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APPENDIX A

Numeric value

A.1 Constant value

Anode volume(V_{an})	=	0.005 m^3
Anode flow constant (k_{an})	=	0.065 mol/s atm
Cathode Volume (V_{cat})	=	0.01 m^3
Cathode flow constant (k_{cat})	=	0.065 mol/s atm
Faraday's constant (F)	=	96485 A.s/mol
Gas constant (R)	=	8.206×10^{-5} m^3 atm/mol K
Hydrogen pressure inlet ($P_{H_2,in}$)	=	3 atm
Number of fuel cell stack (N)	=	35
Oxygen pressure outlet ($P_{O_2,out}$)	=	1 atm

A.2 Normal operating value

Current load (I)	=	100 A
Hydrogen flowrate ($\dot{m}_{H_2,in}$)	=	0.008 mol/s
Oxygen flowrate ($\dot{m}_{O_2,in}$)	=	0.02 mol/s

A.3 Steady state value

Cell voltage ($\overline{V}_{cell}$)	=	27.5 V
Hydrogen pressure ($\overline{P}_{H_2}$)	=	2.74 atm
Oxygen pressure ($\overline{P}_{O_2}$)	=	3.16 atm
Stack temperature ($\overline{T}_s$)	=	75 C

APPENDIX B

Source code

B.1 The steady state value

This section is used to find the steady state values used in the proton exchange membrane fuel cell.

```
clc
clear
format long

%%The partial pressure of hydrogen
syms pH2
H2inlet=0.008;              %mol/s
kan=0.065;                  %mol/s.atm
pH2in=2.9;                  %atm
N=35;
I=100;                      %A
F=96485;                    %A.s/mol
ANS=solve(H2inlet-kan*(pH2-pH2in)-(N*I/2/F));
pH2_ss=double(ANS)

%%The partial pressure of oxygen
syms pO2
O2inlet=0.02;               %mol/s
kcat=0.065;                 %mol/s.atm
patm=3 ;                    %atm
N=35;
I=100;                      %A
F=96485;                    %A.s/mol
ANS=solve(O2inlet-kcat*(pO2-patm)-(N*I/4/F));
pO2_ss=double(ANS)

%%The stack temperature
syms T
CpH2=14209;                 %J/kg.K
CpO2=922;                   %J/kg.K
Cpw=4184;                   %J/kg.K
Ct=17.9*1000;               %J/K
T0=298.15;                  %K
Tamb=298.15;                %K
Tc=298.15;                  %K
MwH2=2.016;                 %kg/kmol
MwO2=32;                    %kg/kmol
MwH2O=18;                   %kg/kmol
kconv_an=2;                 %J/s.K
kconv_cat=10;               %J/s.K
```

```
kconv_amb=17;                %J/s.K
Vcell=0.7552;                %V
Hreact=285.5*1000;           %J/mol
Mcw=0.45;                    %mol/s
Van=0.005;                   %m^3
Vcat=0.01;                   %m^3
R=8.205746e-5;               %m^3 atm/mol K
Tstack=(kconv_amb*(Tamb-T))+(N*I/2/F*Hreact)-(35*Vcell*I)-
(Mcw*Cpw*MwH2O*(T-Tc)/1000);
ANS=solve(Tstack);
T_ss=double(ANS)
T_ss_celcius=T_ss-273.15

%%The cell voltage and power
e1=-0.944;
e2=3.54E-3;
e3=-1.96e-4;
e4=7.8e-5;
e5=3.3e-3;
e6=-7.55e-6;
e7=1.1e-6;
I=100;                       %A
Enert=1.229-((8.5E-4)*(T_ss-298.15))+((4.308E-
5)*T_ss*(log(pH2_ss)+log(0.5*pO2_ss)));
cO2=pO2_ss/((5.08e6)*exp(-498/T_ss));
act=e1+(e2*T_ss)+(e3*T_ss*log(I))+(e4*T_ss*log(cO2));
ohm=-I*(e5+(e6*T_ss)+(e7*I));
Vcell=Enert+act+ohm
Power=Vcell*I                %watt
```

B.2 Finding A,B,C,D, E and system transfer function matrix

```
%%Finding A matrix
A11=-kan*R*T_ss/Van;
A13=R/Van*(H2inlet-kan*(pH2_ss-pH2in)-(N*I/2/F));
A22=-kcat*R*T_ss/Vcat;
A23=R/Vcat*(O2inlet-kcat*(pO2_ss-patm)-(N*I/4/F));
A33=-(kconv_amb+(Mcw*Cpw*MwH2O/1000))/Ct;
A=[A11 0 A13;0 A22 A23;0 0 A33]

%%Finding B matrix
B11=R*T_ss/Van;
B22=R*T_ss/Vcat;
B33=-Cpw*MwH2O/1000*(T_ss-Tc)/Ct;
```

```
B=[B11 0 0;0 B22 0;0 0 B33]

%%Finding C matrix
C=[1 0 0;0 1 0;0 0 1]

%%Finding D matrix
D_process=[0 0 0;0 0 0;0 0 0]
 D_dis=[0;0;0]

%%Finding E matrix
E11=-R*T_ss*N/Van/2/F;
E21=-R*T_ss*N/Vcat/4/F;
E31=((N*Hreact/2/F)-(35*Vcell))/Ct;
E=[E11;E21;E31]

%%Finding transfer function matrix
system=ss(A,B,C,D)
 g=tf(system)
```

B.3 Finding passivity index

```
%%Finding passivity index
w = logspace(-4,4,200)
s = w * sqrt(-1)                        %s = jw
I = eye(length (A))                     %I = identity matrix
for z=1:length(s)
g= C*((s(z)*I-A)\B)+D;                  %g = transfer function
greal = real(Gplus);
gimag = imag(Gplus);

%size of new matrix
[nAr,nAc] = size(greal);
[nBr,nBc] = size(gimag);

% --------LMI-#1--------
setlmis([]);
DE = lmivar(1,[ones(nAr,2)]);
lmiterm([1 1 1 0],eye(nAr));            %LMI #1: eye(nAr)
lmiterm([-1 1 1 DE],1,1);               %LMI #1: DE
lmiterm([1 1 1 DE],greal,-1,'s');       %LMI #2: -greal*DE-DE*greal'
lmiterm([1 2 1 DE],1,gimag');           %LMI #2: DE*gimag'
lmiterm([1 2 1 DE],gimag,-1);           %LMI #2: -gimag*DE
lmiterm([1 2 2 DE],greal,-1,'s');       %LMI #2: -greal*DE-DE*greal'
lmiterm([-1 1 1 DE],1,1);               %LMI #2: DE
```

```
lmiterm([-1 2 1 0],zeros(nAr));          %LMI #2: zeros(nAr)
lmiterm([-1 2 2 DE],1,1);                %LMI #2  DE
sys = getlmis;

% ------ Generalized Eigenvalue Minimization of t #2--------
[tmin,Xopt] = gevp(sys,1);
Dm = dec2mat(sys,Xopt,DE);
DNew = sqrt(Dm);
DGp = inv(DNew)*Gplus*DNew;
nu= -min(real(eig(DGp+DGp'))/2);
end
disp('t is '); disp(tmin);
disp('Dm is '); disp(Dm);

% %Plot passivity index
figure(1);
semilogx(w,nu,'bl-.');
xlabel('Frequency (rad/hr)');
ylabel('Passivity Index');
legend('Passivity index of PEMFC system');
grid;
```

B.4 Finding the weighting function

```
%%Finding weighting function of PEMFC system
%To find the parameters of weighting function (a,b,c and k), this file must be opened
open with
   myfun.m and mynoncon.m file
options = optimset('Display','iter','MaxFunEvals',100000);
[x,fval] = fmincon(@myfun,[9.234 3.2687 0.081 0.5558],[],[],[],[],[0 0 0 0],[100 100
100 1],@mynoncon,options,nu,w)
ww01 = x(1)
ww02 = x(2)
ww03 = x(3)
ww04 = x(4)
%%The transfer function and passivity index of weighting function
WtfnG=tf({[x(4) x(1)*x(4) 0]}, {[1 x(2)+x(3) x(2)*x(3)]});
for z=1:length(w)
Gwt = freqresp(WtfnG,w(z));
nu1(z)= -min(real(eig(Gwt+Gwt'))/2);
end

%%Plot passivity index of weighting function
figure(2);
```

```
semilogx(w,nu1,'r-');
xlabel('Frequency (rad/hr)');
ylabel('Passivity index');
legend('passivity Index of weighting functionp');
% grid;
```

B.5 Making the PEMFC is passive process

After the weighting function is obtained, it is added into the non-passive system to make it passive system

```
%%Add the weighting into the PEMFC system
 for z = 1:length(w)
 Gp1 = C*(((s(z)*I)-A)\B) + D ;
Gplus = Gp1;
wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
Wtfn = wf*I;                          %weighting function matrix
H =Gplus+Wtfn                         %Transfer function after adding the weighting
function
nu2(z)= -min(real(eig(H+H'))/2);      %passivity index of passive system
end

 %%Finding passivity index of passive system
 for z = 1:length(w)
 Gp1 = C*(((s(z)*I)-A)\B) + D ;
 Gplus = Gp1;
 wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
 Wtfn = wf*I;                          %weighting function matrix
 H =Gplus+Wtfn                         %Transfer function after adding the weighting
 function
 nu2(z)= -min(real(eig(H+H'))/2);      %passivity index of passive system
 ggreal = real(H);
 ggimag = imag(H);


  %size of new matrix
  [nAAr,nAAc] = size(ggreal);
  [nBBr,nBBc] = size(ggimag);
  setlmis([]);
  DE = lmivar(1,[ones(nAAr,2)]);
  lmiterm([1 1 1 0],eye(nAAr));         %LMI #1: eye(nAAr)
  lmiterm([-1 1 1 DE],1,1);             %LMI #1: DE
  lmiterm([1 1 1 DE],ggreal,-1,'s');    %LMI #2: -ggreal*DE-DE*ggreal'
  lmiterm([1 2 1 DE],1,ggimag');        %LMI #2: DE*ggimag'
  lmiterm([1 2 1 DE],ggimag,-1);        %LMI #2: -ggimag*DE
```

```
lmiterm([1 2 2 DE],ggreal,-1,'s');      %LMI #2: -ggreal*DE-DE*ggreal'
lmiterm([-1 1 1 DE],1,1);               %LMI #2: DE
lmiterm([-1 2 1 0],zeros(nAAr));        %LMI #2: zeros(nAAr)
lmiterm([-1 2 2 DE],1,1);               %LMI #2  DE
sys2 = getlmis;

%------ Generalized Eigenvalue Minimization of t #2
[tmin2,Xopt2] = gevp(sys2,1);
Dm2 = dec2mat(sys2,Xopt2,DE);
DNew2 = sqrt(Dm2);
DGp2 = inv(DNew2)*H*DNew2;
nu3(z) = -min(real(eig(DGp2+DGp2'))/2);
end

%%Plot passivity index of passive system
figure(3);
semilogx(w,nu3,'r-');
xlabel('Frequency (rad/hr)');
ylabel('Passivity index of passive system');
legend('');
grid;
```

B.6 Finding the PI-controller

%To find the parameters of PI-controller ($k^{+}_{c,i}$ and $\tau_{I,i}$), this file must be opened open with
myfunpi.m and mynonconpi.m file

%% The parameters for loop1 $\dot{m}_{H_2,in} - P_{H_2}$

```
for z =1:length(w)
gPI = C*((((w(z).*i).*I)-A)\B) + D;
wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
Wtfn = wf*I;
U = [1 0 0;0 1 0;0 0 1];
gPIplus = gPI.*U;
H =gPIplus+Wtfn ;
GLoop1=H(1,1);
options = optimset('Display','iter','MaxFunEvals',10000);
[p,fval] =fmincon(@myfunpi,[0.00005 0.001 20],[],[],[],[],[0.0000000001 0.001
19],[1500 0.001 20],@mynonconpi,options,nu3,GLoop1,w,z);
gamma1=[p(1)]
kcplus1=[p(2)]
ti1=[p(3)]
```

```
end
```

%% The parameters for loop2 $\dot{m}_{O_{2,in}} - P_{O_2}$

```
for z =1:length(w)
gPI = C*((((w(z).*i).*I)-A)\B) + D;
wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
% Wtfn = wf*I;
U = [1 0 0;0 1 0;0 0 1];
gPIplus = gPI.*U;
H =gPIplus+Wtfn ;  %Transfer function after adding the weighting function
GLoop1=H(2,2);
options = optimset('Display','iter','MaxFunEvals',10000);
[p,fval] =fmincon(@myfunpi,[0.00005 0.001 20],[],[],[],[],[0.0000000001 0.001
19],[1500 0.001 20],@mynonconpi,options,nu3,GLoop1,w,z);
gamma2=[p(1)]
kcplus2=[p(2)]
ti2=[p(3)]
end
```

%% The parameters for loop3 $\dot{m}_{cw} - T_s$

```
for z =1:length(w)
gPI = C*((((w(z).*i).*I)-A)\B) + D;
wf = (x(4).*(w(z).*i).*((w(z).*i)+x(1)))/(((w(z).*i)+x(2)).*((w(z).*i)+x(3)));
Wtfn = wf*I;
U = [1 0 0;0 1 0;0 0 1];
gPIplus = gPI.*U;
H =gPIplus+Wtfn ;
GLoop1=H(3,3);
options = optimset('Display','iter','MaxFunEvals',10000);
[p,fval] =fmincon(@myfunpi,[0.00005 0.011487 0.000907],[],[],[],[],[0.0000000001
0.011387 0.000907],[1500 0.011487
0.000917],@mynonconpi,options,nu3,GLoop1,w,z);
gamma3=[p(1)]
kcplus3=[p(2)]
ti3=[p(3)]
end
```

B.7 The added files for finding the weighting function

```
%%Myfun
function f = myfun(x,nu,w)
for z=1:length(w)
s = w(z)*i;
```

```
wp(z) = (x(4)*s*(s+x(1)))/((s+x(2))*(s+x(3)));
fnction(z) = (real(wp(z))-nu2(z))^2;
end
f = sum(fnction);
end

%%Mynoncon
function [c, ceq] = mynoncon(x,nu,w)
for z=1:length(w)
s = w(z)*i;
wp(z) = (x(4)*s*(s+x(1)))/((s+x(2))*(s+x(3)));
end
c = nu2-real(wp);        %inequality constraint
ceq=[];
end
```

B.8 The added files for finding the PI-controller

```
%%Myfunpi
function f = myfunpi(p,nu,GLoop1,w,z)
f= -1*p(1);
end

%%Mynonconpi
function [c, ceq] = mynonconpi(p,nu,GLoop1,w,z)
a = 1+GLoop1.*p(2).*(1+(1./(p(3).*w.*i)));
c(1)=max(abs(p(1)./(a.*w.*i)))-1;
c(2)=-min(real((p(2).*(1+(1./(p(3).*w.*i)))).(1-((p(2).*(1+(1./(p(3).*w.*i)))).*nu))));
ceq=[];
end
```

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King Mongkut's University of Technology Thonburi
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Date 27 April 2011

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