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APPENDIX A

The computer program is implemented using MATLAB for numerical experiment in Chapter 4.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Clear All Previous Temporary Variables                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Parameters Declaration                                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
alpha = 1; beta  = 1; gamma = 0.001; delta = 1; theta = 1;
M = 10; N = 2000;
LEFT_BOUNDARY  = 0; RIGHT_BOUNDARY = 1;
START_TIME = 0;        END_TIME    = 20;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Initialize Grid Values                                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
X = zeros(M+1,N+1);
T = zeros(M+1,N+1);

X(1,1)   = LEFT_BOUNDARY;
X(M+1,1) = RIGHT_BOUNDARY;
delx     = ( X(M+1,1) - X(1,1) )/M;

T(1,1)   = START_TIME;
T(1,N+1) = END_TIME;
delt     = ( T(1,N+1) - T(1,1) )/N;

for j=1:N+1
  for i=1:M+1
    X(i,j) = X(1,1) + (i-1)*delx;
    T(i,j) = T(1,1) + (j-1)*delt;
  end
```

```
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Initialize Variables a0, a1, mu and c                                       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
p1 = sqrt( (alpha^2) + (4*beta*(delta+1)) ); p2 = delta+1;

mu = delta*gamma*(p1-alpha)/(4*p2);
c  = ( (alpha-p1)*gamma + (alpha+p1)*(delta+1) )/(2*p2);

a0 = gamma/2;  a1 = gamma/2;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Begin Computation of Exact Solution                                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
U_EXC = zeros(M+1,N+1);
for j=1:N+1
  for i=1:M+1
    U_EXC(i,j) = ( a0 + a1*tanh(mu*( X(i,j) - c*T(i,j) ) ) ) ...
                    ^(1/delta);
  end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  End Computation of Exact Solution                                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Begin Computation of Method M1                                              %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A1 = zeros(M-1,M-1);
B1 = zeros(M-1,M-1);
C1 = zeros(M-1,1);

U_NUM1 = zeros(M+1,N+1);
for i=1:(M+1)
  U_NUM1(i,1) = ( a0 + a1*tanh(mu*X(i,1)) )^(1/delta);
end

p = delt/(delx^2); q = (alpha*delt)/(2*delx);
for j=1:N
```

```
lbound = ( a0 + a1*tanh(mu*(LEFT_BOUNDARY-c*T(1,j+1))) ) ...
            ^(1/delta);
rbound = ( a0 + a1*tanh(mu*(RIGHT_BOUNDARY-c*T(M+1,j+1))) ) ...
            ^(1/delta);

U_NUM1(1,j+1)   = lbound;
U_NUM1(M+1,j+1) = rbound;

for i=1:M-1
  A1(i,i) = 1 + (2*p*theta);
  B1(i,i) = 1 - (2*p*(1-theta)) + delt*beta* ...
              ( 1 - (U_NUM1(i+1,j)^delta) )* ...
              ( (U_NUM1(i+1,j)^delta) - gamma);
end

for i=1:M-2
  A1(i,i+1) = -(p*theta);
  A1(i+1,i) = -(p*theta);
  B1(i,i+1) = -(q*((U_NUM1(i+1,j)^delta))) + p*(1-theta);
  B1(i+1,i) = q*((U_NUM1(i+2,j)^delta)) + p*(1-theta);
end

C1(1)   = p*theta*U_NUM1(1,j+1) + ...
            ( (q*(U_NUM1(2,j)^delta)) + ...
          p*(1-theta) )*U_NUM1(1,j);
C1(M-1) = p*theta*U_NUM1(M+1,j+1) + ...
            ( -(q*(U_NUM1(M,j)^delta)) + ...
            p*(1-theta) )*U_NUM1(M+1,j);

 RESULT = inv(A1)*(B1*U_NUM1(2:M,j)+C1);

 U_NUM1(:,j+1) = [lbound; RESULT; rbound];
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  End Computation of Method M1                                              %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Begin Computation of Method M2                                            %
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A2 = zeros(M-1,M-1);
B2 = zeros(M-1,M-1);
C2 = zeros(M-1,1);

U_NUM2 = zeros(M+1,N+1);
for i=1:(M+1)
  U_NUM2(i,1) = ( a0 + a1*tanh(mu*X(i,1)) )^(1/delta);
end

p = delt/(delx^2); q = (alpha*delt)/(2*delx);
for j=1:N

   lbound = ( a0 + a1*tanh(mu*(LEFT_BOUNDARY-c*T(1,j+1))) ) ...
               ^(1/delta);
   rbound = ( a0 + a1*tanh(mu*(RIGHT_BOUNDARY-c*T(M+1,j+1))) ) ...
               ^(1/delta);

   U_NUM2(1,j+1)   = lbound;
   U_NUM2(M+1,j+1) = rbound;

   for i=1:M-1
      A2(i,i) = 1 + (2*p*theta) + ...
                  delt*beta*( gamma + (U_NUM2(i+1,j))^(2*delta) );
      B2(i,i) = 1 - (2*p*(1-theta)) + ...
                  delt*beta*( (U_NUM2(i+1,j)^delta) + ...
                  gamma*(U_NUM2(i+1,j)^delta) );
   end

   for i=1:M-2
      A2(i,i+1) = -(p*theta);
      A2(i+1,i) = -(p*theta);
      B2(i,i+1) = -(q*(U_NUM2(i+1,j)^delta)) + p*(1-theta);
      B2(i+1,i) = q*(U_NUM2(i+2,j)^delta) + p*(1-theta);
   end

   C2(1)   = p*theta*U_NUM2(1,j+1) + ...
                ( (q*(U_NUM2(2,j)^delta)) + ...
              p*(1-theta) )*U_NUM2(1,j);
   C2(M-1) = p*theta*U_NUM2(M+1,j+1) +  ...
```

```
                ( -(q*(U_NUM2(M,j)^delta)) + ...
                p*(1-theta) )*U_NUM2(M+1,j);

  RESULT = inv(A2)*(B2*U_NUM2(2:M,j)+C2);

  U_NUM2(:,j+1) = [lbound; RESULT; rbound];
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  End Computation of Method M2                                                  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Absolute Error Computation                                                    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ERROR1 = abs(U_EXC - U_NUM1);
ERROR2 = abs(U_EXC - U_NUM2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Relative Error Computation                                                    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ERROR_REL1 = zeros(M+1,N+1);
for i=1:M+1
  for j=1:N+1
    ERROR_REL1(i,j) = (ERROR1(i,j) * 100) / U_EXC(i,j);
  end
end

ERROR_REL2 = zeros(M+1,N+1);
for i=1:M+1
  for j=1:N+1
    ERROR_REL2(i,j) = (ERROR2(i,j) * 100) / U_EXC(i,j);
  end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  L2 Norm Computation                                                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L2_NORM1 = zeros(1,N+1);
for j=1:N+1
  NORM1 = 0;
```

```
  for i=1:M+1
    NORM1 = NORM1 + (ERROR1(i,j)^2);
  end
  L2_NORM1(1,j) = sqrt(delx * NORM1);
end

L2_NORM2 = zeros(1,N+1);
for j=1:N+1
  NORM2 = 0;
  for i=1:M+1
    NORM2 = NORM2 + (ERROR2(i,j)^2);
  end
  L2_NORM2(1,j) = sqrt(delx * NORM2);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  L Infinity Norm Computation                                                  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L_INF_NORM1 = zeros(1,N+1);
for j=1:N+1
  L_INF_NORM1(1,j) = max(ERROR1(:,j));
end

L_INF_NORM2 = zeros(1,N+1);
for j=1:N+1
  L_INF_NORM2(1,j) = max(ERROR2(:,j));
end
```

BIOGRAPHY

NAME	Mr. Moke Krisnangkura
DATE OF BIRTH	10 April 1984
EDUCATIONAL RECORD	
High school	Mathayom Watsing School, 2000
Bachelor's Degree	Bachelor of Engineering (Computer Engineering) King Mongkut's University of Technology Thonburi, 2005
Master's Degree	Master of Science (Applied Mathematics) King Mongkut's University of Technology Thonburi, 2010
PUBLICATION	Krisnangkura, M. and Chinviriyasit, W., "Traveling Wave Solutions of the Generalized Burgers-Huxley Equation", **International Conference in Mathematics and Applications (ICMA-MU 2009)**, 17-19 December 2009, Bangkok, pp. 203-207.

มหาวิทยาลัยเทคโนโลยีพระจอมเกล้าธนบุรี
ข้อตกลงว่าด้วยการโอนลิขสิทธิ์ในวิทยานิพนธ์

วันที่ ..10.. เดือน ..พ.ย.. พ.ศ. ..54..

ข้าพเจ้า (นาย / ~~นาง~~ / ~~นางสาว~~) โมกข์ กฤษณังกูร รหัสประจำตัว 50401405
เป็นนักศึกษาของมหาวิทยาลัยเทคโนโลยีพระจอมเกล้าธนบุรี ระดับปริญญา (/) โท ( ) เอก หลักสูตร วท.ม. / ~~ปร.ด.~~
สาขาวิชา...คณิตศาสตร์ประยุกต์... คณะวิทยาศาสตร์ อยู่บ้านเลขที่...8/9...ถนน ...วุฒากาศ.... ตำบล / แขวง ...บางค้อ... เขต ...จอมทอง...จังหวัด...กรุงเทพมหานคร ...รหัสไปรษณีย์ ...10150.... ขอโอนลิขสิทธิ์ในวิทยานิพนธ์ให้ไว้กับมหาวิทยาลัยเทคโนโลยีพระจอมเกล้าธนบุรี โดยมี ผศ.ดร.วรนุช เกิดสินธ์ชัย ตำแหน่ง คณบดีคณะวิทยาศาสตร์ เป็นผู้รับโอนลิขสิทธิ์และมีข้อตกลงดังนี้

1. ข้าพเจ้าได้จัดทำวิทยานิพนธ์เรื่อง...การศึกษาเชิงวิเคราะห์ของรูปทั่วไปของสมการเบอร์เกอร์-ฮัคเลย์ (Analytic Study of the Generalized Burgers–Huxley Equation)..... ซึ่งอยู่ในความควบคุมของ ผศ.ดร. วิราวรรณ ชินวิริยสิทธิ์ ตามมาตรา 14 แห่ง พ.ร.บ.ลิขสิทธิ์ พ.ศ. 2537 และถือว่าเป็นส่วนหนึ่งของการศึกษาตามหลักสูตรของมหาวิทยาลัยเทคโนโลยีพระจอมเกล้าธนบุรี

2. ข้าพเจ้าตกลงโอนลิขสิทธิ์จากผลงานทั้งหมดที่เกิดขึ้นจากการสร้างสรรค์ของข้าพเจ้าในวิทยานิพนธ์ให้กับมหาวิทยาลัยเทคโนโลยีพระจอมเกล้าธนบุรี ตลอดอายุแห่งการคุ้มครองลิขสิทธิ์ตามมาตรา 23 แห่งพระราชบัญญัติลิขสิทธิ์ พ.ศ. 2537 ตั้งแต่วันที่ได้รับอนุมัติโครงร่างวิทยานิพนธ์จากมหาวิทยาลัย

3. ในกรณีที่ข้าพเจ้าประสงค์จะนำวิทยานิพนธ์ไปใช้ในการเผยแพร่ในสื่อใดๆ ก็ตาม ข้าพเจ้าจะต้องระบุว่าวิทยานิพนธ์เป็นผลงานของมหาวิทยาลัยเทคโนโลยีพระจอมเกล้าธนบุรีทุก ๆ ครั้งที่มีการเผยแพร่

4. ในกรณีที่ข้าพเจ้าประสงค์จะนำวิทยานิพนธ์ไปเผยแพร่ หรืออนุญาตให้ผู้อื่นทำซ้ำ หรือดัดแปลง หรือเผยแพร่ต่อสาธารณชน หรือกระทำการอื่นใด ตามมาตรา 27, มาตรา 28 , มาตรา 29 และมาตรา 30 แห่งพระราชบัญญัติลิขสิทธิ์ พ.ศ.2537 โดยมีค่าตอบแทนในเชิงธุรกิจ ข้าพเจ้าจะกระทำได้เมื่อได้รับความยินยอมเป็นลายลักษณ์อักษรจากมหาวิทยาลัยเทคโนโลยีพระจอมเกล้าธนบุรี

ลงชื่อ..โมกข์ กฤษณังกูร.. ผู้โอนลิขสิทธิ์
(นายโมกข์ กฤษณังกูร)

ลงชื่อ ..... ผู้รับโอนลิขสิทธิ์
(ผศ.ดร.วรนุช เกิดสินธ์ชัย)

ลงชื่อ ..... พยาน
(ผศ.ดร.วิราวรรณ ชินวิริยสิทธิ์)

ลงชื่อ ..... พยาน
(ดร.คุษฎี ศุขวัฒน์)