



## A Dynamic General Equilibrium Model of Food and Energy Crop\*

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This essay develops a dynamic general equilibrium model of an energy crop. The model can guide policy to resolve the 'food or fuel' dilemma. Data from Thailand were employed in calibrating the model, with cassava as the energy crop. The stationary state solution gave the set of optimal consumption, production, and allocation of resources in the economy. An approximation of optimal policy function and optimal time paths was derived by using the linear approximation method and the Runke-Kutta reverse shooting method. The results of the model provide the basic information for decision makers in optimal allocation of resources for the production of crops for food and biofuel.

*Keywords:* energy crop, optimal control, sustainable development

*JEL Classification:* C61, Q40

### Introduction

The emergence of ethanol production for biofuel has created the 'food or fuel' dilemma. This is because most biofuels, particularly ethanol, are produced from a variety of food crops, called 'energy crops', such as corn, sugar cane, cassava, sweet sorghum and others. In addition, energy crops use the same land and resources as those used for food crops. This affects availability and allocation of resources for food and biofuel production.

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The global production of ethanol has increased more than fourfold between 2000 and 2008; the United States currently produces more than 50% of global output and Brazil about 37% (Renewable Fuels Association, 2010). A significant increase in ethanol production implies a significant increase in the use of land and other resources and/or competition over the same resources with food production. Moreover the use of food crops to supply bio-energy has been directly linked to the increase in food prices. These two issues of competition over resources and rise in food prices can become more serious as countries continue to pursue a policy of energy security using biofuel along with food security.

The allocation of a country's resources to produce food or biofuel needs to be carefully planned in order to strike a balance between energy and food security. The purpose of this paper is to provide a basis for the allocation of a country's resources over a period of time for the production of an energy crop which does not jeopardize the achievement of the country's sustainable development goal. A dynamic general equilibrium model is developed to describe the consumption, production and allocation of resources in Thai economy. The model is analyzed in a continuous time optimal control framework. It is calibrated to the Thai economy with cassava as the energy crop. Thailand became sixth in the world in ethanol production only four years after starting domestic production (Licht, 2008). As it is a relatively low-income small country that has its own share of problems inflicted by high food prices, it presents an interesting case for this kind of study. The government has placed biofuel production high on the national agenda, which has made biofuel production from food crops an issue of great interest among stakeholders.

The presentation in next section describes the concepts and model which comprise the optimal control methodology and the compositions and functions of the model. The data section explains the set of value used to estimate parameters in calibration and simulation. The results provide the set of stationary state solution from the calibration and the optimal time paths. The final section concludes with comments and suggestion for extension of the model in future research.

## Concepts and Model

This study applies the dynamic optimal control as the main methodology. The method formulates the general equilibrium of energy crop production and international trade. A number of studies on energy crop production applied dynamic optimization. For instance, Chakravorty *et al.* (2008) studied the allocation of land to produce ethanol from corn to meet the clean air standards. The study applied the extended Hotelling model to consider clean fuel as a substitute to fossil fuel. The utility function of an economy at any given time of their study is an additive utility function of food and energy. Chen *et al.* (2009) evaluated the economic potential of biofuel in the dynamic land use model. They assumed imperfect substitution between ethanol and gasoline. The utility of economy is the sum of utility from miles driven and food consumed. The model maximizes the choice between fuel and food production by constraining both the constant elasticity of substitution (CES) production function and land use.

In this study, the dynamic optimization model differs from previous studies. The model applies dynamic general equilibrium optimal control that maximizes utility (composed of the composite commodity and food). The primary factors of production are capital, labor, land and energy. The production functions are specified as Cobb-Douglas functions to ensure a strictly convex utility curve. The model assumes that ethanol and fossil energy are perfect substitutes.

A dynamic general equilibrium optimal control model of cassava is constructed following the optimal control theory. The objective of the model is to determine the values of all relevant variables of the model over a continuous time period; it reflects the concern of decision makers, which is to maximize the sum of the discounted utility function of society under various constraints over an infinite time horizon. The arguments of the utility function are domestic consumption of food and of the composite commodity. In this study, 'food' is defined as food produced from cassava. The continuous time utility function is given by equation (1). The objective function is:

$$\text{Maximize} \quad W = \int_0^{\infty} e^{-rt} u(y_t^d, f_t^d) dt \quad (1)$$

For the continuous time model, we consider an infinite time horizon from time period 0 to  $\infty$  and the sum of utility assuming the form of an integral. The objective function

is to maximize utility of an economy ( $W$ ) which is the summation of the discounted value of utility from time periods 0 to  $\infty$ , where  $u(y_t^d, f_t^d)$  is the society's utility function at time  $t$  of domestic composite commodity consumption ( $y_t^d$ ) and domestic food consumption ( $f_t^d$ ). We assume  $u(y_t^d, f_t^d)$  is increasing and concave in consumption of  $y_t^d$  and  $f_t^d$ . The term  $e^{-rt}$  is a discount factor where  $r$  is the real interest rate and assumed to be constant along the time horizon. The utilized energy and energy produced from cassava are assumed to be perfect substitutes and are called 'energy'. The composite commodity is composed of all other commodities not specifically modeled and is the numeraire commodity.

The constraints for the problem are:

$$\frac{dk_t}{dt} = I_t - \delta k_t \quad (2)$$

$$y_t^d + y_t^{\text{ex}} + I_t - F_y(k_t^y, N_t^y, L_t^y, U_t^y) = 0 \quad (3)$$

$$f_t^d + f_t^{\text{ex}} - F_f(k_t^f, N_t^f, C_t^f, U_t^f) \leq 0 \quad (4)$$

$$C_t^f + C_t^e - F_c(k_t^c, N_t^c, L_t^c, U_t^c) \leq 0 \quad (5)$$

$$E_t^c - F_e(k_t^e, N_t^e, C_t^e, U_t^e) \leq 0 \quad (6)$$

$$k_t - k_t^y - k_t^f - k_t^c - k_t^e = 0 \quad (7)$$

$$\bar{N} - N_t^y - N_t^f - N_t^c - N_t^e = 0 \quad (8)$$

$$\bar{L} - L_t^y - L_t^c = 0 \quad (9)$$

$$U_t + E_t^c - U_t^y - U_t^f - U_t^c - U_t^e = 0 \quad (10)$$

$$p^f f_t^{\text{ex}} + p^y y_t^{\text{ex}} - p^u U_t = 0 \quad (11)$$

$$k_0 = k^0 \quad (12)$$

The descriptions of the above constraints are as follows:

In equation (2),  $\frac{dk_t}{dt} = I_t - \delta k_t$  the net increase in the stock of physical capital at a point in time equals the gross investment ( $I_t$ ) less its depreciation ( $\delta k_t$ ), where  $\delta$  is the depreciation rate and  $k_t$  is the stock of physical capital.

In equation (3), the production function for the composite commodity ( $F_y(k_t^y, N_t^y, L_t^y, U_t^y)$ ) where  $k_t^y, N_t^y, L_t^y$ , and  $U_t^y$  are capital, labor, land, and energy, respectively. The production function is expected to equal the sum of its domestic consumption ( $y_t^d$ ), its export ( $y_t^{\text{ex}}$ ) and gross investment ( $I_t$ ).

In equation (4), the production function for food from cassava ( $F_f(k_t^f, N_t^f, L_t^f, U_t^f)$ ) where  $C_t^f$  is the raw cassava which is used as feedstock for producing food. The food

production is expected to be greater or equal to its domestic consumption ( $f_t^d$ ) plus its export ( $f_t^{ex}$ ).

In equation (5), the production function for raw cassava ( $F_c(k_t^c, N_t^c, L_t^c, U_t^c)$ ) is greater or equal to the sum of raw cassava used in food production ( $C_t^f$ ) and raw cassava used in energy production ( $C_t^e$ ).

In equation (6), the production function for energy produced from cassava ( $F_e(k_t^e, N_t^e, L_t^e, U_t^e)$ ) is greater or equal to energy produced from cassava ( $E_t^c$ ).

In equation (7),  $k_t - k_t^y - k_t^f - k_t^c - k_t^e = 0$  is a full employment constraint of capital used, that is, total capital stock equals the sum of stock of capital used in the composite commodity production ( $k_t^y$ ), in food production ( $k_t^f$ ), in raw cassava production ( $k_t^c$ ) and in energy production ( $k_t^e$ ).

In equation (8),  $\bar{N} - N_t^y - N_t^f - N_t^c - N_t^e = 0$  can be interpreted as the total labor used in all industries. The sum of labor used in the composite commodity production ( $N_t^y$ ), labor used in food production ( $N_t^f$ ), labor used in raw cassava production ( $N_t^c$ ), and labor used in energy from cassava production ( $N_t^e$ ) equals total labor available in the economy ( $\bar{N}$ ).

In equation (9),  $\bar{L} - L_t^y - L_t^c = 0$  is the total land available ( $\bar{L}$ ) which is the sum of land used in the composite commodity production ( $L_t^y$ ) and land used in raw cassava production ( $L_t^c$ ).

In equation (10),  $U_t + E_t^c - U_t^y - U_t^f - U_t^c - U_t^e = 0$  is a total energy constraint that is the total energy import ( $U_t$ ) plus the energy produced from cassava ( $E_t^c$ ) and equal to the sum of energy used in the composite commodity production ( $U_t^y$ ), energy used in food production ( $U_t^f$ ), energy used in raw cassava production ( $U_t^c$ ) and energy used in energy produced from cassava production ( $U_t^e$ ).

In equation (11),  $p^f f_t^{ex} + p^y y_t^{ex} - p^u U_t = 0$  is the trade balance equation where  $p^f f_t^{ex}$  is the value of food export where  $p^f$  is the relative price between food price and the composite commodity price, and  $f_t^{ex}$  is the food export quantity.  $p^y y_t^{ex}$  is the net value of the composite commodity export. We consider the term of  $p^y y_t^{ex}$  as the net value of the composite commodity export for covering the rest of the economy total exports minus the value of its total imports.  $p^y$  is a numeraire price and equal to one, and  $y_t^{ex}$  is the net export quantity of the composite commodity (the composite commodity export minus its import).

$p^u U_t$  is the value of energy import.  $p^u$  is the relative price between energy price and the composite commodity price, and  $U_t$  is the energy import quantity.

In equation (12),  $k_0 = k^0$  is the given value of initial stock of capital.

All production functions are assumed to be increasing and concave. In addition, total land and total labor available are fixed. The objective of the model is to maximize  $W$  over time period 0 to  $\infty$  subject to the constraints (2) to (12).

The present value Hamiltonian for the problem is:

$$H = e^{-rt} u(y_t^d, f_t^d) + \varphi_t (F^y(k_t^y, N_t^y, L_t^y, U_t^y) - y_t^d - y_t^{\text{ex}} - \delta k_t) \quad (13)$$

where  $\varphi_t$  is the costate variable of the state variable.

The present value Lagrangian is:

$$\begin{aligned} L = & e^{-rt} u(y_t^d, f_t^d) + \varphi_t (F^y(k_t^y, N_t^y, L_t^y, U_t^y) - y_t^d - y_t^{\text{ex}} - \delta k_t) \\ & - \mu_f [f_t^d + f_t^{\text{ex}} - F^f(k_t^f, N_t^f, C_t^f, U_t^y)] \\ & - \mu_c [C_t^f + C_t^e - F^c(k_t^c, N_t^c, L_t^c, U_t^c)] \\ & - \mu_e [E_t^c - F^e(k_t^e, N_t^e, C_t^e, U_t^e)] \\ & + \eta_k [k_t - k_t^y - k_t^f - k_t^c - k_t^e] \\ & + \eta_N [\bar{N} - N_t^y - N_t^f - N_t^c - N_t^e] \\ & + \eta_L [\bar{L} - L_t^y - L_t^c] \\ & + \eta_U [U_t + E_t^c - U_t^y - U_t^f - U_t^c - U_t^e] \\ & + \eta_T [p^f + f_t^{\text{ex}} + p^y y_t^{\text{ex}} - p^u U_t] \end{aligned} \quad (14)$$

where  $\mu_f, \mu_c, \mu_e, \eta_k, \eta_N, \eta_L, \eta_U,$  and  $\eta_T$  are the Lagrangian multipliers for food production, raw cassava production, energy production, capital supply, labor supply, land supply, energy supply, and trade balance, respectively.

The current value necessary condition, the costate variable and Lagrangian multipliers are developed and defined as follows:

For costate variable,  $\lambda_t = e^{rt} \varphi_t$  and  $\dot{\lambda}_t = r e^{rt} \varphi_t + e^{rt} \dot{\varphi}_t$  where  $\lambda_t$  is the current value of costate variable. The current value Lagrangian multipliers are defined as:  $\phi_k = e^{rt} \eta_k$ ,  $\phi_N = e^{rt} \eta_N$ ,  $\phi_L = e^{rt} \eta_L$ ,  $\phi_U = e^{rt} \eta_U$ ,  $\phi_T = e^{rt} \eta_T$ ,  $v_c = e^{rt} \mu_c$ ,  $v_f = e^{rt} \mu_f$ , and  $v_e = e^{rt} \mu_e$ .

The current value necessary conditions for an internal solution are:

$$\frac{\partial u_t^* (y_t^d, f_t^d)}{\partial y_t^d} - \lambda_t^* = 0 \quad (15)$$

$$\dot{\lambda}_t = (r + \delta)\lambda_t^* - \phi_k^* \quad (17)$$

$$v_f^* \frac{\partial F^f (k_t^f, N_t^f, C_t^f, U_t^f)}{\partial C_t^f} - v_c^* = 0 \quad (19)$$

$$\lambda_t^* \frac{\partial F^{y^*} (k_t^y, N_t^y, L_t^y, U_t^y)}{\partial k_t^y} - \phi_k^* = 0 \quad (21)$$

$$v_c^* \frac{\partial F^{c^*} (k_t^c, N_t^c, L_t^c, U_t^c)}{\partial k_t^c} - \phi_k^* = 0 \quad (23)$$

$$\lambda_t^* \frac{\partial F^{y^*} (k_t^y, N_t^y, L_t^y, U_t^y)}{\partial N_t^y} - \phi_N^* = 0 \quad (25)$$

$$v_c^* \frac{\partial F^{c^*} (k_t^c, N_t^c, L_t^c, U_t^c)}{\partial N_t^c} - \phi_N^* = 0 \quad (27)$$

$$\lambda_t^* \frac{\partial F^{y^*} (k_t^y, N_t^y, L_t^y, U_t^y)}{\partial L_t^y} - \phi_L^* = 0 \quad (29)$$

$$\lambda_t^* \frac{\partial F^{y^*} (k_t^y, N_t^y, L_t^y, U_t^y)}{\partial U_t^y} - \phi_U^* = 0 \quad (31)$$

$$v_c^* \frac{\partial F^{c^*} (k_t^c, N_t^c, L_t^c, U_t^c)}{\partial U_t^c} - \phi_U^* = 0 \quad (33)$$

$$v_f^* (f_t^{d^*} + f_t^{ex^*} - F^f (k_t^f, N_t^f, C_t^f, U_t^f)) = 0 \quad (35)$$

$$f_t^{d^*} + f_t^{ex^*} - F^f (k_t^f, N_t^f, C_t^f, U_t^f) \leq 0, v_f^* \geq 0$$

$$v_c^* (C_t^{f^*} + C_t^{e^*} - F^{c^*} (k_t^c, N_t^c, L_t^c, U_t^c)) = 0 \quad (36)$$

$$C_t^{f^*} + C_t^{e^*} - F^{c^*} (k_t^c, N_t^c, L_t^c, U_t^c) \leq 0, v_c^* \geq 0$$

$$v_e^* (E_t^{c^*} + F^{e^*} (k_t^e, N_t^e, C_t^e, U_t^e)) = 0 \quad (37)$$

$$E_t^{c^*} - F^{e^*} (k_t^e, N_t^e, C_t^e, U_t^e) \leq 0, v_e^* \geq 0$$

$$\bar{L} - L_t^{y^*} - L_t^{c^*} = 0 \quad (38)$$

$$\bar{N} - N_t^{y^*} - N_t^{f^*} - N_t^{c^*} - N_t^{e^*} = 0 \quad (40)$$

$$-v_e^* + \phi_U^* = 0 \quad (42)$$

$$v_f^* = p^f \phi_t^* \quad (44)$$

$$p^f f_t^{ex^*} + p^y y_t^{ex^*} = p^U U_t^* \quad (46)$$

$$\frac{\partial u_t^* (y_t^d, f_t^d)}{\partial f_t^d} - v_f^* = 0 \quad (16)$$

$$\dot{k}_t = F^{y^*} (.) - y_t^{d^*} - y_t^{ex^*} - \delta k_t^* \quad (18)$$

$$v_e^* \frac{\partial F^{e^*} (k_t^e, N_t^e, C_t^e, U_t^e)}{\partial C_t^e} - v_c^* = 0 \quad (20)$$

$$v_f^* \frac{\partial F^f (k_t^f, N_t^f, C_t^f, U_t^f)}{\partial k_t^f} - \phi_k^* = 0 \quad (22)$$

$$v_e^* \frac{\partial F^{e^*} (k_t^e, N_t^e, C_t^e, U_t^e)}{\partial k_t^e} - \phi_k^* = 0 \quad (24)$$

$$v_f^* \frac{\partial F^f (k_t^f, N_t^f, C_t^f, U_t^f)}{\partial N_t^f} - \phi_N^* = 0 \quad (26)$$

$$v_e^* \frac{\partial F^{e^*} (k_t^e, N_t^e, C_t^e, U_t^e)}{\partial N_t^e} - \phi_N^* = 0 \quad (28)$$

$$v_c^* \frac{\partial F^{c^*} (k_t^c, N_t^c, L_t^c, U_t^c)}{\partial L_t^c} - \phi_L^* = 0 \quad (30)$$

$$v_f^* \frac{\partial F^f (k_t^f, N_t^f, C_t^f, U_t^f)}{\partial U_t^f} - \phi_U^* = 0 \quad (32)$$

$$v_e^* \frac{\partial F^{e^*} (k_t^e, N_t^e, C_t^e, U_t^e)}{\partial U_t^e} - \phi_U^* = 0 \quad (34)$$

$$k_t^* - k_t^{y^*} - k_t^{f^*} - k_t^{c^*} - k_t^{e^*} = 0 \quad (39)$$

$$U_t^* + E_t^{c^*} - U_t^{y^*} - U_t^{f^*} - U_t^{c^*} - U_t^{e^*} = 0 \quad (41)$$

$$\lambda_t^* = p^y \phi_t^* \quad (43)$$

$$\phi_U^* = p^U \phi_t^* \quad (45)$$

where equation (17) is the law of motion of the costate variable,  $\lambda_t^*$ , that is, the rate of change in value of capital stock equals the net value of the marginal product of capital. The costate variable can be interpreted as the shadow value of stock of capital in each time period. In equation (18) the rate of change in the state variable (stock of capital) is the gross investment minus its depreciation.

The maximum principle in the optimal control theory is stated as a set of conditions that exist along the optimal path. Equations (15)-(46) state these conditions and were derived from the Lagrangian function (14). The set of solutions that satisfies the current value necessary equations (15)-(46) consists of one state variable ( $k_t^*$ ), one costate variable ( $\lambda_t^*$ ), eight Lagrangian multipliers ( $\phi_k^*$ ,  $\phi_N^*$ ,  $\phi_L^*$ ,  $\phi_U^*$ ,  $\phi_T^*$ ,  $v_c^*$ ,  $v_f^*$  and  $v_e^*$ ), and control variables (the other twenty-two variables). In this section, we will discuss some selected variables in the set of solutions that satisfies the maximum principle.

In this study, the dynamic general equilibrium optimal control model of cassava is developed to determine the set of all relevant variables in the economy. The economy is assumed to have four production sectors, which are the composite commodity, food, raw cassava, and energy. With optimal control theory as the analytical tool, the model maximizes the sum of discounted utility of society over an infinite time horizon.

The assumptions of the model in this study are: 1) perfect substitution between utilized energy and ethanol from cassava, and 2) fixed interest rate and fixed depreciation rate over the time horizon. In the calibration, the model assumes Cobb-Douglas functions for the utility function and the production functions for the economy. It has the advantage of finding a unique solution from increasing and concave function qualification. The model has one utility function in equation (1) and four production functions in equations (3)-(6) of the composite commodity, food, raw cassava and energy, respectively. The model can be specified in Cobb-Douglas functions as:

$$W = \int_0^T e^{-rt} \left[ (Y_t^d)^\alpha \cdot (f_t^d)^\beta \right] dt \quad (47)$$

$$y_t^d + y_t^{ex} + I_t - \left[ (k_t^y)^{\gamma 1} \cdot (N_t^y)^{\rho 1} \cdot (L_t^y)^{\sigma 1} \cdot (U_t^y)^{\delta 1} \right] = 0 \quad (48)$$

$$f_t^d + f_t^{ex} - \left[ (k_t^f)^{\gamma 2} \cdot (N_t^f)^{\rho 2} \cdot (C_t^f)^{\tau 2} \cdot (U_t^f)^{\delta 2} \right] \leq 0 \quad (49)$$

$$C_t^f + C_t^e - \left[ (k_t^c)^{\gamma 3} \cdot (N_t^c)^{\rho 3} \cdot (L_t^c)^{\sigma 3} \cdot (U_t^c)^{\delta 3} \right] \leq 0 \quad (50)$$

$$E_t^c - \left[ (k_t^e)^{\gamma 4} \cdot (N_t^e)^{\rho 4} \cdot (C_t^e)^{\tau 4} \cdot (U_t^e)^{\delta 4} \right] \leq 0 \quad (51)$$

where  $\alpha$  and  $\beta$  are the preference parameters of the utility function for the composite commodity and for food, respectively.  $\gamma_1 - \gamma_4$  are output elasticity of capital for the composite commodity, for food, for cassava, and for utilized energy, respectively.  $\rho_1 - \rho_4$  are output elasticity of labor for the composite commodity, for food, for cassava, and for utilized energy, respectively.  $\sigma_1$  and  $\sigma_3$  are output elasticity of land for the composite commodity and for cassava.  $\tau_2$  and  $\tau_4$  are output elasticity of cassava for food and for utilized energy. And  $\delta_1 - \delta_4$  are output elasticity of utilized energy for the composite commodity, for food, for cassava, and for utilized energy, respectively.

## Data

The model is calibrated using the data of Thailand. The data used in the estimation were the national yearly data in 2007 for some parameters and the existing research data for other parameters. In the utility function  $\alpha$  and  $\beta$  are the preference parameters of the utility function for the composite commodity and for food respectively. The assigned values for  $\alpha$  and  $\beta$  in this study were estimated by using expenditure share in the total expenditure. The share of the composite commodity in national consumption is set to  $\alpha = 0.9889$  and it implies that the share of food from cassava in national consumption is set to  $\beta = 0.01106$ . The data was obtained from the Office of National Economic and Social Development Board (NESDB).

The output elasticity can be obtained by using factor share of the output as follows:

1) *The composite commodity production.* The factors used in the composite commodity production are capital, labor, land, and energy. The data for factor shares was obtained from NESDB. The capital share in the composite commodity production is set at  $\gamma_1 = 0.65316$ , labor in the composite commodity production at  $\rho_1 = 0.2832$ , land share in the composite commodity production at  $\sigma_1 = 0.038$  and energy share in the composite commodity production at  $\delta_1 = 0.02564$ .

2) *Food production.* The production function of food has four factors: capital, labor, raw cassava, and energy. The data for factor shares was obtained from the research study of the Export-Import Bank of Thailand. The capital share in food production is set at  $\gamma_2 = 0.1372$ , labor share in food production at  $\rho_2 = 0.0696$ , raw cassava share in food production at  $\tau_2 = 0.600$  and energy share in food production at  $\delta_2 = 0.1932$ .

3) *Raw cassava production*. The raw cassava production function has the same factors as the composite commodity production function (capital, labor, land and energy). The data for factor shares was obtained from the Office of Agricultural Economics. The capital share in raw cassava production is set at  $\gamma_3 = 0.3041$ , labor share in raw cassava production at  $\rho_3 = 0.5444$ , land share in raw cassava production at  $\sigma_3 = 0.1108$  and energy share in raw cassava production at  $\delta_3 = 0.0403$ , and 4) energy production. The energy production function has four factors: capital, labor, raw cassava, and energy. The data of factor shares was obtained from the research study of Yoosin and Sorapipatana (2007). The capital share in energy production is set at  $\gamma_4 = 0.449$ , labor share in energy production at  $\rho_4 = 0.0079$ , raw cassava share in energy production at  $\tau_4 = 0.5415$  and energy share in energy production at  $\delta_4 = 0.00151$ .

The parameter  $\bar{L}$  is the total land area available for all agricultural production. The data for land parameters was obtained from the Ministry of Agriculture and Cooperatives.  $\bar{L}$  is set to 20.82 million hectares.  $\bar{N}$  is the total labor available in economy. The data for the labor parameter was obtained from NESDB.  $\bar{N}$  is set at 37.7 million persons.

The relative export and import prices are assigned in the relative price by using the composite commodity as the numeraire. The export price of the composite commodity is set at 1, the relative export price of food at  $\frac{V_f}{\lambda_t}$ , and the relative import price of energy at  $\frac{\phi_U}{\lambda_t}$ .

Last, the designated interest rate and depreciation rate are 3.5% and 5%, respectively. The interest rate is assigned following the description in the World Fact Book (CIA, 2010). The Central Bank discount rate is defined as the interest rate that the Central Bank charges commercial banks for loans to meet temporary shortage of funds. In this study, the discount rate or interest rate is set at 3.5% for the average years of 2007 and 2008. The depreciation rate is assigned following the study of Tanboon (2008). The annual depreciation rate is set from the annual depreciation divided by gross capital stock at 1988 prices for use in the structural model for The Bank of Thailand policy analysis. In this study, the depreciation rate is calculated by using Tanboon's concept and data of year 2007. The average annual depreciation rate of real sector (agriculture sector and industrial sector) in 2007 is set at 5%.

## Results and Discussion

This section describes the stationary state solution, the stationary state evaluation, and the optimal time path for the set of variables by solving the system of equations (15)-(46) and substituting the above defined parameter values.

### *Calibration Results*

The stationary state solution of the model is obtained by solving the system of current value necessary equations (15)-(46) with  $\frac{dk}{dt} = \frac{d\lambda}{dt} = 0$  and using functions of equations (47)-(51). The set of solutions for all variables in the stationary state have a large difference between the solution values of variables from calibration and the defined parameter values from the Thai data. Thus the model is applied with a new value of  $\tau_2$  to reduce the difference between them. The explanation of the calibration evaluation is presented in the next section. The old parameter value of  $\tau_2$  is set as 0.600 and the new value for  $\tau_2$  is changed to 0.47.

The system of equations generates values of variables at the maximum point (Table 1). The model generates the quantities of consumption, the quantities of production, the quantities of factors allocation, the quantities of export and import, and the shadow prices. The equilibrium value of the composite commodity is 1540.483 units, which includes 910.483 units of domestic composite commodity consumption, 34.948 units of the composite commodity for export and 595.052 units of gross investment. The optimal production of food is 1.840 units, which are 1.005 units for domestic consumption and 0.835 units for export. The optimal value of raw cassava production for food is 1.482 units and for energy it is 0.054 units. The energy production from cassava is 0.3048 units while imported energy is 21.389 units.

Table 1 Stationary state solution for endogenous variables

Variable	Value	Description
$y_t^d$	910.4835	The composite commodity
$f_t^d$	1.0051	Food from cassava
$y_t^{ex}$	34.9488	Net composite commodity for export
$f_t^{ex}$	0.8350	Food from cassava for export
$U_t$	21.3899	Imported energy
$\left[ (k_t^y)^{\gamma_1} \cdot (N_t^y)^{\rho_1} \cdot (L_t^y)^{\sigma_1} \cdot (U_t^y)^{\delta_1} \right]$	1,540.4830	The composite commodity production
$k_t^y$	11,834.5340	Capital for the composite commodity
$N_t^y$	37.1702	Labor for the composite commodity
$L_t^y$	20.5000	Land for the composite commodity
$U_t^y$	19.7365	Energy for the composite commodity
$\left[ (k_t^f)^{\gamma_2} \cdot (N_t^f)^{\rho_2} \cdot (C_t^f)^{\tau_2} \cdot (U_t^f)^{\delta_2} \right]$	1.8401	Food from cassava production
$k_t^f$	30.0900	Capital for food
$N_t^f$	0.1106	Labor for food
$C_t^f$	1.4828	Raw cassava for food
$U_t^f$	1.7747	Energy for food
$\left[ (k_t^c)^{\gamma_3} \cdot (N_t^c)^{\rho_3} \cdot (L_t^c)^{\sigma_3} \cdot (U_t^c)^{\delta_3} \right]$	1.5367	Cassava production
$k_t^c$	33.1160	Capital for cassava
$N_t^c$	0.4181	Labor for cassava
$L_t^c$	0.3498	Land for cassava
$U_t^c$	0.1789	Energy for cassava
$\left[ (k_t^e)^{\gamma_4} \cdot (N_t^e)^{\rho_4} \cdot (C_t^e)^{\tau_4} \cdot (U_t^e)^{\delta_4} \right]$	0.3048	Energy production
$k_t^e$	3.2750	Capital for energy
$N_t^e$	0.0011	Labor for energy
$C_t^e$	0.0539	Raw cassava for energy
$U_t^e$	0.0046	Energy for energy
$\lambda_t$	1.0000	Shadow price of the composite commodity
$\phi_k$	0.0849	Shadow price of capital
$\phi_N$	11.7281	Shadow price of labor
$\phi_L$	2.8554	Shadow price of land

Table 1 (Continued)

$\phi_U$	2.0290	Shadow price of utilize energy
$\phi_t$	1.0000	Shadow price of trade
$v_f$	10.1308	Shadow price of food
$v_C$	5.9085	Shadow price of raw cassava
$v_e$	2.0290	Shadow price of energy from cassava

### Calibration Evaluation

In calibrating the model it was necessary to adjust some of the parameters from those stated above. First, the parameters from the actual data are substituted and computed for generating the stationary state solution. Second, the difference between solutions and actual data was computed in absolute percentage differences. Third, the new set of parameters is used for generating stationary state solution and computing the difference between the solution and actual data again. After that, we compare the difference between two values of the absolute percentage difference and select the lesser one. We then repeat the above steps until the set of parameters generates the minimum value of the average absolute percentage difference.

After comparing the results of the sets of parameters in calibration, the model that gives the closest solution to capture the actual Thai data in the year 2007 is selected.

The set of parameters that generates the minimum value of percentage difference is the set that has  $\tau_2 = 0.470$ . The first data set that is defined by Thailand data gives 9.8423% of the average absolute percentage difference while the new data set with  $\tau_2 = 0.470$  gives 9.4509%. The values of absolute percentage from the data set with  $\tau_2 = 0.470$  express the lesser value in the average absolute percentage difference. The new data set affects the structure of food production function by changing it from a constant return to scale function to a decreasing return to scale function.

### Simulation Results

The objective of the simulation is to find a policy function or a time independent numerical decision rule that solves the model described above. An infinite time horizon optimal control model for the Thai economy has two differential equations and thirty algebraic equations in the first order necessary conditions. Due to the complex structure of the model and many variables in the model, the solution time path cannot be directly solved for the state variable ( $k_t^*$ ) and the costate variables ( $\lambda_t^*$ ). In this section, the solution time paths are solved by two numerical methods:

1) *Linear approximation method* (Léonard and Long, 1992)

The general solution to linear equations is given by:

$$k(t) = C_1 v_{11} e^{\pi_1 t} + C_2 v_{12} e^{\pi_2 t} + k_{ss} \tag{52}$$

$$\lambda(t) = C_1 v_{21} e^{\pi_1 t} + C_2 v_{22} e^{\pi_2 t} + \lambda_{ss} \tag{53}$$

where  $C_i$  are the constant of integration and  $v_{ij}$  are the elements of Eigen vectors. The numerical results of the problem are  $C_1 = 0$ ,  $C_2 = 1450.016$ ,  $v_{12} = -0.9999$ ,  $v_{22} = 1.389e-06$ ,  $\pi_2 = -1.7505$ ,  $k_{ss} = 11,901.016$ , and  $\lambda_{ss} = 0.9172$ . The relationship between  $k(t)$  and  $\lambda(t)$  in equations (52) and (53) shown by solid line in Figure 1.

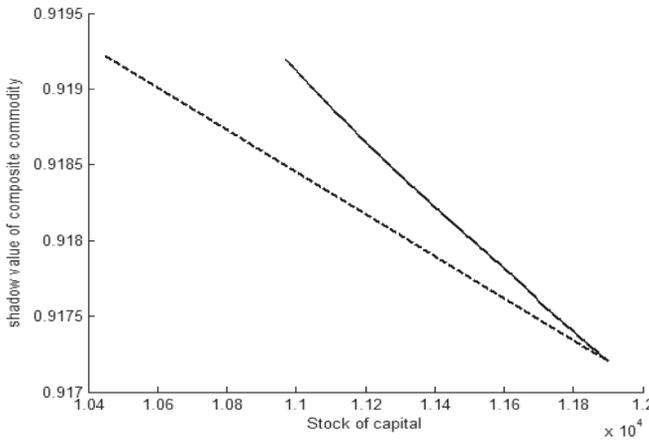


Figure 1 The reverse shooting of the policy function and the linear approximation relationship between  $k(t)$  and  $\lambda(t)$

2) *Runke-Kutta reverse shooting method* (Judd, 1998)

Since the time path solution from the linear approximation method has a high speed of adjustment to stationary state, the method of Runke-Kutta reverse shooting is used to solve the numerical policy function solution to capture the adjustment of the solution time path at a very small step size to get more accuracy. An approximation of the capital stock policy function  $K'(\lambda(t))$  in equation (54) is generated by using reverse shooting method.

$$K'(\lambda(t)) = \frac{\dot{k}}{\dot{\lambda}} = \frac{g(k_t, \lambda_t)}{f(k_t, \lambda_t)} \tag{54}$$

The solution for the policy function is generated from a point close to stationary state and move backwards to a beginning point. The result is shown by the dashed line in Figure 1. The line represents policy function approximation from the reverse shooting method and the solid line shows the linear approximation relationship between  $k(t)$  and  $\lambda(t)$  in equations (60) and (61). The vertical axis is the value of the costate variable or the shadow value of the

composite commodity. The horizontal axis is the value of the stock of capital. The policy function begins (in dashed line) at a stationary state point where stock of capital equals 11,901.016 and the shadow price of the composite commodity (the relative shadow price is 1) is 0.9172. From the initial point, the shadow price of the composite commodity decreases and the stock of capital increases through time until they reach the stationary state.

In Figure 2, the first graph (left) shows the approximation of the optimal path for stock of capital and the shadow value of the capital (right) by linear approximation method and reverse shooting method. The linear approximation method is simulated 200 iterations with the beginning stock of capital  $k_0 = 10,401.016$ . After six iterations, the stock of capital goes to a long run optimal level (the solid line). The reverse shooting method is set at the beginning stock of capital to  $k_0 = 11,161.016$  and it requires 73 iterations through time to reach the long run optimal path (dashed line).

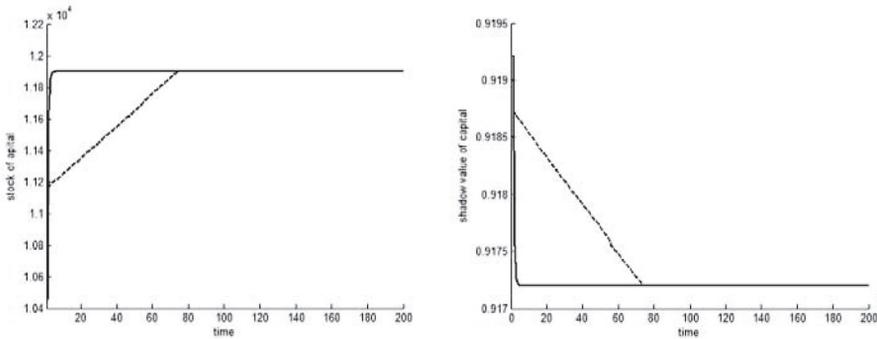


Figure 2 Approximation of the optimal time path of the stock of capital (left) and the shadow value of the capital stock (right)

The beginning stock of capital of the reverse shooting method is larger than that of the linear approximation method because after 73 iterations the solution ceases to be meaningful. Thus the simulation from the reverse shooting method needs to stop at about  $k_t = 11,161.016$  and is set at the beginning point of  $k_0$ . In Figure 2, the second graph (right) is the time path shadow value of the capital stock. Due to the difference in the beginning stock of capital, they have different beginning values of shadow value of capital. The shadow values decrease through time until they reach long run optimal path at 0.9172.

Figure 3 shows the approximated time path of gross investment and consumption. The time path of gross investment decreases through time until it reaches long run optimal level at 383.824. The dashed line in both graphs represents the solution time path for the reverse shooting method, while the solid line represents the solution for the linear approximation. An adjustment for consumption increases when time increases and reaches the optimal long run consumption at 1,118.290. The optimal path for consumption is the domestic total consumption in the economy which is composed of domestic composite commodity and domestic food.

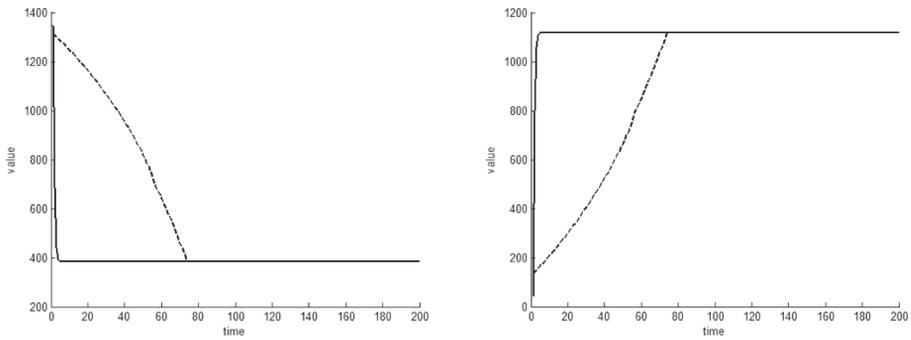


Figure 3 Approximation of the optimal time path of the gross investment (left) and consumption (right)

Figure 4 shows the optimal path of domestic composite commodity and domestic food consumption. Both graphs of optimal path of consumption increase through time until they reach long run optimal level where the composite commodity consumption equals 1,117.057 and food consumption equals 1.233.

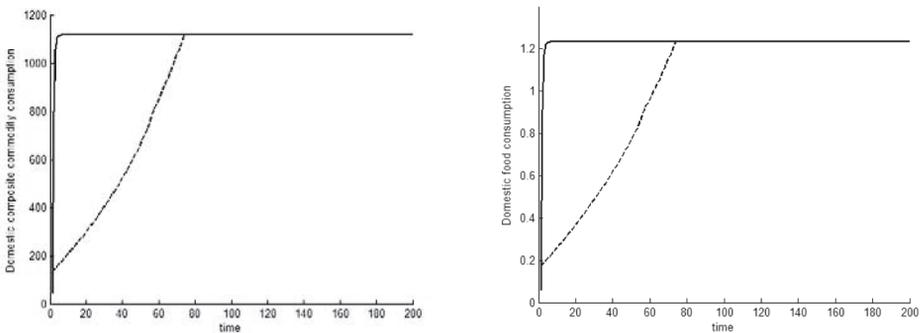


Figure 4 Approximation of the optimal time path of the domestic composite commodity consumption (left) and domestic food consumption (right)

Figure 5 shows the optimal path of domestic ethanol production. The time path of ethanol production starts from time zero and increases in production through time until reaching long run optimal level of production at 17.084.

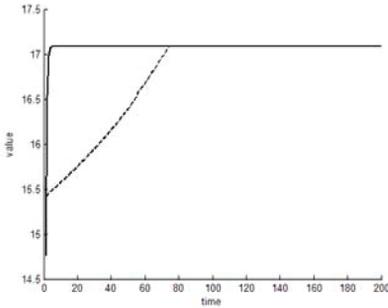


Figure 5 Approximation of the optimal time path of the domestic ethanol production

## Conclusions

The study analyzes a dynamic general equilibrium model of cassava based on the optimal control problem for an infinite time horizon. The model is developed and calibrated using the data of Thailand. The objective of the model is to maximize total utility of society, and it highlights food from cassava as a separate commodity. The constraints of the model are defined to capture all activities in the economy to represent food consumption and energy production from cassava. The model is simulated to find the optimal time path using two methods which are the linear approximation method and the reverse shooting method. The linear approximation quickly reaches the long run equilibrium while the reverse shooting of the policy function has the advantage of generating adjustments on the way to reaching the long run equilibrium. Thus the approximation of the policy function for capital is generated by using Runke-Kutta reverse shooting method. The results show the optimal consumption, production, and allocation of resources in the economy in case of producing cassava for food and energy. The calibration results are close to the actual data when we multiply the results with a scale factor.

A dynamic general equilibrium model is developed to analyze the conflict of using cassava for food and energy. The model allocates resources in the economy over a period of time using the necessary conditions from an application of the maximum principle.

Regarding the land, labor, capital, and energy competitions, the model can allocate the energy crop production to balance the energy and food demands while maintaining the levels of consumption and ethanol production. The results indicate that an increase in ethanol production through time can reach a long run optimal path and significantly contribute to the sustainability of the economy. This implies that the model can be used as a tool to provide a guide for policy makers in economic planning, particularly in the allocation of production resources. Achieving sustainable development with biofuel production requires appropriate energy crop production planning for long run resource allocation.

Several limitations to this model and recommendations based on its use and results are noted. First, the specified assumption for basic model and optimization was set up as simple as possible. Furthermore the model provides an optimal set of solutions that was not derived from a stochastic process. These issues represent the gap between the model and the real world. Second, population growth and labor growth are not taken into account in this basic model. Thus, for any extension, labor can be allowed to increase over time; doing so would affect the demand for food which drives the increase in food production or results in lower food exports. Finally, the model can be considered for several possible extensions based on the agricultural and energy issues that relate to an increase in food price, rising energy price, increase in the use of energy, and technological innovation.

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