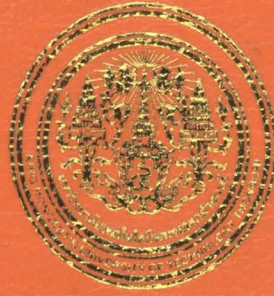


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E47231



ANALYTIC STUDY OF THE GENERALIZED BURGERS-HUXLEY EQUATION

MR. MOKE KRISNANGKURA

**A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF SCIENCE (APPLIED MATHEMATICS)
DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
KING MONCKUT'S UNIVERSITY OF TECHNOLOGY THONBURI**

2010

600254152

ห้องสมุดงานวิจัย สำนักงานคณะกรรมการวิจัยแห่งชาติ



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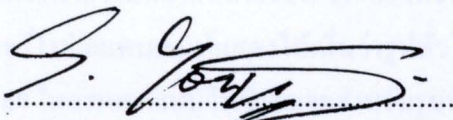
Faculty of Science

King Mongkut's University of Technology Thonburi

2010

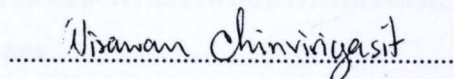
Thesis Committee





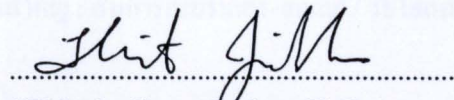
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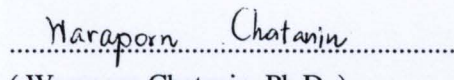
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หัวข้อวิทยานิพนธ์	การศึกษาเชิงวิเคราะห์ของรูปทั่วไปของสมการเบอร์เกอร์-ฮักเลย์
หน่วยกิต	12
ผู้เขียน	นายโมกซ์ กฤษณังกูร
อาจารย์ที่ปรึกษา	ผศ.ดร. วีราวรรณ ชินวิริยสิทธิ์
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พ.ศ.	2553

บทคัดย่อ

E 47231

วิทยานิพนธ์นี้ได้ศึกษารูปทั่วไปของสมการเบอร์เกอร์-ฮักเลย์ วิธีไฮเพอร์โบลิกแทนเจนต์เป็นวิธีที่ใช้ในการหาความหลากหลายของผลเฉลยแท้จริงสำหรับรูปทั่วไปของสมการเบอร์เกอร์-ฮักเลย์ วิธีไฮเพอร์โบลิกแทนเจนต์แสดงให้เห็นว่ารูปทั่วไปของสมการเบอร์เกอร์-ฮักเลย์มีผลเฉลยแท้จริงสี่ผลเฉลย ในการออกแบบวิธีเชิงตัวเลขสำหรับรูปทั่วไปของสมการเบอร์เกอร์-ฮักเลย์ได้สร้างวิธีเชิงตัวเลขโดยยึดตามหลักวิธีผลต่างอันดับ จากการจำลองเชิงตัวเลขพบว่า วิธีเชิงตัวเลขที่สร้างขึ้นมีประสิทธิภาพในแง่ของเสถียรภาพเชิงตัวเลขมากกว่าวิธีมาตรฐานภายใต้พารามิเตอร์ที่ใช้ในการจำลอง

คำสำคัญ : สมการเบอร์เกอร์-ฮักเลย์ / วิธีไฮเพอร์โบลิกแทนเจนต์ / วิธีผลต่างอันดับ

Thesis Title	Analytic Study of the Generalized Burgers–Huxley Equation
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Thesis Advisor	Asst. Prof. Dr. Wirawan Chinviriyasit
Program	Master of Science
Field of Study	Applied Mathematics
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Abstract

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In this thesis, the generalized Burgers–Huxley equation is studied. The hyperbolic tangent method is used to obtain a variety of the exact solutions of this equation. This method reveals that the generalized Burgers–Huxley equation has four exact solutions. To design the numerical method for this equation, a numerical method based on finite difference method is constructed. Numerical simulations reveal that the constructed method is more efficient in terms of numerical stability than the standard method based on the parameter values used in simulations.

Keywords : Burgers–Huxley Equation / Tanh Method / Finite Difference Method

ACKNOWLEDGEMENTS

This thesis would be incomplete without the invaluable contribution of many people. I would like to use this opportunity to express my gratitude to these people individually. First of all, I am very indebted to Asst.Prof.Dr. Wirawan Chinviriyasit for her patient, time, encouragement and guidance throughout this study as my supervisor. My successful completion of this study would not possible without her practical advice and support. I would like to thank for her kindness. I am fortunate and honoured to be her student. I am also very grateful to the committee members, Dr. Thiradet Jiarasuksakun and Dr. Waraporn Chatanin, for their time, valuable comments, patience and suggestions. Special thanks to Asst.Prof.Dr. Settapat Chinviriyasit for suggesting in the numerical part.

I appreciate all lecturers who taught me in the graduate course, all staffs and all friends at the Department of Mathematics, KMUTT for encouragement and suggestion. Finally, I also go to thanks my family for their love, encouraging and understanding throughout the duration of this thesis; this could not have been done without you.

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