

# Chapter 1

## Executive Summary

The most important and powerful tools guaranteeing the existence of a solution of a given nonlinear system are Banach's contraction principle and Schauder's fixed point theorem. The application of the first theorem requires only the completeness of a space but a rather heavy assumption on a mapping, i.e., a contraction. For the latter one, it requires a weak assumption on a mapping, i.e., a continuous one, while its domain is compact and convex. In recent years, many mathematicians have investigated several new fixed point theorems requiring only that the mapping and its domain be weaker than the contraction and the compactness, respectively.

Let  $E$  be a real Banach space with the dual  $E^*$ . The duality mapping  $J : E \rightarrow 2^{E^*}$  is defined by

$$Jx := \{x^* \in E^* : \langle x, x^* \rangle = \|x\|^2 = \|x^*\|^2\} \quad (x \in E).$$

In this report, we study various types of mappings in Banach spaces. Let  $C$  be a nonempty subset of a Banach space  $E$ . A mapping  $T : C \rightarrow C$  is said to be

- (1) *nonexpansive* if  $\|Tx - Ty\| \leq \|x - y\|$  for all  $x, y \in C$ ;
- (2) *Lipschitzian* if there exists a real number  $L > 0$  such that

$$\|Tx - Ty\| \leq L\|x - y\|$$

for all  $x, y \in C$ ;

- (3) *strictly pseudo-contractive (in the terminology of Browder and Petryshyn)* if there exists a real number  $\lambda > 0$  such that for all  $x, y \in C$  there is  $j \in J(x - y)$  such that

$$\langle Tx - Ty, j \rangle \leq \|x - y\|^2 - \lambda \|x - Tx - (y - Ty)\|^2.$$

It is clear that  $T : C \rightarrow C$  is strictly pseudo-contractive if and only if there exists a real number  $\lambda > 0$  such that for all  $x, y \in C$  there is  $j \in J(x - y)$  such that

$$\langle x - Tx - (y - Ty), j \rangle \geq \lambda \|x - Tx - (y - Ty)\|^2.$$

We are mainly interested in the problem of finding a common fixed point of a family of mappings above so we always assume that there exists at least one common element in the set of fixed points of all mappings in the family. The purpose of this research is to introduce and study a new iteration scheme for approximating such a common fixed point. Moreover, we prove several weak and strong convergence theorems of such schemes.