REFERENCES

[1] L. Aggoun and R. J. Elliod. **Measure theory and filtering: Introduction and applications**. New York: Cambridge University Press; 2004.

[2] T. M. Apostol. **Mathematical analysis**. (2nd ed.). Reading, MA: Addison-Wesley; 1974.

[3] Z. Brzeźniak and T. Zastawniak. **Basic stochastic processes**. London: Springer-Verlag; 1999.

[4] M. Capiński and E. Kopp. **Measure integral and probability**. London: Springer-Verlag; 2004.

[5] W. S. Chan and L. Zhang. Direct derivation of finite-time ruin probabilities in the discrete risk model with exponential or geometric claims. **North American Actuarial Journal** 2006; 10(4): 269-79.

[6] R. Durrett. **Probability : Theory and example.** U.S.A.: Brooks/Cole; 1991.

[7] D. C. M. Dickson. **Insurance risk and ruin**. New York: Cambridge University Press; 2005.

[8] E. Kreyszig. **Introductory functional analysis with applications**. USA: John Wiley & Sons.; 1978.

[9] S. Li. On a class of discrete time renewal risk models. **Scandinavian Actuarial Journal** 2005; 4: 241-260.

[10] S. Li. Distributions of the surplus before ruin, the deficit at ruin and the claim causing ruin in a class of discrete time risk model. **Scandinavian Actuarial Journal** 2005; 4: 271-284.

[11] M. Mikosch. **Non-life insurance mathematics: An introduction with the Possion process**. London: Springer-Verlag; 2004.

[12] K. P. Pavlova G. E. and Willmot. The discrete stationary renewal risk model and the Gerber-Shiu discounted penalty function. **Insurance: Mathematics and Economics** 2004; 35(2): 267-77.

[13] R. G. Laha and V. K. Rohatgi. **Probability Theory.** NewYork: John Wiley & son; 1979.

[14] W. Klongdee, P. Sattayatham and K. Sangaroon. A value function of discrete-time surplus process in insurance under investment and reinsurance credit risk. **Far East Journal of Theoretical Statistics** 2010; 32(2): 183–198.

APPENDICES

APPENDIX A

Notations

$(\Omega, \mathcal{F}, \mathrm{P})$	Probability space
Ω	Outcome space
$\mathcal{F}$	σ-field
P	Probability measure
$\sigma(X)$	σ-field generated by random variable X
$(X \in B)$	$\{\omega \in \Omega : X(\omega) \in B\}$
$(X \leq x)$	$\{\omega \in \Omega : X(\omega) \leq x\}$
$\mathrm{E}[X]$	Expectation of the random variable X
I_A	Indicator of set A
$\mathbb{N}$	Set of positive integers
$\mathbb{R}$	Real line
max	Maximum
min	Minimum

APPENDIX B

Probability Theory

We recall some definition and theorem in probability theory. Most of these results can be found in Brzeźniak and Zastawniak (1999), Capiński and Kopp (2004), and Aggoun and Elliott (2004).

Definition B.1 Let Ω be a non-empty set. A *σ-field* $\mathcal{F}$ on Ω is a family of a subsets of Ω such that

1. the empty set $\emptyset$ belong to $\mathcal{F}$;
2. if A belong to $\mathcal{F}$, then so does the complement $\Omega \setminus A$;
3. if $A_1, A_2, \cdots$ is a sequence of sets in $\mathcal{F}$, then their union $A_1 \cup A_2 \cup \cdots$ also belong to $\mathcal{F}$.

Definition B.2 Let $\mathcal{F}$ be a σ-field on Ω. A *probability measure* P is a function

$$\mathrm{P} : \mathcal{F} \to [0, 1]$$

such that

1. $\mathrm{P}(\Omega) = 1$;
2. if $A_1, A_2, \cdots$ are pairwise disjoint set (that is, $A_i \cap A_j = \emptyset$ for $i \neq j$) belong to $\mathcal{F}$, then

$$\mathrm{P}(A_1 \cup A_2 \cup \cdots) = \mathrm{P}(A_1) + \mathrm{P}(A_2) + \cdots .$$

The triple $(\Omega, \mathcal{F}, \mathrm{P})$ is called a *probability space.* The sets belonging to $\mathcal{F}$ is called *events.* An event A is said to occur *almost surely* (a.s.) whenever $\mathrm{P}(A) = 1$.

Definition B.3 If $\mathcal{F}$ is a σ-field on Ω, then a function $X : \Omega \to \mathbb{R}$ is said to be *$\mathcal{F}$-measurable* if

$$(X \in B) := \{\omega \in \Omega : X(\omega) \in B\} = X^{-1}(B) \in \mathcal{F}$$

for every Borel set $B \in \mathcal{B}(\mathbb{R})$. If $(\Omega, \mathcal{F}, \mathrm{P})$ is a probability space, then such a function X is called a *random variable.*

Definition B.4 The σ-field $\sigma(X)$ *generated* by a random variable $X : \Omega \to \mathbb{R}$ consists of all sets of the form $(X \in B)$, where B is a Borel set in $\mathbb{R}$.

Lemma B.1 (Doob-Dynkin) Let X be a random variable. Then each $\sigma(X)$-measurable random variable Y can be written as

$$Y = f(X)$$

for some Borel function $f : \mathbb{R} \to \mathbb{R}$.

Definition B.5 Every random variable $X : \Omega \to \mathbb{R}$ gives rise to a probability measure

$$\mathrm{P}_X(B) = \mathrm{P}(X \in B)$$

on $\mathbb{R}$ defined on the σ-field of Berel sets $B \in \mathcal{B}(\mathbb{R})$. We call P_X the distribution of X. The function $F_X : \mathbb{R} \to [0, 1]$ defined by

$$F_X(x) = \mathrm{P}(X \leq x)$$

is called the *distribution function* of X.

Definition B.6 If there is a Borel function $f_X : \mathbb{R} \to \mathbb{R}$ such that for any Borel set $B \subset \mathbb{R}$

$$\mathrm{P}(X \in B) = \int_B f_X(x)dx$$

then X is said to be a random variable with *absolutely continuous distribution* and f_X is called *density* of X. If there is a (finite or infinite) sequence of pairwise distinct real numbers $x_1, x_2, \cdots$ such that for any Borel set $B \subset \mathbb{R}$

$$\mathrm{P}(X \in B) = \sum_{x_i \in B} \mathrm{P}(X = x_i),$$

then X is said to have *discrete distribution* with value $x_1, x_2, \cdots$ and *mass* $\mathrm{P}(X = x_i)$ at x_i.

Definition B.7 A random variable $X : \Omega \to \mathbb{R}$ is said to be *integrable* if

$$\int_\Omega |X| d\mathrm{P} < \infty.$$

Then

$$\mathrm{E}[X] := \int_\Omega X d\mathrm{P}$$

exist and is called the *expectation* of X.

Definition B.8 Two events $A, B \in \mathcal{F}$ are called *independent* if

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B).$$

In general, we say that n events $A_1, A_2, \cdots, A_n \in \mathcal{F}$ are *independent* if

$$\mathrm{P}(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = \mathrm{P}(A_{i_1})\mathrm{P}(A_{i_2}) \cdots \mathrm{P}(A_{i_k})$$

for any indices $1 \leq i_1 < i_2 < \cdots < t_k \leq n$.

Definition B.9 Two random variable X and Y are called *independent* if for any Borel sets $A, B \in \mathcal{B}(\mathbb{R})$ the two events

$$(X \in A) \text{ and } (Y \in B)$$

are independent. We say that n random variable $X_1, X_2, \cdots, X_n$ are *independent* if for any Borel sets $B_1, B_2, \cdots, B_n \in \mathcal{B}(\mathbb{R})$ the events

$$(X_1 \in B_1),\ (X_2 \in B_2), \cdots, (X_n \in B_n)$$

are independent.

Definition B.10 Two σ-fields $\mathcal{G}$ and $\mathcal{H}$ contain in $\mathcal{F}$ are called *independent* if any two events $A \in \mathcal{G}$ and $B \in \mathcal{H}$ are independent. Similarly, any finite number of σ-fields $\mathcal{G}_1, \mathcal{G}_2, \cdots, \mathcal{G}_n$ contained in $\mathcal{F}$ are *independent* if any n events

$$A_1 \in \mathcal{G}_1, A_2 \in \mathcal{G}_2, \cdots, A_n \in \mathcal{G}_n$$

are independent.

Definition B.11 We say that a random variable X is *independent* of σ-field $\mathcal{G}$ if the σ-fields $\sigma(X)$ and $\mathcal{G}$ are independent.

Definition B.12 A *Stochastic process* is a family of random variable $X(t)$ parametrized by $t \in T$, where $T \subset \mathbb{N}$. When $T = \mathbb{N}$, we shall say that $X(t)$ is a stochastic process in *discrete time* (i.e., a sequence of random variable). When T is an interval in $\mathbb{R}$ (typically $T = [0, \infty)$), we shall say that $X(t)$ is a stochastic process in *continuous time.*

Theorem B.2 (Lebesgue's Dominated Convergence Theorem) Suppose $\{X_n, n \in \mathbb{N}\}$ is a sequence of random variables such that $|X_n| \leq Y$ a.s. where Y is an integrable random variable. If X_n converses to X a.s., then X_n and X are integrable,

$$\lim_{n\to\infty} \int_\Omega X_n d\mathrm{P} = \lim_{n\to\infty} \int_\Omega X d\mathrm{P}$$

and

$$\lim_{n\to\infty} \int_\Omega |X_n - X| d\mathrm{P} = 0.$$

Theorem B.3 Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space. Given a random variable $X : \Omega \to \mathbb{R}$,

$$\int_\Omega g(X(\omega)) d\mathrm{P}(\omega) = \int_\mathbb{R} g(x) d\mathrm{P}_X(x).$$

Theorem B.4 If P_X defined on $\mathbb{R}^n$ is absolutely continuous with density f_X, $g : \mathbb{R}^n \to \mathbb{R}$ is integrable with respect to P_X, then

$$\int_{\mathbb{R}^n} g(x) d\mathrm{P}_X(x) = \int_{\mathbb{R}^n} f_X(x) g(x) dx.$$

Corollary B.5 In the situation of the previous theorem we have

$$\int_\Omega g(X) d\mathrm{P} = \int_{\mathbb{R}^n} f_X(x) g(x) dx.$$

Theorem B.6 Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space. Let X be a real random variable and B a Borel set. Then

$$\int_B g(x) dF_X(x) = \int_{X^{-1}(B)} g(X(\omega)) d\mathrm{P}(\omega).$$

Here g is a Borel function and where $B = \mathbb{R}$

$$\int_\mathbb{R} g(x) dF_X(x) = \int_\Omega g(X(\omega)) d\mathrm{P}(\omega).$$

Proposition B.7 Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space.

(i) F_X is non-decreasing ($y_1 \leq y_2$ implies $F_X(y_1) \leq F_X(y_2)$);

(ii) $\lim\limits_{y\to\infty} F_X(y) = 1, \ \lim\limits_{y\to-\infty} F_X(y) = 0$;

(iii) F_X is right continuous (if $y \to y_0, \ y \geq y_0$, then $F_X(y) \to F_X(y_0)$).

APPENDIX C

Computer Programs

This appendix contains a copy of the program written in **MatLab** to implement the approximation in chapter III.

Here we choose parameters; $u = 0$, $p = 0.5$, $\lambda = 1.0$ and $\theta = 0.1$ (i.e., $c_0 = 1.1$). In the following code, u, c_0 and λ is represented by s, c and lambda, respectively.

——————— Table 3.1 in Chapter III ———————

```
clc; clear;
% Set the parameters
s=0; p=0.5; c=1.1;  lambda=1;
R=[ ];

% Ruin probabilities of the first and second times
R(1) = p*exp(-lambda*(s+c));
R(2) = p*exp(-lambda*(s+2*c))*(exp(lambda*c)+(1-p)+lambda*p*(s+c));
syms u
f = exp(lambda*c)+(1-p)+lambda*p*(u+c)
% Ruin probabilities for n = 3 to 82
for n = 3:82
        f0 = f;
        f00 = (1-p)*subs(f0,u,'u+c') + lambda*p*int(f0,u,[0],[u+c]);
        f = exp((n-1)*lamda*c) + f00;
        g = p*exp(-lambda*(u+c)) + p*exp(-lambda*n*c-lambda*u)*f00;
        T = subs(g,u,'0');
        T = subs(T);
        n
        R(n) = T
end
```

VITAE

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