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APPENDIX A

APPENDIX

MODELING OF POPULATION WITH STAGE-STRUCTURE

Recent interest in a mathematical description of the age-structure of growing cell populations has centered on the use of an equation proposed by von-Foerster. This equation is a first order partial differential equation for a function $n(t,a)$ defined so that $n(t,a)$ is the number of organisms at time t whose ages lie in the interval $(a, a+da)$. Note that most details in this part are collected and adapted from [2] and [18].

The age a that appears as an independent variable in $n(t,a)$ is always interpreted as the chronological age of the organism. In consequence, the properties of two organisms belonging to a single age-group and subjected to identical environments are identical. Thus, the important feature of biological variability is omitted from the description of the age-structure of a single cohort.

The derivative of this kind of models is from the von-Foerster or the McKendrick equation. Let $n(t,a)$ be the density of an age-structure population at time t of age a. Consider the partial differential equation

$$\frac{\partial n}{\partial t}+\frac{\partial n}{\partial a}=-\mu(a)n,$$

for $t > a$. Integrate along a characteristic curve given by $t = a + \xi$, and define

$$n_{\xi}(a) = n(a+\xi, a).$$

Differentiate n_ξ with respect to a, we have

$$\frac{dn_\xi}{da} = \left[\frac{\partial n}{\partial t} + \frac{\partial n}{\partial a}\right]_{t=a+\xi}$$
$$= -\mu(a)n(a+\xi, a)$$
$$= -\mu(a)n_\xi(a).$$

Hence,

$$n_\xi(a) = n_\xi(0)e^{-\int_0^a \mu(s)ds},$$

or

$$n(a+\xi, a) = n(\xi, 0)e^{-\int_0^a \mu(s)ds}.$$

Putting $a = \tau$ and $\xi = t - \tau$ gives

$$n(t, \tau) = n(t-\tau, 0)e^{-\int_0^a \mu(s)ds},$$

provided $t > \tau$.

Suppose that the total number of adults is $N(t)$, where

$$N(t) = \int_\tau^\infty n(t, a)da.$$

Then

$$\frac{dN}{dt} = \int_\tau^\infty \frac{\partial n}{\partial t} da$$
$$= \int_\tau^\infty \left(-\frac{\partial n}{\partial a} - \mu(a)n(t, a)\right) da.$$

We can see that

$$\frac{dN}{dt} = n(t, \tau) - n(t, \infty) - \mu_M N(t), \quad \text{if } \mu(a) = \mu_M \text{ for all } a \geq \tau,$$

and

$$\frac{dN}{dt} = n(t-\tau,0)e^{-\int_0^\tau \mu(s)ds} - \mu_M N(t), \text{ if } n(t,\infty) = 0.$$

But $n(t,0)$ is the birth rate. If this is a function of the total number of adults, then $n(t,0) = B(N(t))$. So that

$$\frac{dN}{dt} = e^{-\int_0^\tau \mu(s)ds} B(N(t-\tau)) - \mu_M N(t).$$

We can see that term of age-structure $e^{-\int_0^\tau \mu(s)ds}$ is absorbed. In my thesis, we assume that $\mu(s)$ is a constant. The equation become

$$\frac{dN}{dt} = e^{-\delta\tau} B(N(t-\tau)) - \mu_M N(t),$$

where $\mu(s) = \delta$. Then, the above equation is similar to the selected model for analysis in Chapter IV:

$$N'(t) = -\gamma N(t) + \beta e^{-\delta\tau} f(N(t-\tau))N(t-\tau),$$

where $\mu_M = \gamma$. Note that the equation above is the model which we selected to study for the case of age-structure population models.

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