CHAPTER 2 THEORY

2.1 The Equation of Motion

The equation of motion makes expressed of the principle of conservation of momentum. The forces acting on the atmosphere of the motion compound pressure gradient force (Equation 2.1), Coriolis force (Equation 2.2) and gravity force (Equation 2.3).

$$\frac{\vec{F}_{PGF}}{m_f} = -\frac{1}{\rho}\nabla p \qquad (2.1)$$

$$\frac{\vec{F}_{CF}}{m_f} = -2\vec{\Omega}\times\vec{V} \qquad (2.2)$$

$$\frac{\vec{F}_{GF}}{m_f} = -\vec{g} \qquad (2.3)$$

where m_f is mass.

ρ is air density.

p is pressure.

Ω is the angular velocity of the rotation.

$\vec{V} = u\vec{i} + v\vec{j}$ is the horizontal velocity.

$\vec{g}$ is gravity force.

$$\nabla p = \frac{\partial p}{\partial x}\vec{i} + \frac{\partial p}{\partial y}\vec{j}$$

From Newton's second law for the atmosphere

$$\frac{d\vec{V}}{dt} = -\frac{1}{\rho}\nabla p - 2\vec{\Omega}\times\vec{V} + \vec{g} \qquad (2.4)$$

after applying scale analysis to Equation (2.4),

$$\frac{du}{dt} = -\frac{1}{\rho}\left(\frac{df}{dx}\right) + fv \qquad (2.5.1)$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\left(\frac{df}{dy}\right) + fu \qquad (2.5.2)$$

where $f = 2\Omega \sin\phi$ is the Coriolis parameter, ϕ is latitude.

Therefore, from Equation (2.4)

$$\frac{d\vec{V}}{dt} = -\frac{1}{\rho}\nabla p - f\vec{k}\times\vec{V} \tag{2.6}$$

or

$$\frac{d\vec{V}}{dt} = -\nabla_p \Phi - f\vec{k}\times\vec{V} \tag{2.7}$$

where ∇_p is the horizontal gradient operator applied with pressure held constant.

2.2 The Equation of Wind Collision

Majumdar (2003) proposes that a weak vortex that decays upward from the surface had to be superimposed on the basic flow for a tropical storm to form. The upward decaying effect may be ignored if the disturbance is considered to be limited within a single layer of the atmosphere. In addition, it is also assumed that the wind drag toward the center of the vortex will be along log-spiral paths similar to that near the eye of a matured tropical cyclone. In this model tropical cyclone occurrence can be considered as a result of wind moving spirally into a very low pressure area. The collision of two jet flows can be explained in polar coordinates (r,θ) as shown in Figure 2.1. The collision of fluid jet 1 and fluid jet 2 results in the radial wind component and the cross-radial (tangential) wind component.

In this model, which is in polar coordinates, define the followings:

r	distance from the center
$V_r = dr/dt$	radial wind component
$V_t = r(d\theta/dt)$	cross-radial (tangential) wind component
dz/dt	vertical wind component

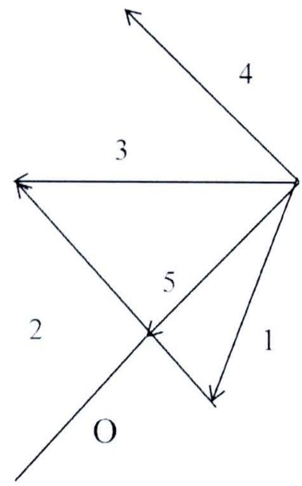


Figure 2.1 A collision of two fluid jets. Define O as the center of a tropical cyclone, 1 is stream number 1, 2 is stream number 2, 3 is the resultant vector of streams 1 and 2, 4 is the tangential wind and 5 is the radial wind.

To get a log-spiral shape of the vortex, the following equation must be satisfied

$$\frac{\frac{dr}{dt}}{r\frac{d\theta}{dt}} = m \tag{2.8}$$

or

$$\frac{dr}{dt} = mr\frac{d\theta}{dt} \tag{2.9}$$

where the wind speed ratio, m, is a constant (Majumdar, 2003).

It is also assumed that the disturbances propagate parallel to the ground, i.e., $dz/dt = 0$.

The exact values of m and r for a particular cyclone cannot be measured because of very severe weather condition associated with the cyclone. However, statistical data from many cyclones can be used to define the possible interval of m and r as in Figures 2.2 and 2.3.

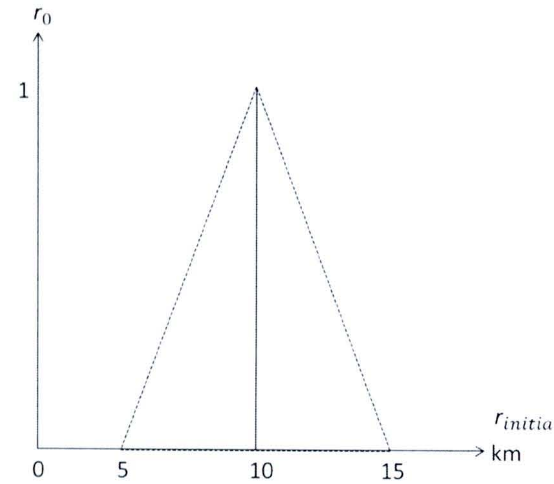


Figure 2.2 Possible values of initial radius (r_{initial}) and associated probability, r_0, $0 \leq r_0 \leq 1$ (Majumdar, 2003).

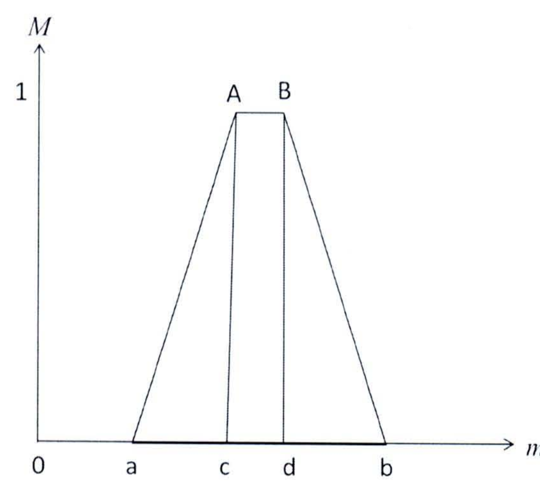


Figure 2.3 The value of $m \in [a, b]$ and associated probability of tropical cyclone occurrence, M, where $0 \leq M \leq 1$ (Majumdar, 2003).

From statistical data, the initial radius of tropical cyclone are in the interval [5,15] km (Figure 2.2). At the radius of 10 km the probability that the initial vortex will be intensified to a tropical cyclone is 1 (Majumdar, 2003).

To include uncertaines in the value of m, Equation (2.8) is now written as

$$r'(\theta) \in [mr(\theta)]^{\alpha}, r(\theta) \in [r(0)]^{\alpha} \tag{2.10}$$

where $r' = \dfrac{dr}{d\theta}$

$r(0)$ is the initial value of $r(\theta)$

α is α-cut or α-level subset.

α is defined as follows:

Let μ:(a, b) $\rightarrow$ [0, 1], then $\mu^{\alpha} = \{x|\mu(x) \geq \alpha\}$.

The values of m from various cyclones are shown in Table 2.1 and Figure 2.3, $m \in [M]^{\alpha}$.

M is the probability of tropical storm formation. The probability is 1 for $m \in$[c, d] (Figure 2.3).

Table 2.1 The value of m (Majumdar, 2003).

a	c	d	b
-0.2	-0.1	-0.05	-0.001

From Equation (2.8)

$$\int_{r_1}^{r_2} \frac{1}{r} dr = \int_{\theta_1}^{\theta_2} m d\theta$$

$$\ln \frac{r_2}{r_1} = m(\theta_2 - \theta_1)$$

Let $\theta_2 - \theta_1 = \Delta\theta$

$$r_2 = e^{m(\Delta\theta)} r_1 \tag{2.11}$$

For $\theta_2 > \theta_1$, negative m implies that $r_1 > r_2$ and the wind moves spirally into the center of cyclone. The spiral has to match the corresponding satellite image as shown in Figure 2.4.

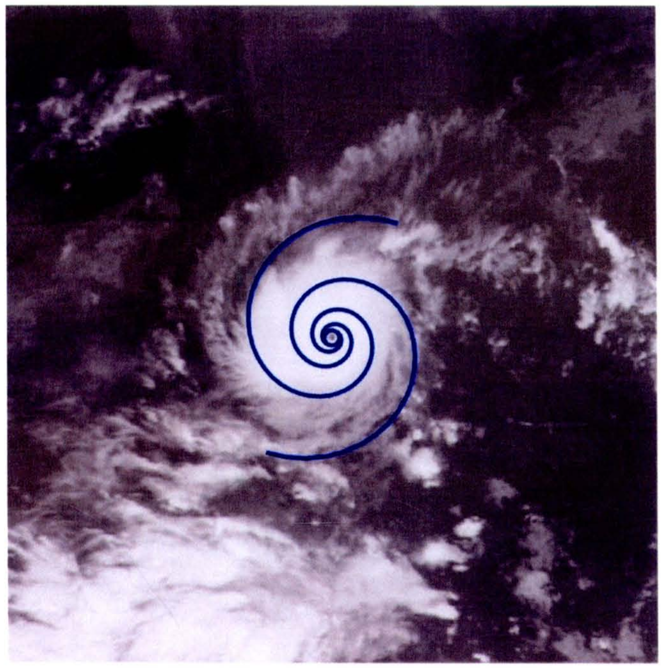


Figure 2.4 Satellite image on 04 November 1989 at 0000UTC of typhoon Gay and the spiral pattern with m = -0.21.

2.3 Symmetric Wind Model

The symmetric wind model is based on the Rankine vortex. In this model, tropical cyclone wind is symmetric around the storm center.

Giaiotti et al. (2006) propose a Rankine vortex model in cylindral coordinates. It can be expressed in a simple two-equation parametric description of a swirling flow, characterized by a forced vortex in the central core. The wind near the center of vortex circulates faster than the wind far from the center. The speed along the circular path of flow decreases as move out from the center. At the same time the inner streamlines have a shorter distance to travel to complete a ring. Wind speed of a vortex increasing with radial distance from the vortex center. The tangential speed (V_t) varies inversely with the distance r from the center of rotation, so the angular momentum rV_t is uniform everywhere throughout the flow. The two parameters of this model are the radius of maximum wind and the maximum wind speed.

The Rankine vortex model can be written as

$$V_t r + \frac{fr^2}{2} = 0 \qquad (2.12)$$

where V_t is the tangential wind components.

r is the radial distance.

f is the Coriolis parameter.

At the storm center, $r = 0$, then

$$V_t r = 0 \qquad (2.13)$$

$$\frac{dV_t r}{dr} = 0$$

$$r\frac{dV_t}{dr} + V_t\frac{dr}{dr} = 0$$

$$\int_r^{R_{max}} \frac{1}{V_t} dV_t = -\int_r^{R_{max}} \frac{1}{r} dr$$

$$\ln\frac{V_t(R_{max})}{V_t(r)} = -\ln\frac{R_{max}}{r}$$

$$V_t(r) = V_t(R_{max})\left(\frac{R_{max}}{r}\right) \qquad (2.14)$$

where $V_t(r)$ is the tangential speed at the distance radial r.

r is radial distance from the center of the storm.

$V_t(R_{max})$ is the maximum speed of tangential wind.

R_{max} is the radius of the maximum wind (V_t is maximum at R_{max}).

The symmetric wind model is defined as follows

$$V_t = 0 \qquad for \quad r = 0 \qquad (2.15.1)$$

$$V_t = V_{\max}\left(\frac{r}{R_{\max}}\right) \qquad for \quad r < R_{\max} \qquad (2.15.2)$$

$$V_t = V_{\max} \qquad for \quad r = R_{\max} \qquad (2.15.3)$$

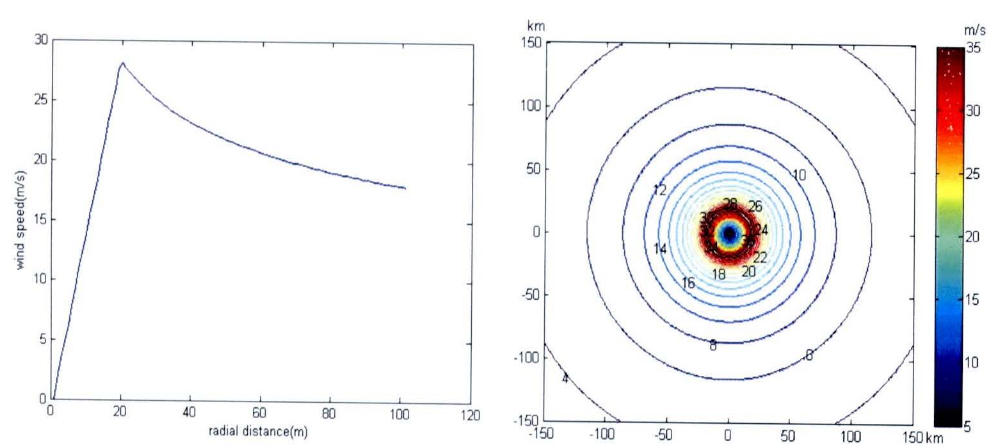
Outside the eyewall,

$$V_t = V_{\max}\left(\frac{R_{\max}}{r}\right)^{\gamma} \qquad for \quad r > R_{\max} \qquad (2.15.4)$$

where γ is the shape parameter.

The value of γ has been determined empirically from observed tropical cyclones to have a range of $0.5 < \gamma < 0.65$ for the United States of America (Miller, 1967).

The result from Equation (2.15) is shown as an example in Figure 2.5. The wind speed equals to zero at the center from Equation (2.15.1). Equation (2.15.2) shows that wind speed increases linearly from the center to R_{max}. Equation (2.15.3) defines that at R_{max} the wind has the maximum value, Equation (2.15.4) shows that wind speed decreases exponentially outside the eye.



(a) (b)

Figure 2.5 (a) Wind speed (m/s) as a function of distance from the storm center with $\gamma = 0.5$. (b) Wind speed (m/s) of typhoon Gay, 04 November 1989 at 0000UTC.

In this reseeach the value of γ is determined from Equation (2.15.4) as follows,

$$V_t = V_{max}\left(\frac{R_{max}}{r}\right)^{\gamma}$$

$$\frac{V_t}{V_{max}} = \left(\frac{R_{max}}{r}\right)^{\gamma}$$

$$\log\left(\frac{V_t}{V_{max}}\right) = \log\left(\frac{R_{max}}{r}\right)^{\gamma}$$

$$\log\left(\frac{V_t}{V_{max}}\right) = \gamma \log\left(\frac{R_{max}}{r}\right)$$

$$\gamma = \frac{\log\left(\frac{V_t}{V_{max}}\right)}{\log\left(\frac{R_{max}}{r}\right)} \quad (2.16)$$

2.4 Asymmetric Wind Model

The asymmetric wind model is formulated by combining the movement of tropical cyclone to the symmetric wind model.

Kong-ied (2005) presents the asymmetric wind model as

$$V(r,\theta) = \sqrt{V_t^2(r) + V_{center}^2 + 2V_t(r)V_{center}\cos\theta} \quad (2.17)$$

where r is radial distance.

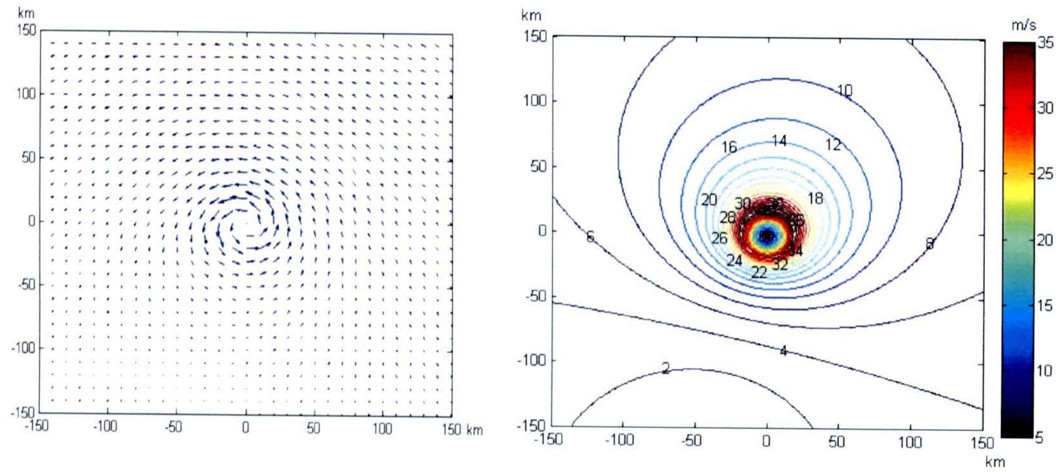
θ is the angle between the radius and the reference axis.

$V(r,\theta)$ is the wind speed of a moving cyclone at distance r from the center and the angle θ.

$V_t(r)$ is the Rankine vortex wind speed at distance r from the center.

V_{center} is the translation speed of the center.

Figure 2.6 shows examples of asymmetric wind vector and speed.



(a) (b)

Figure 2.6 Asymmetric wind of typhoon Gay on 04 November 1989 0000UTC with V_{center} = 4.2 m/s, (a) wind vector, (b) wind speed (m/s).

2.5 Wind Direction along the Spiral

There are 3 steps in determining the wind direction along the spiral flow in a tropical cyclone. The first step is to find the tangential wind direction at a point in the symmetric wind model. The second step is finding the direction angle of the spiral flow from the tangential wind at the point. The last step is to determine the wind direction along the spiral at the point form steps 1 and 2.

2.5.1 Tangential Wind Direction

This part consists of the following steps.

1. Select a point in the tropical cyclone at the radial distance r.
2. Draw the circle that has the selected point on its circumference (Figure 2.7).
3. Find the angle of the tangential line at the selected point (Figure 2.8).

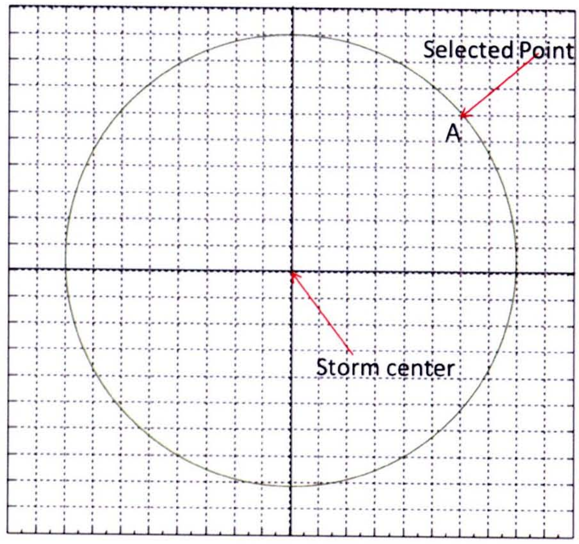


Figure 2.7 Draw a circle that passes trough the selected point A.

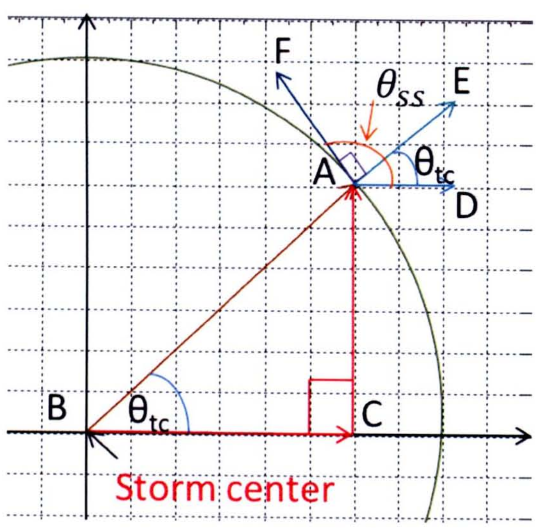


Figure 2.8 Finding the angle of tangent line that passes trough the selected point A.

From Figure 2.8

$$\theta_{tc} = \arctan(CA/BC) \tag{2.18}$$

$$\theta_{ss} = 90° + \theta_{tc} \tag{2.19}$$

where θ_{tc} is the angle of the line BA.

θ_{ss} is the tangential wind direction at point A (symmetric wind).

2.5.2 Direction of the Spiral Wind

1. At the selected point A, determine the change in wind direction, $\Delta\theta$, from Equation (2.11).
2. Add $\Delta\theta$ to the direction of tangential wind, θ_{ss}, form Equation (2.20). The direction of the spiral wind, θ_{vv}, is obtained (Figure 2.9).

$$\theta_{vv} = \theta_{ss} + \Delta\theta \qquad (2.20)$$

where θ_{vv} is the spiral wind direction.

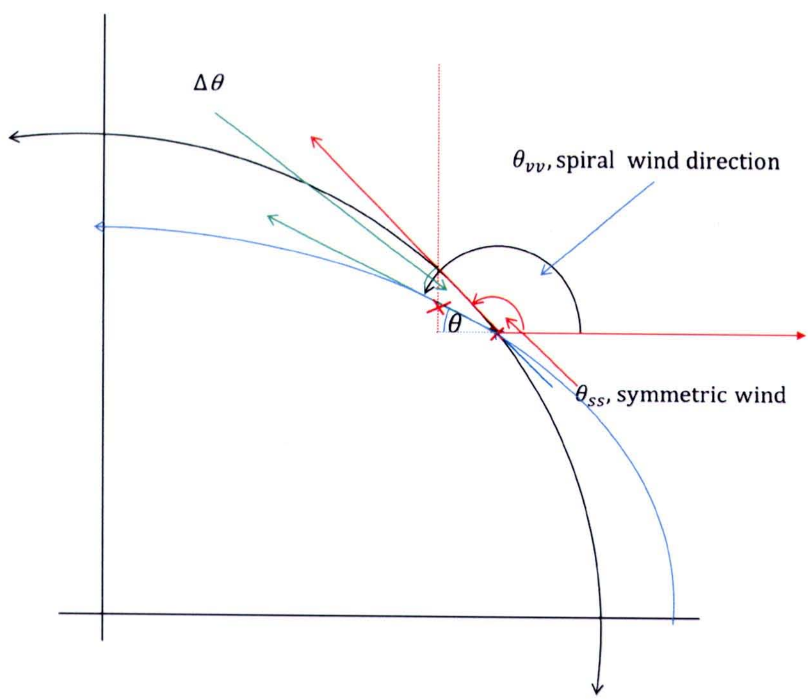


Figure 2.9 Angles used in finding the direction of spiral flow.