CHAPTER 3 METHODOLOGY

3.1 Univariate control chart

3.1.1 Establishment of control chart for mean using simple random sampling

Step 1 calculate $\bar{\bar{X}}$

$$\bar{\bar{X}} = \frac{1}{r}\sum_{j=1}^{r} \bar{X}_j \tag{3.1}$$

when $\bar{X}_j$ is the samples mean of the j^{th} cycle which can be calculated from

$$\bar{X}_j = \frac{1}{n}\sum_{i=1}^{n} X_{ij}, \qquad j = 1, 2, ..., r \tag{3.2}$$

when X_{ij} is the i^{th} observed value in the j^{th} cycle which is chosen by using simple random sampling.

Step 2 estimate for $\bar{S}$

$$\bar{S} = \frac{1}{r}\sum_{j=1}^{r} X_j \tag{3.3}$$

when S_j is the samples variance of the j^{th} cycle which can be calculated from

$$S_j = \left[\frac{1}{n-1}\sum_{i=1}^{n} \left(X_{ij} - \bar{X}_j\right)^2\right]^{1/2} \tag{3.4}$$

is an biased estimate for σ. We can use $\bar{S}/c_4$ as the unbiased estimate for σ where c_4 is calculated from

$$c_4 = \left(\frac{2}{n} - 1\right)\frac{\Gamma(n-2)}{\Gamma\left[(n-1)/2\right]} \tag{3.5}$$

Step 3 estimate the control limit before establishing control chart for the case that out of control will occur when there is at least one value that $\bar{X}_j > UCL$ or $\bar{X}_j < LCL$ as following;

Step 3.1 estimate the control limit

$$\begin{aligned} UCL &= \bar{\bar{X}} + 3\frac{\bar{S}}{c_4\sqrt{n}} \\ CL &= \bar{\bar{X}} \\ LCL &= \bar{\bar{X}} - 3\frac{\bar{S}}{c_4\sqrt{n}} \end{aligned} \tag{3.6}$$

Step 3.2 compare $\bar{X}_j$ with the estimated UCL and LCL
if $\bar{X}_j > UCL$ or $\bar{X}_j < LCL$ at least one value, then out of control.

3.1.2 Establishment of control chart for mean using ranked set sampling

Step 1 calculate $\bar{X}_{rss}$

$$\bar{X}_{rss} = \frac{1}{r}\sum_{j=1}^{r}\bar{X}_{rss,j} \tag{3.7}$$

when $\bar{X}_{rss,j}$ is the samples mean of the j^{th} cycle which can be calculated from

$$\bar{X}_{rss,j} = \frac{1}{n}\sum_{i=1}^{n}X_{(i:n)j}, \qquad j = 1,2,...,r \tag{3.8}$$

when $X_{(i:n)j}$ is the i^{th} observed value in the j^{th} cycle which is chosen by using ranked set sampling.

Step 2 estimate for $\sigma_{\bar{X}_{rss}}$

$$\hat{\sigma}_{\bar{x}_{rss}} = \left[\frac{1}{n}\hat{\sigma}^2_{rss} - \frac{1}{n^2}\sum_{i=1}^{n}\left(\bar{X}_{(i)} - \bar{X}_{rss}\right)^2\right]^{1/2} \tag{3.9}$$

when

$$\hat{\sigma}^2_{rss} = \frac{1}{nr-1}\sum_{i=1}^{n}\sum_{j=1}^{r}\left(X_{(i:n)j} - \bar{X}_{rss}\right)^2 \tag{3.10}$$

and

$$\bar{X}_{(i)} = \frac{1}{r}\sum_{j=1}^{r}X_{(i:n)j}, \qquad i = 1,2,...,n \tag{3.11}$$

where $\bar{X}_{(i)}$ is the estimate for population mean of the i^{th} order statistic.

Step 3 estimate the control limit before establishing control chart for the case that out of control will occur when there is at least one value that $\bar{X}_{rss,j} > UCL$ or $\bar{X}_{rss,j} < LCL$ as following;

Step 3.1 estimate the control limit

$$\begin{aligned} UCL &= \bar{\bar{X}}_{rss} + 3\hat{\sigma}_{\bar{x}_{rss}} \\ CL &= \bar{\bar{X}}_{rss} \\ LCL &= \bar{\bar{X}}_{rss} - 3\hat{\sigma}_{\bar{x}_{rss}} \end{aligned} \tag{3.12}$$

Step 3.2 compare $\bar{X}_{rss,j}$ with the estimated UCL and LCL
if $\bar{X}_{rss,j} > UCL$ or $\bar{X}_{rss,j} < LCL$ at least one value, then out of control.

3.1.3 Establishment of control chart for mean using median ranked set sampling

Step 1 calculate $\bar{X}_{mrss}$

$$\bar{X}_{mrss} = \frac{1}{r}\sum_{j=1}^{r}\bar{X}_{mrss,j} \tag{3.13}$$

when $\bar{X}_{mrss,j}$ is the samples mean of the j^{th} cycle which can be calculated from

$$\bar{X}_{mrss,j} = \frac{1}{n}\sum_{i=1}^{n}X_{(i:m)j}, \qquad j = 1,2,...,r \tag{3.14}$$

when $X_{(i:m)j}$ is the i^{th} observed value in the j^{th} cycle which is chosen by using median ranked set sampling.

Step 2 estimate for $\sigma_{\bar{X}_{mrss}}$

$$\hat{\sigma}_{\bar{x}_{mrss}} = \left[\frac{1}{n(nr-1)}\sum_{j=1}^{r}\sum_{i=1}^{n}\left(X_{(i:m)j} - \bar{X}_{mrss}\right)^2\right]^{1/2} \tag{3.15}$$

Step 3 estimate the control limit before establishing control chart for the case that out of control will occur when there is at least one value that $\bar{X}_{mrss,j} > UCL$ or $\bar{X}_{mrss,j} < LCL$ as following;

Step 3.1 estimate the control limit

$$\begin{aligned} UCL &= \bar{\bar{X}}_{mrss} + 3\hat{\sigma}_{\bar{x}_{mrss}} \\ CL &= \bar{\bar{X}}_{mrss} \\ LCL &= \bar{\bar{X}}_{mrss} - 3\hat{\sigma}_{\bar{x}_{mrss}} \end{aligned} \tag{3.16}$$

Step 3.2 compare $\bar{X}_{mrss,j}$ with the estimated UCL and LCL if $\bar{X}_{mrss,j} > UCL$ or $\bar{X}_{mrss,j} < LCL$ at least one value, then out of control.

3.1.4 Establishment of control chart for mean using ranked set sampling for multiple characteristics

In general, for ranked set sampling only one criterion will be used in sampling ranking in which errors can occur easily. Thus more than one criterion should be taken into the account in order to reduce the errors which have several steps in selection.
For this study, the number of criteria was increased into $f = 1, f = 2, ..., f = p$, in which new values was estimated from $V_1, V_2, ..., V_p$ which were the value of each set sample. The minimum value of $V = V_1 + V_2 + ... + V_p$ was then chosen, and random choosing was carried out when there was more than one value.

Step 1 calculate $\bar{X}_{rssmc}$

$$\bar{X}_{rssmc} = \frac{1}{r}\sum_{j=1}^{r}\bar{X}_{rssmc,j} \tag{3.17}$$

when $\bar{X}_{rssmc,j}$ is the samples mean of the j^{th} cycle which can be calculated from

$$\bar{X}_{rssmc,j} = \frac{1}{n}\sum_{i=1}^{n}X_{(i:mc)j}, \qquad j = 1, 2, ..., r \tag{3.18}$$

when $X_{(i:mc)j}$ is the i^{th} observed value in the j^{th} cycle which is chosen by using median ranked set sampling.

Step 2 estimate for $\sigma_{\bar{X}_{rssmc}}$

$$\hat{\sigma}_{\bar{x}_{rssmc}} = \left[\frac{1}{n(nr-1)}\sum_{j=1}^{r}\sum_{i=1}^{n}\left(X_{(i:mc)j} - \bar{X}_{rssmc}\right)^2\right]^{1/2} \tag{3.19}$$

Step 3 estimate the control limit before establishing control chart for the case that out of control will occur when there is at least one value that $\bar{X}_{rssmc,j} > UCL$

or $\bar{X}_{rssmc,j} < LCL$ as following;

Step 3.1 estimate the control limit

$$\begin{aligned} UCL &= \bar{\bar{X}}_{rssmc} + 3\hat{\sigma}_{\bar{x}_{rssmc}} \\ CL &= \bar{\bar{X}}_{rssmc} \\ LCL &= \bar{\bar{X}}_{rssmc} - 3\hat{\sigma}_{\bar{x}_{rssmc}} \end{aligned} \tag{3.20}$$

Step 3.2 compare $\bar{X}_{rssmc,j}$ with the estimated UCL and LCL if $\bar{X}_{rssmc,j} > UCL$ or $\bar{X}_{rssmc,j} < LCL$ at least one value, then out of control.

3.2 Multivariate control chart

3.2.1 Establishment of bivariate control chart for simple random sampling

Step 1 calculate T_j^2 using the steps as following;

Step 1.1 estimate the vector of the sample means by given

$$\bar{X}_j = \begin{bmatrix} \bar{X}_{1j} \\ \bar{X}_{2j} \end{bmatrix}, \qquad j = 1, 2, ..., r \tag{3.21}$$

when $\bar{X}_{ij}$ is the samples mean of the i^{th} criterion for the j^{th} cycle which can be calculated from

$$\bar{X}_{ij} = \frac{1}{n}\sum_{k=1}^{n} X_{ijk}, \qquad i = 1, 2; j = 1, 2, ..., r \tag{3.22}$$

when X_{ijk} is the value of the k^{th} observed value on the i^{th} criterion in the j^{th} cycle.

Step 1.2 estimate sample variance of the i^{th} criterion in the i^{th} cycle by given

$$S_{ij}^2 = \frac{1}{n-1}\sum_{k=1}^{n} \left(X_{ijk} - \bar{X}_{ij}\right)^2, \qquad i = 1, 2; j = 1, 2, ..., r \tag{3.23}$$

for covariance between criteria i and h of the j^{th} cycle, it can be calculated from

$$S_{ihj} = \frac{1}{n-1}\sum_{k=1}^{n} \left(X_{ijk} - \bar{X}_{ij}\right)\left(X_{hjk} - \bar{X}_{hj}\right), \qquad i \neq h; j = 1, 2, ..., r \tag{3.24}$$

vector of $\bar{\bar{X}}$ for target means of individual criterion in r cycles is estimated by

$$\bar{\bar{X}}_i = \frac{1}{r}\sum_{j=1}^{r} \bar{X}_{ij}, \qquad i = 1, 2 \tag{3.25}$$

Step 1.3 estimate the member of the variance-covariance matrix S from the mean of r cycles

$$S_i^2 = \frac{1}{r}\sum_{j=1}^{r} S_{ij}^2, \qquad i = 1, 2 \text{ and } S_{ih} = \frac{1}{r}\sum_{j=1}^{r} S_{ihj}, \qquad i \neq h \tag{3.26}$$

finally, the vector $\bar{\bar{X}}$ is estimated by using the member $\left(\bar{X}_i\right)$, and the matrix S is calculated as following;

$$S = \begin{bmatrix} S_1^2 & S_{12} \\ S_{12} & S_2^2 \end{bmatrix} \tag{3.27}$$

Step 1.4 calculate T_j^2 from statistic value T^2

$$T^2 = \frac{n}{(S_1^2 S_2^2 - S_{12}^2)} \left[S_2^2 \left(\bar{X}_1 - \bar{\bar{X}}_1\right)^2 + S_1^2 \left(\bar{X}_2 - \bar{\bar{X}}_2\right)^2 - 2S_{12} \left(\bar{X}_1 - \bar{\bar{X}}_1\right)\left(\bar{X}_2 - \bar{\bar{X}}_2\right) \right] \tag{3.28}$$

Step 2 estimate the upper control limit before establishing control chart for the case that out of control will occur when there is at least one value that $T_j^2 > UCL$, as following;

Step 2.1 estimate the upper control limit from the calculation of $T^2_{\alpha,2,(n-1)}$

$$UCL = \chi^2_{\alpha,p} \tag{3.29}$$

Step 2.2 compare T_j^2 with the estimated UCL

if $T_j^2 > UCL$, $\quad j = 1, 2, ..., r$ at least one value, then out of control.

3.2.2 Establishment of bivariate control chart for ranked set sampling with multiple characteristics

Step 1 calculate $T^2_{j:rssmc}$ using the steps as following;

Step 1.1 estimate the vector of the sample means by given

$$\bar{X}_{j:rssmc} = \begin{bmatrix} \bar{X}_{1j:rssmc} \\ \bar{X}_{2j:rssmc} \end{bmatrix}, \qquad j = 1, 2, ..., r \tag{3.30}$$

when $\bar{X}_{ij:rssmc}$ is the samples mean of the i^{th} criterion for the j^{th} cycle which can be calculated from

$$\bar{X}_{ij:rssmc} = \frac{1}{n} \sum_{k=1}^{n} X_{ijk:rssmc}, \qquad i = 1, 2; j = 1, 2, ..., r \tag{3.31}$$

when $X_{ijk:rssmc}$ is the value of the k^{th} observed value on the i^{th} criterion in the j^{th} cycle.

Step 1.2 estimate sample variance of the i^{th} criterion in the i^{th} cycle by given

$$S^2_{ij:rssmc} = \frac{1}{n-1} \sum_{k=1}^{n} \left(X_{ijk:rssmc} - \bar{X}_{ij:rssmc}\right)^2, \qquad i = 1, 2; j = 1, 2, ..., r \tag{3.32}$$

for covariance between criteria i and h of the j^{th} cycle, it can be calculated from

$$S_{ihj:rssmc} = \frac{1}{n-1} \sum_{k=1}^{n} \left(X_{ijk:rssmc} - \bar{X}_{ij:rssmc}\right)\left(X_{hjk:rssmc} - \bar{X}_{hj:rssmc}\right), \quad i \neq h; j = 1, 2, ..., r \tag{3.33}$$

vector of $\bar{\bar{X}}_{rssmc}$ for target means of individual criterion in r cycles is estimated by

$$\bar{\bar{X}}_{i:rssmc} = \frac{1}{r}\sum_{j=1}^{r} \bar{X}_{ij:rssmc}, \qquad i = 1, 2 \tag{3.34}$$

Step 1.3 estimate the member of the variance-covariance matrix S from the mean of r cycles

$$\begin{aligned} S^2_{i:rssmc} &= \frac{1}{r}\sum_{j=1}^{r} S^2_{ij:rssmc}, \qquad i = 1, 2 \\ \text{and } S_{ih:rssmc} &= \frac{1}{r}\sum_{j=1}^{r} S_{ihj:rssmc}, \qquad i \neq h \end{aligned} \tag{3.35}$$

finally, the vector $\bar{\bar{X}}_{rssmc}$ is estimated by using the member $\left(\bar{\bar{X}}_{i:rssmc}\right)$, and the matrix S is calculated as following;

$$S = \begin{bmatrix} S^2_{1:rssmc} & S_{12:rssmc} \\ S_{12:rssmc} & S^2_{2:rssmc} \end{bmatrix} \tag{3.36}$$

Step 1.4 calculate $T^2_{j:rssmc}$ from statistic value T^2

$$\begin{aligned} T^2 = & \frac{n}{(S^2_{1:rssmc}S^2_{2:rssmc} - S^2_{12:rssmc})}\left[S^2_{2:rssmc}\left(\bar{X}_{1:rssmc} - \bar{\bar{X}}_{1:rssmc}\right)^2\right. \\ & + S^2_{1:rssmc}\left(\bar{X}_{2:rssmc} - \bar{\bar{X}}_{2:rssmc}\right)^2 \\ & \left. - 2S_{12:rssmc}\left(\bar{X}_{1:rssmc} - \bar{\bar{X}}_{1:rssmc}\right)\left(\bar{X}_{2:rssmc} - \bar{\bar{X}}_{2:rssmc}\right)\right] \end{aligned} \tag{3.37}$$

Step 2 estimate the upper control limit before establishing control chart for the case that out of control will occur when there is at least one value that $T^2_{j:rssmc} > UCL$, as following;

Step 2.1 estimate the upper control limit from the calculation of $T^2_{\alpha,2,(n-1)}$

$$UCL = \chi^2_{\alpha,p} \tag{3.38}$$

Step 2.2 compare $T^2_{j:rssmc}$ with the estimated UCL

if $T^2_{j:rssmc} > UCL$, $\quad j = 1, 2, ..., r$ at least one value, then out of control.

3.3 Calculation of average run length of production line

Average run length (ARL) is the average sample number that is required to be investigated until the production is out of control, where ARL can be calculated from;

$$ARL = \frac{1}{M}\sum_{i=1}^{M} R_i \tag{3.39}$$

when M is the number of experiment in estimating the average run length and R_i is the number of samples that is investigated until the production is out of control in the experiment i.