CHAPTER IV

CONCLUSIONS

This thesis is devoted to study of the two discrete-time surplus process. The results obtained are separated into two parts.

In the first part, we study the discrete-time surplus process

$$U_n \quad = \quad u \ + \ c_0 n \ - \ \sum_{i=1}^{n} X_i K_i \ , \qquad n = 1, 2, 3, \cdots$$

where $U_0 = u \geq 0$ is the initial capital, $c_0 > 0$ is the premium rate. Let $\{X_n K_n,\ n \in \mathbb{N}\}$ be a mutually independent and identically distributed (i.i.d.) claim size process, $\{X_n, n \in \mathbb{N}\}$ be an positive i.i.d. process and $\{K_n, n \in \mathbb{N}\}$ be an i.i.d. process with bernoulli distribution with $\mathrm{P}(K_1 = 1) = p = 1 - \mathrm{P}(K_1 = 0)$. Let $\varphi_n(u)$ denoted the survival probability at time n is defined by

$$\varphi_n(u) \ := \ \mathrm{P}(U_1 \geq 0, U_2 \geq 0, U_3 \geq 0, \cdots, U_n \geq 0 | U_0 = u)$$

and $\Phi_n(u)$ denoted the ruin probability at one of times $1, 2, 3, \cdots, n$ is defined by

$$\begin{aligned} \Phi_n(u) &= 1 - \varphi_n(u) \\ &= \mathrm{P}(U_i < 0 \text{ for some } i \in \{1, 2, 3, \cdots, n\} | U_0 = u). \end{aligned}$$

We summarize our results as follows:

Let $c_0 > 0, u \geq 0$ and $N \in \mathbb{N}$ be given.

1. The survival probability at time N satisfies the following equation

$$\varphi_N(u) \ = \ (1-p)\varphi_{N-1}(u + c_0) + p \int_{-\infty}^{u+c_0} \varphi_{N-1}(u + c_0 - x) dF_{X_1}(x)$$

   where $\varphi_0(u) = 1$.

2. The ruin probability at one of the times $1, 2, 3, \cdots, N$ satisfies the following equation

$$\Phi_N(u) \ = \ \Phi_1(u) + (1-p)\Phi_{N-1}(u+c_0) + p \int_{-\infty}^{u+c_0} \Phi_{N-1}(u+c_0-x) dF_{X_1}(x)$$

   where $\Phi_0(u) = 0$.

3. If $\{X_n, n \in \mathbb{N}\}$ be an i.i.d. process has exponential distributed with intensity $\lambda > 0$, i.e., X_1 has the probability density function

$$f_{X_1}(x) = \lambda e^{-\lambda x}.$$

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Then the conclusion of 2. is in the following form

$$\Phi_n(u) = \Delta_{n-1}(u) \cdot pe^{-\lambda(u+nc_0)}$$

where

$$\Delta_0(u) = 1, \ \Delta_n(u) = e^{n\lambda c_0} + (1-p)\Delta_{n-1} + \lambda p \int_{-\infty}^{u+c_0} \Delta_{n-1}(x)dx$$

and premium rate $c_0 > \mathrm{E}[X_1] = 1/\lambda$.

If we choose $\mathrm{P}(K_1 = 1) = 1$, then this surplus process becomes to the discrete-time risk process in Chan and Zhang (2006)[5]. The surplus we studied reflects the fact in the sense of time that the claim time more flexibility than the study of Chan and Zhang. This means, thesis studied at time n, the claim size X_n may be occur or not while the claim size must occur at time n in the study of Chan and Zhang.

In the second part, we study the discrete-time surplus process

$$V_0 = u, \quad V_n = V_{n-1} + c_0 Z_n - X_n, \quad n = 1, 2, 3, \cdots$$

where V_n is the surplus at time n, $c_0 > 0$ is the constant premium rate and $\{X_n, n \in \mathbb{N}\}$ is an i.i.d. claim size process and $\{Z_n, n \in \mathbb{N}\}$ is an i.i.d. interarrival time process taking values in $N = \{0, 1, 2, \cdots\}$. Let $\tilde{\varphi}_n(u)$ denoted the survival probability at time n is defined by

$$\tilde{\varphi}_n(u) := \mathrm{P}(V_1 \geq 0, V_2 \geq 0, V_3 \geq 0, \cdots, V_n \geq 0 | V_0 = u)$$

and $\tilde{\Phi}_n(u)$ denoted the ruin probability at one of times $1, 2, 3, \cdots, n$ is defined by

$$\begin{aligned}\tilde{\Phi}_n(u) &= 1 - \tilde{\varphi}_n(u) \\ &= \mathrm{P}(V_i < 0 \text{ for some } i \in \{1, 2, 3, \cdots, n\} | V_0 = u).\end{aligned}$$

We consider the first claim time N of the claim times defined by

$$N = \min\{n \geq 1 | K_n = 1\}.$$

If $\{X_n, n \in \mathbb{N}\}$ has exponential distributed, then we have

$$\tilde{\Phi}_1(u) = pe^{-\lambda(u+c_0)} \cdot \frac{1}{1 - (1-p)e^{-\lambda c_0}}.$$

The object of main interest from the first surplus process is the ruin probability at one of unit time while the object from the second surplus process is the ruin probability at one of the claim times.