CHAPTER IV
NUMERICAL EXPERIMENTS

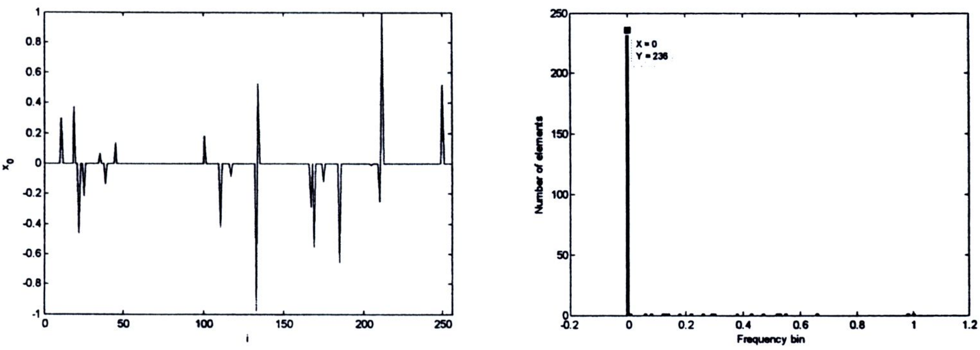
1. Experiment I: Sparse signal recovery

The purpose of this experiment is to demonstrate the HSR and AAR algorithms that recover $K-\text{sparse}$ signals and the increase percentage of reconstruction when comparing with un-weighted and reweighted ℓ_1-minimization for various the choices of fixed parameters $\varepsilon = 0.01,\ 0.1,\ 1,\ 10$ and 100. The analytical geometry in CHARTER II & III is in sparse representation domain s but this experiment considers the representations $K-\text{sparse}$ signals in time domain. We consider a sparse signal x_0 of lengths $N = 256$ with $\|x_0\|_0 = k$ which is randomly generated by a uniform Gaussian distribution on the interval [-1,1] such as Figure 20. A sample of measurement matrix Φ is i.i.d. Gaussian distribution of size $M \times N$ with $M = 100$ generated once for each $K-\text{sparse}$ signals, giving the data $y = \Phi x_0$. Let expanding rate $\mu = 0.01$ for HSR and AAR algorithms and taking 4 reweighted iterations with equality constraints for all reweighted algorithms. Finally, we run 500 trials for various fixed combination of k and classify the exact reconstructed signal which has the peak signal to noise ratio, PSNR ≥ 80 (the ratio between the maximum possible power of a signal and the power of corrupting noise as the formula (16))

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left|x_0 - x^{(\gamma)}\right|^2 \qquad PSNR = 20\log_{10}\left|\frac{\max(x_0)}{\sqrt{MSE}}\right| \tag{16}$$

where is $x^{(\gamma)}$ a reconstructed signal for iteration γ.

Furthermore, the optimization technique as simplex method in linear programming with duality property is applied to solve un-weighted and weighted ℓ_1-minimization problems and is explained in Appendix [9].



(a) (b)

Figure 20 Example of $K-$ sparse signal: (a) its signal form of length $N = 256$ with $k = 20$ and (b) its histogram of absolute signal vector $|x_0|$

Figure 21 shows the results of former reweighted ℓ_1-minimization (RW) when varying parameters $\varepsilon = 0.001,\ 0.01, 0.1, 1,$ and 10 which these can recover the exact $K-$ sparse signal outperforming ℓ_1-minimization by 0.20%, 4.45%, 13.68%, 10.31%, and 1.83%, respectively.

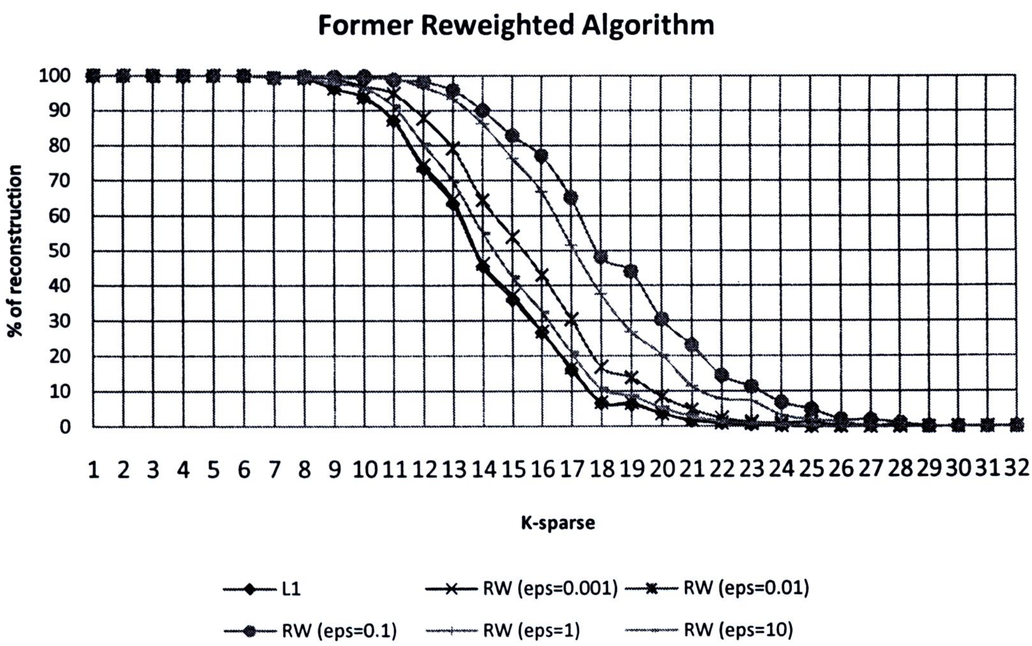


Figure 21 Comparisons of ℓ_1-minimization and former reweighted algorithm when varying parameters ε in compressive sampling recovery

Figure 22 shows the results of HSR algorithm when varying the threshold τ, 0.5τ, 1.5τ, 2τ, 2.5τ, and 3τ which these can recover the exact K – sparse signal outperforming ℓ_1-minimization by 12.66%, 4.90%, 17.19%, 18.31%, 17.50%, and 15.59%, respectively.

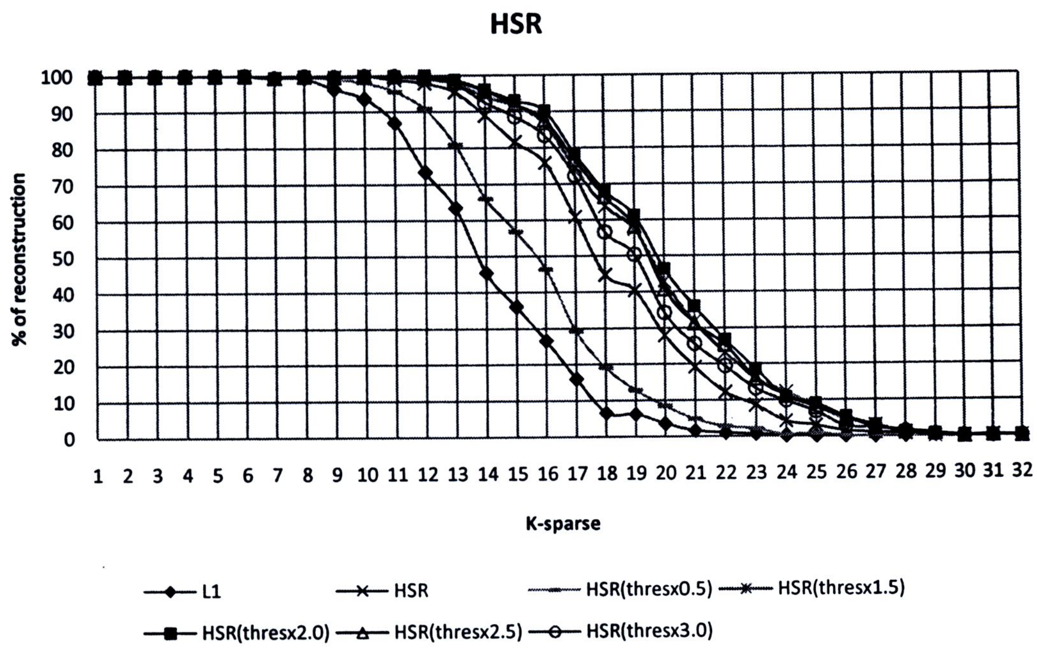


Figure 22 Comparisons of ℓ_1-minimization and HSR algorithm when varying thresholds τ in compressive sampling recovery

Figure 23 shows the results of AAR algorithm when varying the threshold τ, 0.5τ, 1.5τ, 2τ, 2.5τ, and 3τ which these can recover the exact K – sparse signal outperforming ℓ_1-minimization by 13.76%, 7.17%, 15.81%, 15.92%, 15.13%, and 14.11%, respectively.

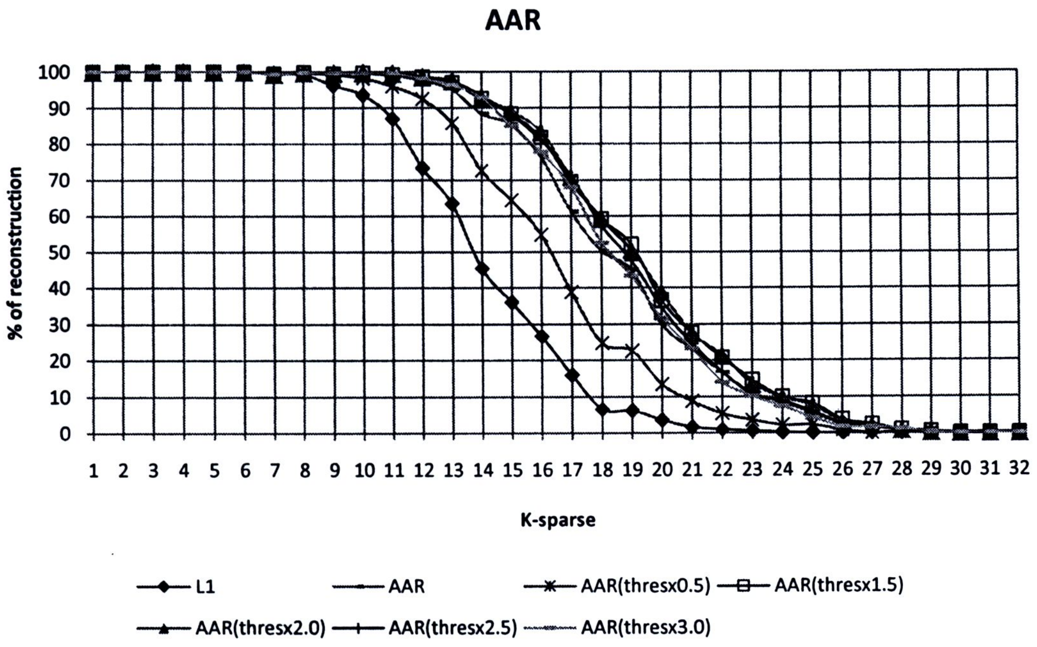


Figure 23 Comparisons of ℓ_1-minimization and AAR algorithm when varying thresholds τ in compressive sampling recovery

Finally, Figure 24 shows the comparison of former reweighted ℓ_1-minimization with parameter $\varepsilon = 0.1$, HSR algorithm with threshold 2τ, and ARR algorithm with threshold 2τ. These algorithms outperform ℓ_1-minimization by 13.68%, 18.31%, and 15.92%.

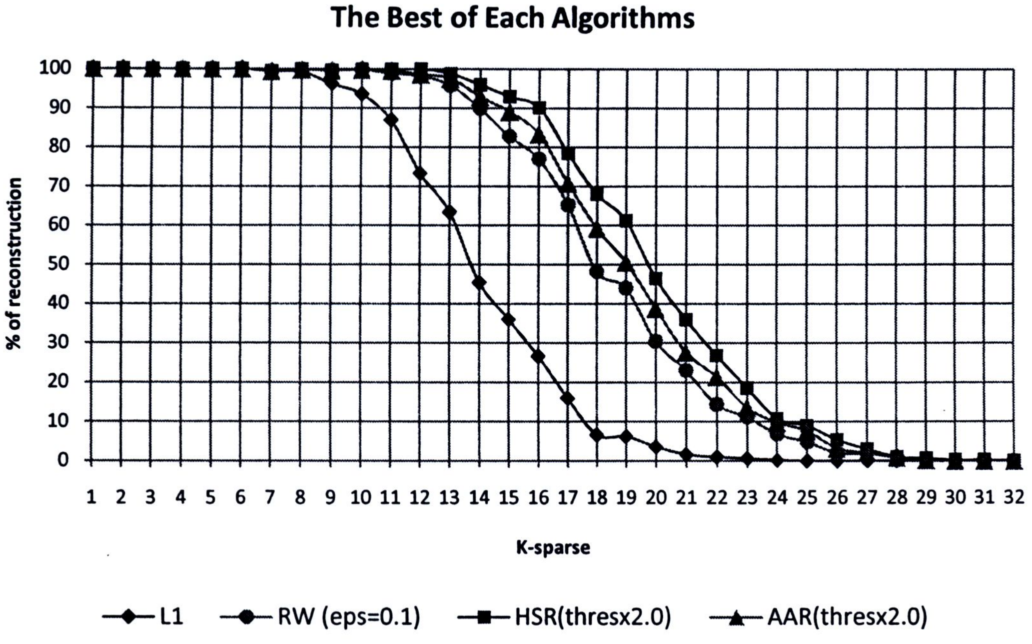


Figure 24 Comparisons of ℓ_1-minimization, former reweighted ℓ_1-minimization with parameter $\varepsilon = 0.1$, HSR algorithm with threshold 2τ, and ARR algorithm with threshold 2τ in compressive sampling recovery

2. Experiment II: Phantom and MRI images recovery

This experiment demonstrates the performance of reweighted recovery algorithms for 2-dimensinal signals: Phantom and MRI images. We use the Fourier-domain sampling pattern on modifying the standard basis pursuit problem $(P1)$ using a primal-dual algorithm from ℓ_1-MAGIC [10] to solve the weighted ℓ_1-minimization $(WP1)$.

Figure 25, 26, 27, 28, and 29 show Phantom, Brain MRI scan, Upside Brain MRI scan, Angiogram MRI scan, and Knee MRI scan images recovery in compressive sampling of size 256x256 with K – sparse 49.46%, 69.35%, 77.68%, 48.62% and, 60.93%, respectively. Define the Fourier sampling mask 44 lines which its compression ratio is reduced to 16.35% approximately and set reweighted iteration at once for all reweighted algorithms.

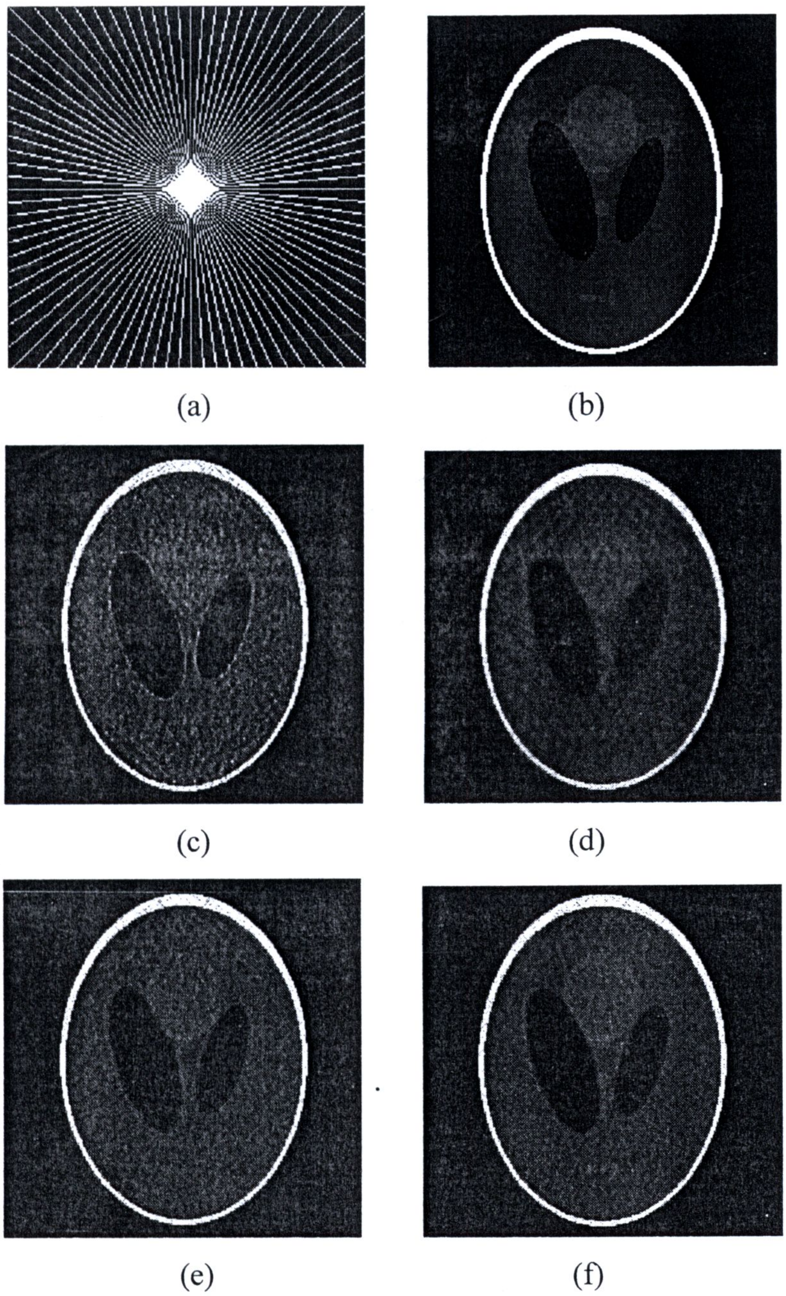


Figure 25 Phantom image recovery: (a) Fourier sampling pattern. (b) Original Phantom image. Its reconstructions via (c) ℓ_1-minimization, PSNR=50.86 dB, (d) former reweighted ℓ_1-minimization with parameter $\varepsilon = 0.1$, PSNR=57.81 dB, (e) HSR algorithm with threshold 2τ, PSNR=60.01 dB, and (f) ARR algorithm with threshold 2τ, PSNR=60.01 dB.

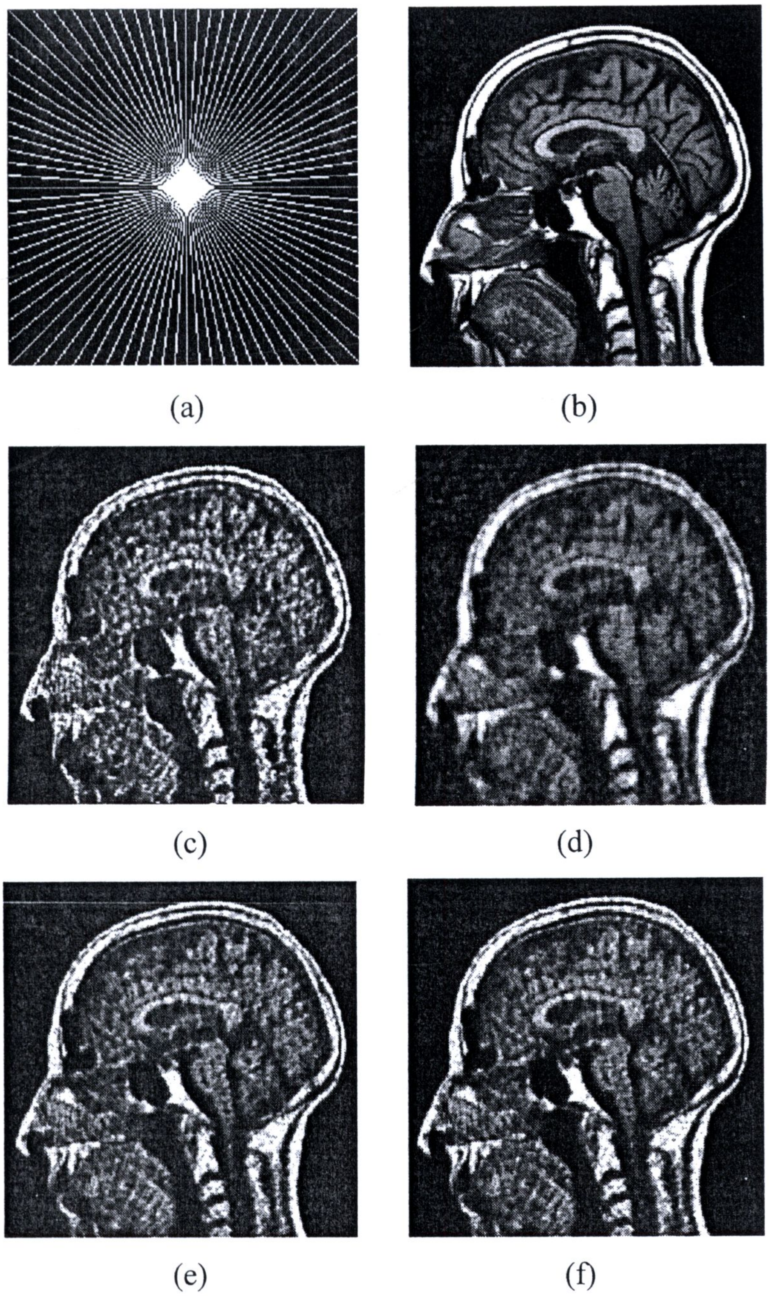


Figure 26 Brain MRI scan recovery: (a) Fourier sampling pattern. (b) Original Brain MRI. Its reconstructions via (c) ℓ_1-minimization, PSNR=47.36 dB, (d) former reweighted ℓ_1-minimization with parameter $\varepsilon = 0.1$, PSNR=60.66 dB, (e) HSR algorithm with threshold 2τ, PSNR=62.96 dB, and (f) ARR algorithm with threshold 2τ, PSNR=62.96 dB.

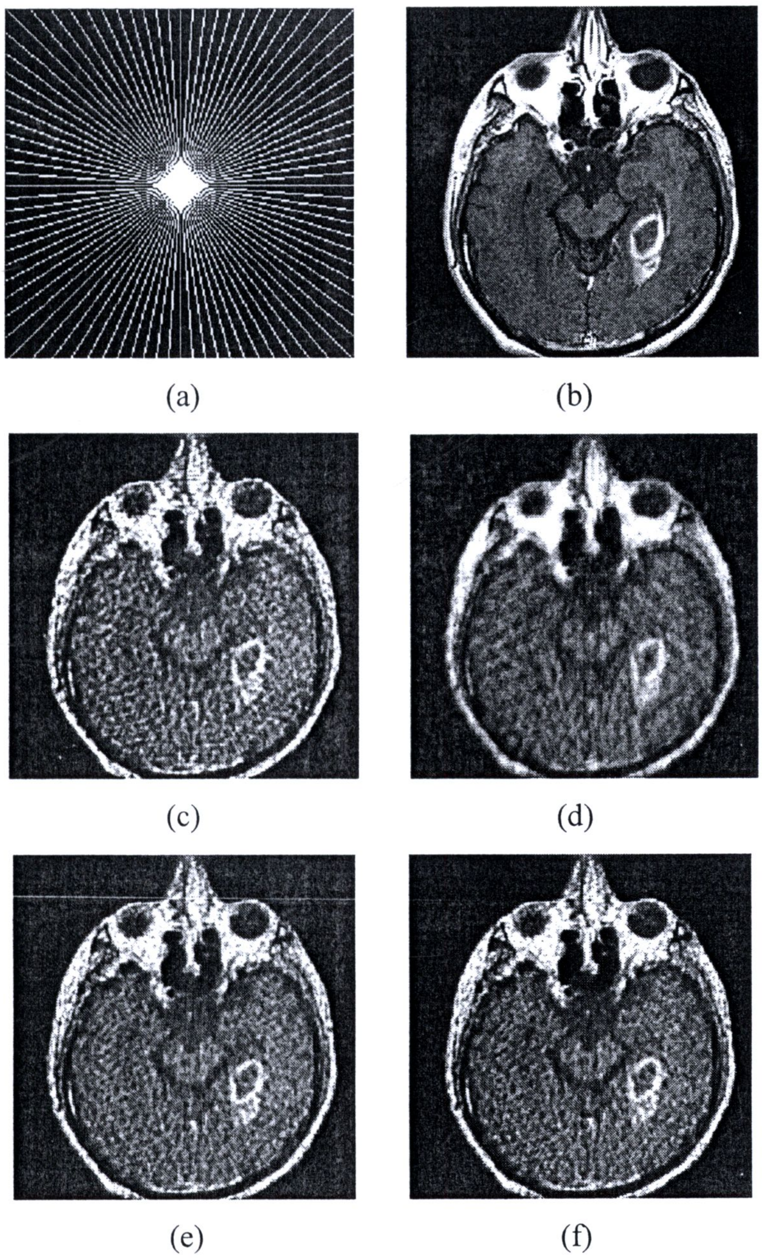


Figure 27 Upside brain MRI scan recovery: (a) Fourier sampling pattern. (b) Original Upside brain MRI. Its reconstructions via (c) ℓ_1-minimization, PSNR=42.79 dB, (d) former reweighted ℓ_1-minimization with parameter $\varepsilon = 0.1$, PSNR=46.28 dB, (e) HSR algorithm with threshold 2τ, PSNR=47.27 dB, and (f) ARR algorithm with threshold 2τ, PSNR=47.27 dB.

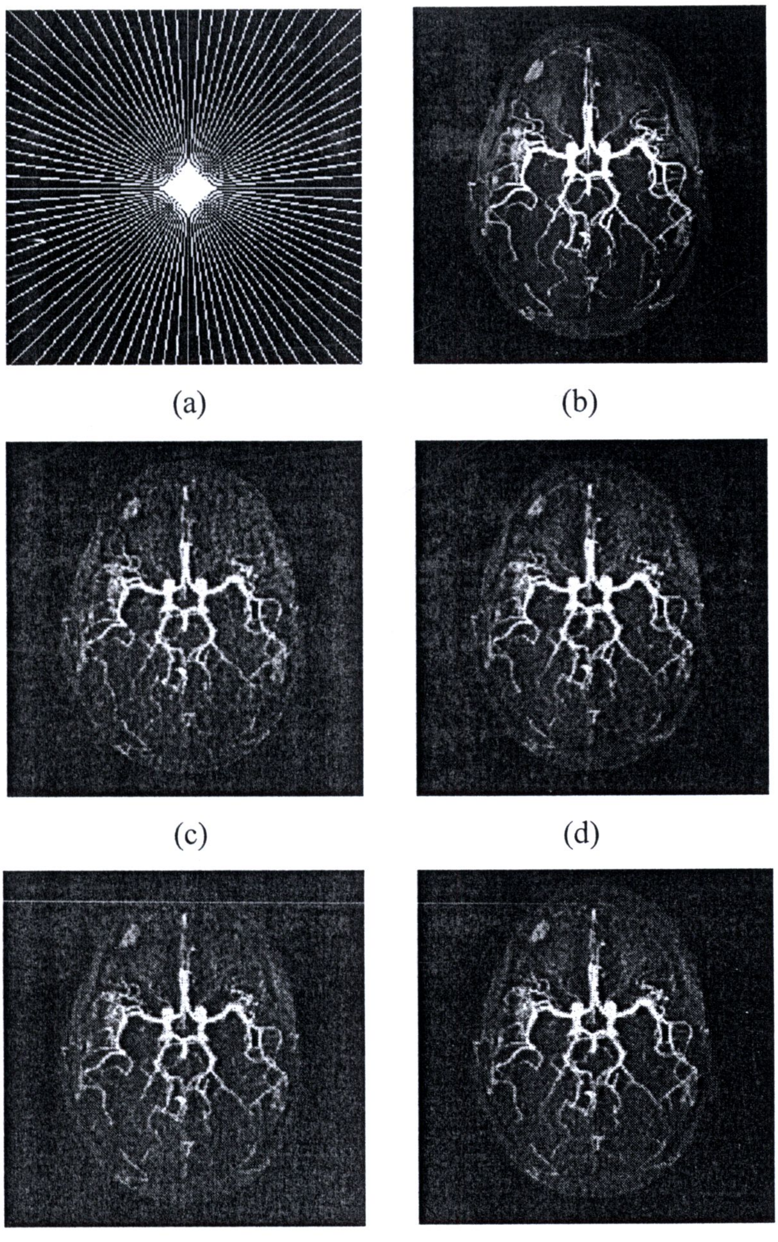

(a) (b) (c) (d) (e) (f)

Figure 28 Angiogram MR scan recovery: (a) Fourier sampling pattern. (b) Original Angiogram MR. Its reconstructions via (c) ℓ_1-minimization, PSNR=59.29 dB, (d) former reweighted ℓ_1-minimization with parameter $\varepsilon = 0.1$, PSNR=62.42 dB, (e) HSR algorithm with threshold 2τ, PSNR=62.42 dB, and (f) ARR algorithm with threshold 2τ, PSNR=62.42 dB.

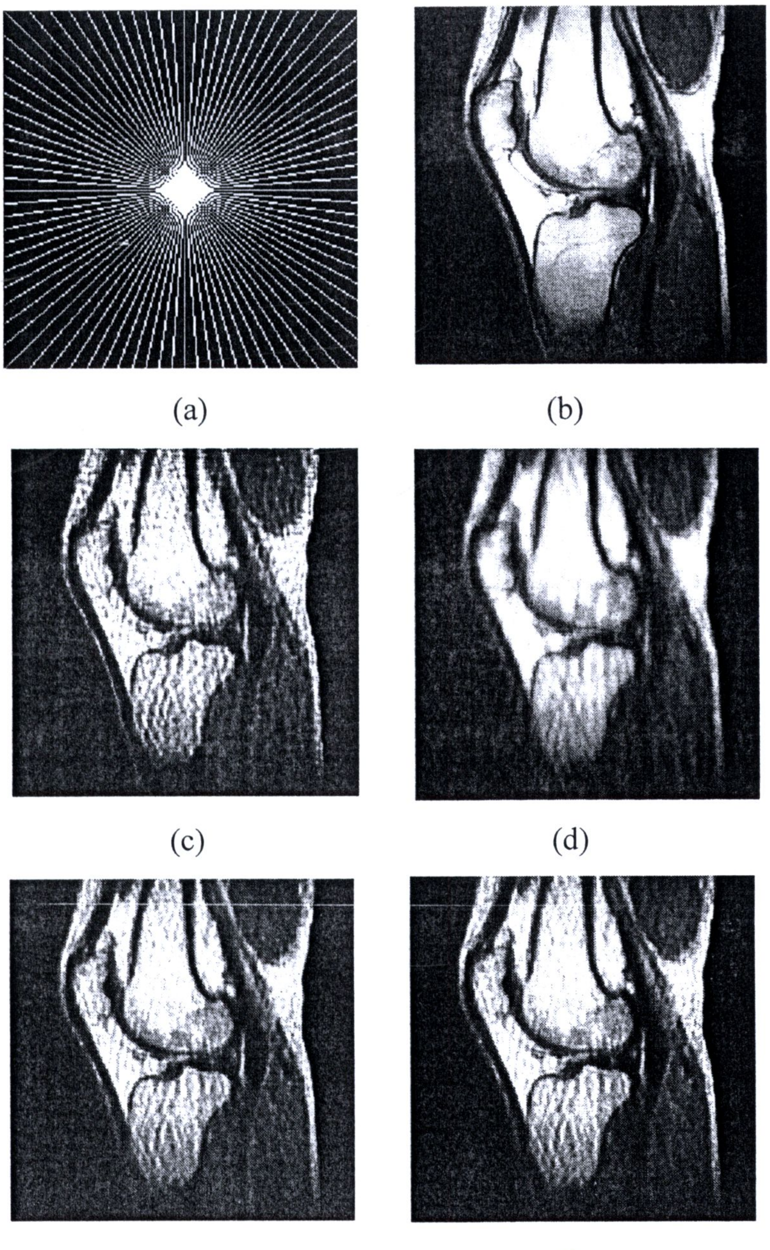


Figure 29 Knee MRI scan recovery: (a) Fourier sampling pattern. (b) Original Knee MRI scan. Its reconstructions via (c) ℓ_1-minimization, PSNR=46.46 dB, (d) former reweighted ℓ_1-minimization with parameter $\varepsilon = 0.1$, PSNR=51.82 dB, (e) HSR algorithm with threshold 2τ, PSNR=52.27 dB, and (f) ARR algorithm with threshold 2τ, PSNR=52.27 dB.