CHAPTER 4 RESULT AND ARL COMPARISONS

4.1 Control chart for mean using RSSMC

The control chart based on RSSMC and the RSSMC estimator was first proposed by Pongpullponsak and Sontisamran (2010). However, performance of the proposed estimator was unsatisfactory. For this reason, the aim of this study is to improve the estimator as could be concluded following.
The RSSMC mean $\bar{X}_{rssmc,j}$ of the j^{th} cycle which can be plotted on the control chart based on RSSMC is calculated by

$$\begin{aligned} UCL &= \mu + 3\sigma_{\bar{x}_{rssmc}} \\ CL &= \mu \\ LCL &= \mu - 3\sigma_{\bar{x}_{rssmc}} \end{aligned} \tag{4.1}$$

where

$$\sigma_{\bar{X}_{rssmc}} = \sqrt{\frac{1}{n^2}\sum_{i=1}^{n}\sigma^2_{(i:mc)}} \tag{4.2}$$

In actual measurement, the μ and $\sigma_{\bar{X}_{rssmc}}$ are unknown so the estimator for μ based on RSSMC data, when the distribution is normal, is given by

$$\bar{X}_{rssmc} = \frac{1}{r}\sum_{j=1}^{r}\bar{X}_{rssmc,j} \tag{4.3}$$

And the estimator for $\sigma_{\bar{X}_{rssmc}}$ will be

$$\hat{\sigma}_{\bar{x}_{rssmc}} = \left[\frac{1}{n(nr-1)}\sum_{j=1}^{r}\sum_{i=1}^{n}\left(X_{(i:mc)j} - \bar{X}_{rssmc}\right)^2\right]^{1/2} \tag{4.4}$$

where $X_{(i:mc)j}$ is the estimate for population mean of the i^{th} order statistic. The control chart can be constructed using the $\bar{X}_{rssmc}$ and $\hat{\sigma}_{\bar{x}_{rssmc}}$ as following.

$$\begin{aligned} UCL &= \bar{X}_{rssmc} + 3\hat{\sigma}_{\bar{x}_{rssmc}} \\ CL &= \bar{X}_{rssmc} \\ LCL &= \bar{X}_{rssmc} - 3\hat{\sigma}_{\bar{x}_{rssmc}} \end{aligned} \tag{4.5}$$

4.2 Hotelling's control chart using RSSMC

Hotelling's control chart using RSSMC is control chart developed for better statistical quality control by considering the variable more than one for controlling. In this paper, we use two variables for controlling. The test statistic plotted on the chi-square control chart for each sample is

$$\chi_0^2 = n\left(\bar{X}_{rssmc} - \mu_{rssmc}\right)' \Sigma_{rssmc}^{-1}\left(\bar{X}_{rssmc} - \mu_{rssmc}\right) \tag{4.6}$$

where $\bar{X}'_{rssmc} = \left[\begin{array}{cc} \bar{X}_{1:rssmc} & \bar{X}_{2:rssmc} \end{array} \right]$ is quality characteristic means which is represented by the 2×1 vector, $\mu'_{rssmc} = \left[\begin{array}{cc} \mu_{1:rssmc} & \mu_{2:rssmc} \end{array} \right]$ is the vector of in-control means for each quality characteristic and Σ_{rssmc} is the covariance matrix. The upper limit on the control chart is

$$UCL = \chi^2_{\alpha,p} \tag{4.7}$$

In practice, it is usually necessary to estimate μ and Σ from the analysis of preliminary samples of size n, taken when the process is assumed to be in control. Now suppose that S_{rssmc} is used to estimated Σ_{rssmc} and that the vector $\bar{\bar{X}}_{rssmc}$ is taken as the in-control value of vector of the process. The test statistic now becomes

$$T^2 = n\left(\bar{X}_{rssmc} - \bar{\bar{X}}_{rssmc}\right)' \Sigma^{-1}_{rssmc}\left(\bar{X}_{rssmc} - \bar{\bar{X}}_{rssmc}\right) \tag{4.8}$$

4.3 The computer simulations and ARL comparisons

We use the average run length (ARL) to compare the SRS control charts to the RSSMC control charts. The ARL assumes that the process is under control with mean μ_0 and standard deviation σ_0, and at some point in time the process may start to get out of control, i.e. the mean is shifted from μ_0 to $\mu_0 + \delta\sigma_0/\sqrt{n} = \mu$. We are assuming that the process is following the normal distribution with mean μ_0 and variance σ_0^2 if the process is under control, and the shift on the process mean is $\delta = (\sqrt{n}/\sigma_0)\,|\mu - \mu_0|$. If $\delta = 0$ the process is under control and in this case if the point is outside the control limits it is a false alarm.
Ranking the variable of interest with errors in ranking the units is called imperfect ranking. The data consist of 3 related variables that are simulated with normal distribution. For each value of ARL we simulate 10,000 replications. The computer simulations are run for $n = 3, 4, 5, 6, 7$ and $\delta = 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0$ in case univariate control chart as soon as $n = 3, 4, 5, 6$ in case multivariate control chart when Type I error probability of α is 0.0027.

4.4 ARL comparisons for univariate control chart

The studying limit of control chart by using SRS, RSS, MRSS and RSSMC show how the constructing RSSMC can reduce the variation due to sampling method. In this study, we use the simulation data which is the normal distribution with mean μ and variance σ by the statistical package R. After simulating data, we select the sample by using SRS, RSS, MRSS and RSSMC from data and use it to construct the quality control chart. From different sampling methods by using the sample size $n = 3, 4$ and the result showed that

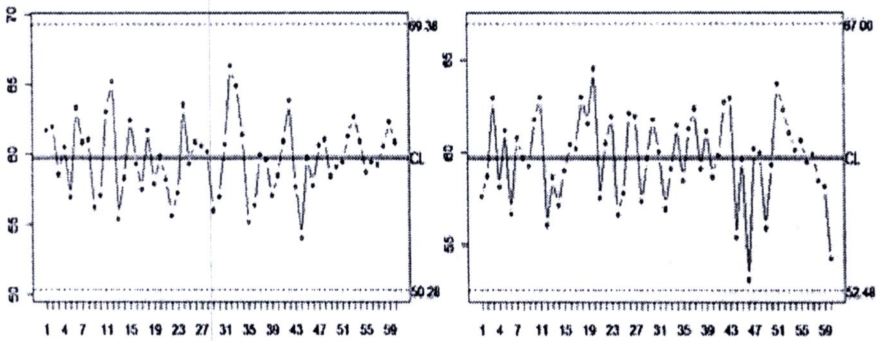
1. When the sample size is increased, the control limit by using SRS, RSS, MRSS and RSSMC is narrowed so it satisfy the principal of constructing control chart for mean.

2. From the comparing limit of control chart by using SRS, RSS, MRSS and RSSMC, the control chart based on SRS has the maximum width of control chart and similar to MRSS. The control chart based on RSSMC has the minimum width of control chart. We can conclude as the central limit which is the same other methods so the other control chart having the mass width of

control chart shows that the control chart has the mass variation. In this study, we find the constructing control chart for RSSMC having the minimum width of control chart. It shows that the control chart based on RSSMC is the control chart having the minimum variation of control chart. The detail following Table 4.1 and Figure 4.1-4.4.

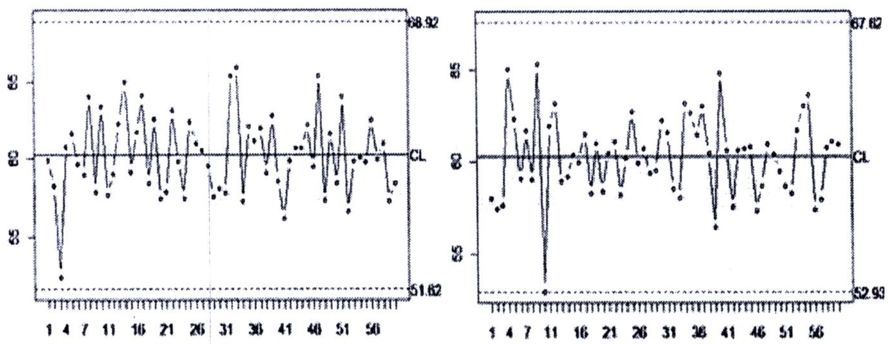
Table 4.1 Control limits of SRS, RSS, MRSS and RSSMC for $n = 3, 4$.

Sample size	Control limit	SRS	RSS	MRSS	RSSMC
$n = 3$	UCL	69.38	68.92	69.34	66.41
	CL	60.00	60.00	60.00	60.00
	LCL	50.28	51.62	51.13	53.36
$n = 4$	UCL	67.00	67.62	67.87	65.32
	CL	60.00	60.00	60.00	60.00
	LCL	52.48	52.39	53.39	54.56



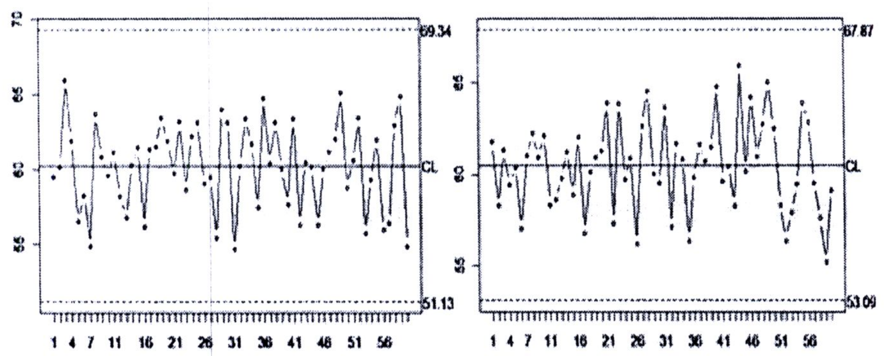
sample size $n=3$. sample size $n=4$.

Figure 4.1 Quality control chart using SRS.



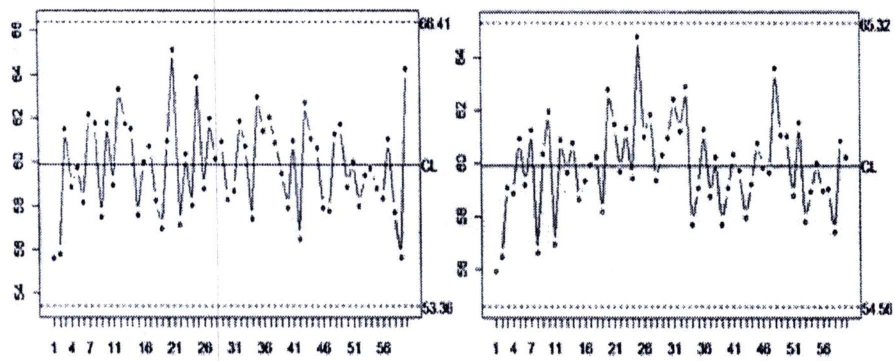
sample size $n = 3$. sample size $n = 4$.

Figure 4.2 Quality control chart using RSS.



sample size $n = 3$. sample size $n = 4$.

Figure 4.3 Quality control chart using MRSS.



sample size $n = 3$. sample size $n = 4$.

Figure 4.4 Quality control chart using RSSMC.

Table 4.2 Show the ARL for $n = 3, 4, 5, 6, 7$ when δ is 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0.

Sample size	δ	SRS	RSS	MRSS	RSSMC
$n = 3$	0.0	346.090	345.470	347.886	341.993
	0.5	161.448	156.354	144.185	107.159
	1.0	45.601	42.542	46.388	22.995
	1.5	15.176	15.276	15.192	6.837
	2.0	6.526	6.466	6.265	2.906
	2.5	3.293	3.508	3.210	1.652
	3.0	2.012	2.030	2.120	1.215
$n = 4$	0.0	346.214	341.513	343.138	336.341
	0.5	161.782	150.134	155.275	86.234
	1.0	45.752	42.001	42.224	15.944
	1.5	14.698	14.043	14.724	4.879
	2.0	6.558	6.490	6.339	2.142
	2.5	3.233	3.259	3.319	1.323
	3.0	2.032	1.968	2.000	1.093
$n = 5$	0.0	348.782	345.915	350.4718	343.823
	0.5	155.978	152.849	150.679	72.119
	1.0	46.690	45.363	39.822	12.335
	1.5	15.144	15.650	14.219	3.555
	2.0	6.611	6.394	5.909	1.697
	2.5	3.224	3.285	3.177	1.178
	3.0	2.044	2.004	2.008	1.037
$n = 6$	0.0	349.488	343.289	343.184	343.91
	0.5	158.048	160.457	142.864	59.858
	1.0	44.872	42.295	43.596	9.322
	1.5	14.394	14.471	14.422	2.808
	2.0	6.726	6.150	6.164	1.436
	2.5	3.394	3.350	3.176	1.089
	3.0	2.008	2.116	2.016	1.012
$n = 7$	0.0	348.246	343.445	354.498	340.772
	0.5	158.124	162.663	147.014	51.718
	1.0	47.674	47.075	39.748	7.809
	1.5	15.818	14.499	14.478	2.316
	2.0	6.762	6.059	5.644	1.321
	2.5	3.366	3.189	2.906	1.043
	3.0	1.994	1.972	2.030	1.004

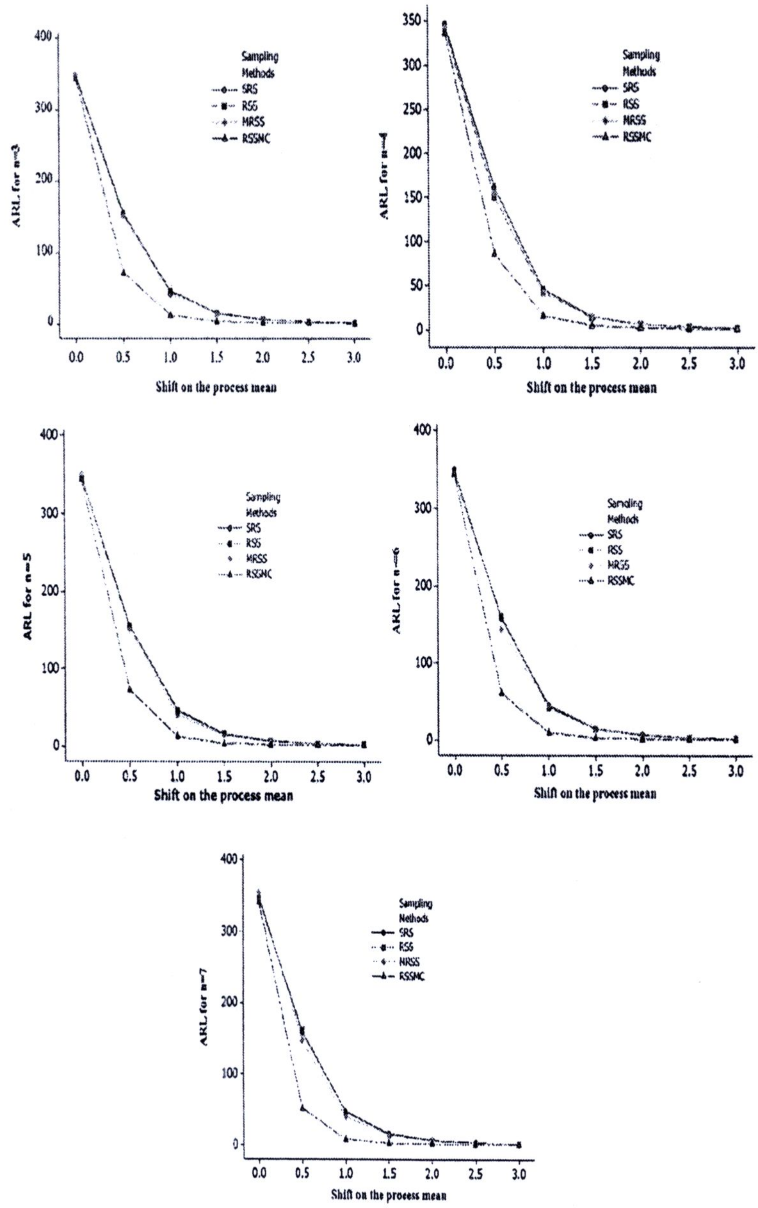


Figure 4.5 Comparison of ARL of univariate control chart in term graph for $n = 3, 4, 5, 6, 7$.

Considering the results in Table 4.2 the following conclusions can be made:

1. In case $n = 3$, ARL value obtained from RSSMC will be minimum value and other methods will be neighbor. In case that $n = 4$, ARL value obtained from RSSMC will be minimum value. RSS and MRSS will be neighbor together, however. They have value less than SRS. In case $n = 5$, ARL value obtained from RSSMC will be minimum value and MRSS will be less value. RSS and SRS will be neighbor together, however. RSS have value less than SRS. In case that $n = 6$, ARL value obtained from RSSMC will minimum value and MRSS will be less value. RSS and SRS will be neighbor together, however. RSS have value less than SRS. In case $n = 7$, ARL value obtained from RSSMC will be minimum value and MRSS will be less value. SRS will be minimum value.

2. If the process under control, i.e.$\delta = 0$, MPRSS will not increase the number of false alarms as compared to SRS, RSS and MRSS if the process is under control. In fact there is a small decrease in the ARL, for example, for $n = 3$, $ARL = 341.993$ as compared to 346.090, 345.470 and 347.866 for SRS, RSS and MRSS respectively.

3. If the sample size increases, the ARL will decrease if $\delta > 0$, for example if the sample size is 4 and $\delta = 0$ the ARL is 15.944 as compared with 22.995 in the case of $n = 3$.

4. The ARL for the RSS will decrease much faster than SRS if δ increases. This increase in ARL will depend on the correlation between the variable of interest and the concomitant variable that we use to estimate the rank of the variable of interest.

5. 5. The ARL for RSSMC while the data have error ranking has less than SRS because some variables used for ranking have related to a variable of interest without actual measurement.

4.5 ARL comparisons for multivariate control chart

Table 4.3 ARL values for SRS and RSSMC methods when $n = 3$.

δ	0.0	0.5	1.0	1.5	2.0	2.5	3.0
SRS	350.934	125.789	25.726	7.183	2.854	1.605	1.180
RSSMC	349.350	57.197	7.593	2.333	1.223	1.023	1.000

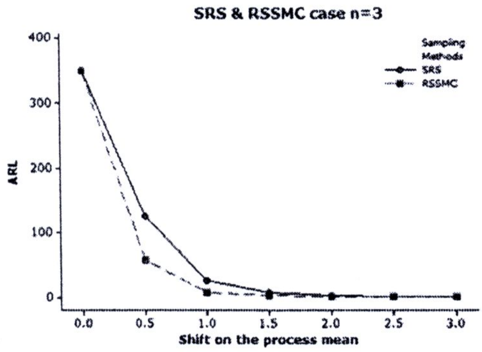


Figure 4.6 Comparison of ARL of multivariate control chart in term graph for $n = 3$.

A comparison between SRS and RSSMS using the Hotelling's control chart when $n = 3$, revealed that in case of no shift on the process mean ($\delta = 0$), no difference of ARL values observes. However, decrease of ARL values of RSSMC appears faster than those of SRS when there is shift in the process mean.

Table 4.4 ARL values for SRS and RSSMC methods when $n = 4$.

δ	0.0	0.5	1.0	1.5	2.0	2.5	3.0
SRS	350.077	125.119	25.669	7.184	2.845	1.600	1.184
RSSMC	348.856	47.408	5.580	1.546	1.100	1.008	1.000

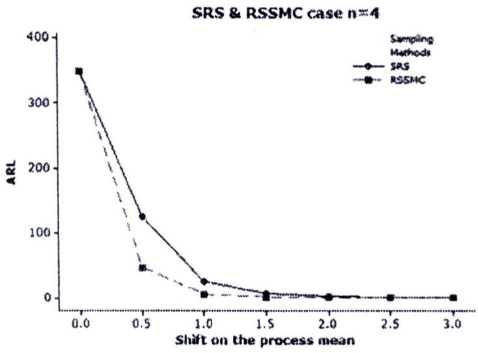


Figure 4.7 Comparison of ARL of multivariate control chart in term graph for $n = 4$.

For $n = 4$, when no shift on the process mean ($\delta = 0$), there is no ARL value difference observed between SRS and RSSMC using the Hotelling's control chart. But when shift in the process mean occurs, the ARL values of RSSMC are decreased faster than those of the SRS cases. Furthermore, the RSSMC value decrease appears to be faster than the case of the $n = 3$.

Table 4.5 ARL values for SRS and RSSMC methods when $n = 5$.

δ	0.0	0.5	1.0	1.5	2.0	2.5	3.0
SRS	349.201	124.952	25.728	7.055	2.865	1.587	1.183
RSSMC	348.567	31.687	3.837	1.303	1.023	1.000	1.000

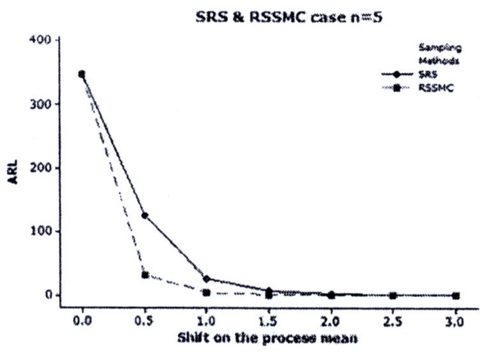


Figure 4.8 Comparison of ARL of multivariate control chart in term graph for $n = 5$.

Like the cases of $n = 3, 4,$, using the Hotelling's control chart there is no difference of ARL values between SRS and RSSMC observed when no shift on the process mean ($\delta = 0$). And when there is shift in the process mean, occurrence of ARL values decrease of RSSMC is faster than those of the SRS values, and also faster than what observed in the cases of $n = 3, 4,$.

Table 4.6 ARL values for SRS and RSSMC methods when $n = 6$.

δ	0.0	0.5	1.0	1.5	2.0	2.5	3.0
SRS	348.015	126.824	26.202	7.217	2.851	1.662	1.181
RSSMC	347.028	25.355	2.750	1.158	1.005	1.000	1.000

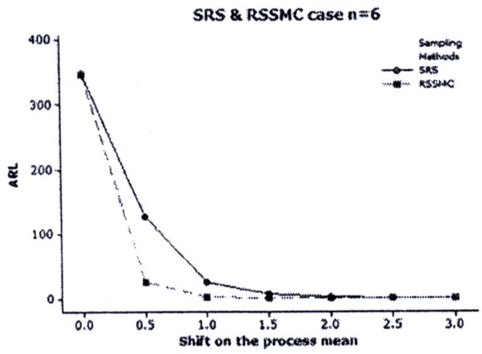


Figure 4.9 Comparison of ARL of multivariate control chart in term graph for $n = 6$.

Similarly, no difference of ALR values, for $n = 6$, between SRS and RSSMC observed in the Hotelling's control chart when there is no shift on the process mean ($\delta = 0$). The ARL values of RSSMC decrease faster than the SRS values when the process mean shift is detected. Decreasing of the RSSMC values occurs faster than what seen in smaller sample size cases ($n = 3, 4, 5$).
From the results described above, it can be concluded that

1. If the process is in control, where $\delta = 0$, the limit number of defective products for SRS will be changed while the RSSMC number is almost the same. For example, when $n = 3$, the ARL value for SRS is 350.934, whereas for RSSMC, $ARL = 349.350$.

2. In case of sample sizes increased, the ARL value of RSSMC will be reduced when $\delta > 0$. For example, if the sample size is 4 and $\delta = 1.0$, the ARL value will be only 5.580, while for $n = 3$ the ARL value is 7.593.

3. The ARL value for RSSMC will be decreased faster than that of SRS, when δ increases. This implies that increase of ARL value depends on the relationship between an interest variable and an associated variable. Therefore we use the allocated variables to estimate a variable of interest.

4. The ARL for RSSMC while the data have error ranking has less than SRS because some variables used for ranking have related to a variable of interest without actual measurement.