Chapter 6

Conclusion

This thesis has presented new results on $PS(q)$; the Baer-Levi semigroups of partial transformations. We have described the existence of the meet and join, the existence of maximal and minimal elements, and the existence of compatible elements with respect to the partial orders $\leq$, $\subseteq$ and Ω on $PS(q)$. We have shown that, although $R(q)$ is a subsemigroup of $PS(q)$ which is also a subsemigroup of $I(X)$, there are surprising differences between the partial orders $\leq$, $\subseteq$ and Ω on these semigroups, concerned with their compatibility. We have also described the existence of the meet and join, and the existence of maximal and minimal elements in $PS(q)$ with respect to each of $\leq$, $\subseteq$ and Ω. Moreover, we have described the group of automorphisms of $R(q)$ and gave the necessary and sufficient conditions when $R(X, p, q)$ and $R(Y, r, s)$ are isomorphic, that is, $p = r$ and $q = s$ which are the same conditions when $PS(X, p, q)$ is isomorphic to $PS(Y, r, s)$.

In the last section of this thesis, we described maximal subsemigroups of $PS(q)$, we got some ideas from [9] which presented a first class of maximal subsemigroup of $BL(q)$, the Baer-Levi semigroups. Since $BL(q)$ is a subsemigroup of $PS(q)$, we applied some arguments in [9] and obtained some maximal subsemigroups of $PS(q)$ for $|X| = q$. In the case that $|X| > q$, we also showed that almost every maximal subsemigroup of $PS(q)$, except one, is induced by a maximal subsemigroup of its subsemigroup S_r for some cardinal r such that $q \leq r < p$. That is, we have given a guideline for determining all maximal subsemigroups of $PS(q)$ for $|X| > q$. In addition, we also determined some maximal subsemigroups of S_r.

However, to determine all maximal subsemigroups of $PS(q)$ for $|X| > q$, it is necessary to describe all maximal subsemigroups of S_r, and we leave this to be an open problem.