CHAPTER 1 INTRODUCTION

The differential equations are very important in science and engineering whereas many phenomena and problems are modelled mathematically by ordinary differential equations (ODEs) or partial differential equations (PDEs). Especially, the nonlinear PDEs frequently appear in various scientific fields such as physics (fluid dynamics, solid state physics, plasma physics), chemistry (chemical kinetics), and mathematical biology (population dynamics) [1, 2].

Because of the interest in these problems, a variety of analytical methods (such as inverse scattering method, Hirota's bilinear technique, Bäcklund transformation and Painlevé analysis) were developed and used to obtain exact solutions [1–3]. Furthermore, the numerical simulations were used to handle those problems as well.

The tanh (hyperbolic tangent) method is one of the mathematical techniques which is a straightforward solution method to find analytic expression of traveling wave of nonlinear PDEs. Recently, much works has been focused on the applications of the method. The tanh technique was used to solve many nonlinear PDEs such as modified KdV (Korteweg–de Vries) equation, Burgers equation and Fisher equation [1–4].

The generalized Burgers–Huxley equation is one of the most famous nonlinear PDE that describes relation between diffusion, convection and reaction process. Although, the exact solutions of this problem were established by using the analytical methods such as relevant nonlinear transformations [5] and first integral method [6] and Exp–function (exponential function) method [7] as well as the numerical solutions by using the numerical methods such as differential quadrature method [8] and meshless method [9]. But the tanh method has not yet to obtain the analytic solution.

The purposes of this research are to study the tanh method and to employ this method for finding the analytic solutions of the generalized Burgers–Huxley equation. Moreover, the numerical methods are developed using finite-difference method to obtain the numerical solution of generalized Burgers–Huxley problem and compared with the analytic solution which is determined by the tanh method.

This thesis is organized as follows.

In the chapter 2, various preliminary definitions and theorems needed in the theoretical analysis and development of numerical methods in later chapter are given.

Chapter 3 deals with the analytic solutions of generalized Burgers–Huxley equation by using hyperbolic tangent method and the development of numerical methods based on finite-difference method.

Chapter 4 demonstrates that the developed methods which are capable of achieving high accuracy for the problems represented by the generalized Burgers–Huxley equation. To verify the effectiveness of the developed methods, the computed results are compared with the exact solution of generalized Burgers–Huxley equation. The computer program using MATLAB for numerical experiments is given in Appendix A.

Finally, chapter 5 gives the discussions and conclusions of this thesis.