CHAPTER I
INTRODUCTION

The general goal of mathematical finance and risk management is to mathematically quantify the behavior of financial instruments today and under different possible environments in the future. This implies that we have some mathematical or empirical procedure of determining values under various circumstances. While the road is long, and while there has been substantial progress, for many reasons, this goal is only partially achievable in the end and must be tempered with good judgment, especially in the case of problematic and rare extreme events, which are difficult to characterize, where most of risk lies.

This chapter provides the background information on which this work is built, and reviews related work from the literature. In Section 1.1, we review concepts of the basic model. In Section 2.3, we review basic results from the literature, including mention to the discrete-time risk process that we are especially interested.

1.1 The Basic Model

In 1903, the Swedish actuary Philip Lundberg laid the foundations of modern risk theory. *Risk Theory* is a synonym for non-life insurance mathematics, which deals with the modeling of claims that arrive in an insurance business and which gives advice on how much premium has to be charged in order to avoid bankruptcy (ruin) of the insurance company.

One of Lundberg's main contributions is the introduction of a simple model which is capable of describing the basic dynamics of a homogeneous insurance portfolio. By this we mean a portfolio of contracts or policies for similar risks such as car insurance for a particular kind of car, insurance against theft in households or insurance against water damage of one-family homes.

There are three assumptions in the model:

1. Claims happen at the times T_i satisfying $0 = T_0 \le T_1 \le T_2 \le \cdots$. We call them *claim arrivals* or *claim times* or *claim arrival times* or, simply, *arrivals*.

2. The i-th claim arriving at time T_i causes the *claim size* or *claim severity* X_i . The sequence (X_i) constitutes an independent identically distribution (i.i.d.) sequence of non-negative random variables.

3. The claim size process (X_i) and the claim arrival process (T_i) are *mutually independent.*

The i.i.d property of the claim sizes, X_i, reflects the fact that there is a homogeneous probabilistic structure in the portfolio. The assumption that claim sizes and claim times be independent is very natural from an intuitive point of view. But the independence of claim sizes and claim arrivals also makes the life of the mathematician much easier, i.e., this assumption is made for mathematical convenience and tractability of the model.

Now we can define the *claim number process*

$$N(t) \quad = \quad \#\{\, i \ge 1 \;:\; T_i \le t \,\}, \qquad t \ge 0,$$

i.e., $N \;=\; (N(t))_{t\ge 0}$ is a counting process on $[0,\infty)$: $N(t)$ is the number of claims which occurred by time t.

The object of main interest from the point of view of an insurance company is the *total claim amount process* or *aggregate claim amount process* which is represented by

$$S(t) \quad = \quad \sum_{i=1}^{N(t)} X_i \quad = \quad \sum_{i=1}^{\infty} X_i I_{[0,t]} T_i \;, \qquad t \ge 0.$$

The process $S = (S(t))_{t\ge 0}$ is a random partial sum process which refers to the fact that the deterministic index n of the partial sums $S_n \;=\; X_1+X_2+\cdots+X_n$ is replaced by the random variables $N(t)$:

$$S(t) \quad = \quad X_1 + X_2 + \cdots + X_n \quad = \quad S_{N(t)} \;, \qquad t \ge 0.$$

It is often called *a compound (sum) process.* The total claim amount process S shares various properties with the partial sum process. For example, asymptotic properties such as the central limit theorem and the strong law of large numbers are analogous for the two processes.

One would like to solve the following problems by means of insurance mathematical method:

(1). Find sufficiently realistic, but simple, probabilistic models for S and N. This means that we have to specify the distribution of the claim sizes X_i and to introduce models for the claim arrival times T_i.

(2). Determine the theoretical properties of the stochastic processes S and N.

(3). Give simulation procedures for the process N and S.

(4). Based on the theoretical properties of N and S, give advice how to choose a premium in order to cover the claims in the portfolio, how to build reserves, how to price insurance products, etc.

1.2 Outline of the Thesis

To attain the major objective, we give a brief outline of how we intend to proceed and what each chapter contain. The thesis is organized as follows.

In Chapter II, we introduce some notation, terminology and some mathematical tools which are used in the main theorems. Further, we mention to the related work of the discrete-time surplus process and also introduce the our model where this thesis is organized.

In Chapter III, we study the discrete-time surplus process with the claim arrival time $T_n, n \in \mathbb{N}$. We find the recursive formulas and prove the survival and the ruin probability of this model. We interest the claim process which has exponential distribution and also illustrate numerical result of these claim. Finally, we study the inter-arrival time process with some well known distribution.

The conclusion of the thesis is presented in the last chapter.