terms of accuracy.

The outline of this thesis is as follows.

Chapter II introduces some background knowledge of numerical simulation. The basic ideas of BEM and meshless method are briefed.

Chapter III presents formulation of BEM which is the way to construct the algorithm matrix systems.

Chapter IV describes the formation of meshless method. Concepts of Kansa's method and Hermite-based method are presented.

Chapter V investigates the accuracy of BEM and meshless method. Results of both methods are compared with analytical solution in a form of tables and graphs.

Chapter VI concludes the whole thesis and suggests significant points for further investigation.