

REFERENCES

1. Aphirukmatakun, C., Dejdumrong, N., 2010, "Multiple Degree Elevation and Constrained Multiple Degree Reduction for DP Curves and Surfaces", **Computers and Mathematics with Applications**. (In Press)
2. Ball, A.A., 1974, "CONSURF, Part one: Introduction to Conic Lifting Title", **Computer-Aided Design** 6, pp. 243-249.
3. Ball, A.A., 1975, "CONSURF, Part two: Description of the Algorithms", **Computer-Aided Design** 7, pp. 237-242.
4. Ball, A.A., 1977, "CONSURF, Part three: How the Program is Used", **Computer-Aided Design** 9, pp. 9-12.
5. Boehm, W, Farin, G., Kahmann J., 1984, "A Survey of Curve and Surface Methods in CAGD", **Computer Aided Geometric Design**, pp. 1-60.
6. Chen J. and Wang G-J., 2008, "Construction of Triangular DP Surface and Its Application", **Journal of Computational and Applied Mathematics**, 219(1), pp. 312-326.
7. Dejdumrong, N., 2001, "Rational Wang-Ball Curves", **International Journal of Mathematical Education in Science and Technology**, 4 July, Vol. 32, pp. 565-584.
8. Dejdumrong, N., 2006, "Rational DP Curves", **Third International Conference on Computer Graphics, Imaging and Visualization (CGIV 2006)**, Sydney, Australia, pp. 478-483.
9. Dejdumrong, N., 2008, "Efficient Algorithms for Non-Rational and Rational Bézier Curves", **The 5th International Conference on Computer Graphics, Imaging and Visualization (CGIV'08)**, Penang, Malaysia, pp. 109-114.
10. Dejdumrong, N., 2010, "A New Bivariate Basis Representation for Bezier-Based Triangular Patches with Quadratic Complexity", **Computers and Mathematics with Applications**. (In Press)
11. Delgado, J. and Penã, J.M., 2003, "A Shape Preserving Representation with an Evaluation Algorithm of Linear Complexity", **Computer Aided Geometric Design**, pp. 1-10.
12. Farin, G., 1985, "Triangular Bernstein-Bézier Patches", **Computer Aided Geometric Design**, 2, pp. 83-127.
13. Farin, G., 2002, "Curves and Surfaces for Computer Aided Geometric Design", **A Practical Guide**, 5th ed., Academic Press Inc., London.
14. Goodman, T.N.T. and Said, H.B., 1989, "Properties of Generalized Ball Curves and Surfaces", **Computer Aided Design**, Vol.23, i8, pp. 554-560.

15. Goodman, T.N.T. and Said, H.B., May 1991, "Shape Preserving Properties of the Generalised Ball Basis", **Computer Aided Geometric Design**, Vol.8 No.2, pp. 115-121.
16. Hu, S.M., Wang, G.J. and Jin, T.G., 1996, "Properties of Two Types of Generalized Ball Curves", **Computer-Aided Design**, Vol. 28, No. 2, pp. 125-133.
17. Hu, S.M., Wang, G.J. and Sun, J.G., 1998, "A Type of Triangular Ball Surface and Its Properties", **J. Comput. Sci. Technol**, Vol.13, i1, pp. 63-72.
18. Itsariyawanich, K., 2007, "**DP Curve and Its Properties**", Master Thesis, King Mongkut's University of Technology Thonburi, Thailand.
19. Itsariyawanich, K. and Dejrumrong, N., 2009, Erratum to: "Conversion and Evaluation for Two Types of Parametric Surface Constructed by NTP Bases", **Computers and Mathematics with Applications**, Vol. 49, No. 2, pp. 321-329.
20. Jiang, S.R. and Wang, G.J., 2005, "Conversion and Evaluation for Two Types of Parametric Surface Constructed by NTP Bases", **Computers and Mathematics with Applications**, Vol. 49, No. 2, pp. 321-329.
21. Phien, H.N. and Dejrumrong, N., "Efficient Algorithms for Bézier Curves", **Computer Aided Geometric Design**, 17, pp. 247-250.
22. Said, H.B., 1989, "A Generalized Ball Curve and Its Recursive Algorithm", **ACM Transactions on Graphics**, Vol. 8, No. 4, pp. 360-371.
23. Tien, H.L., 1997, "**Further Developments on Ball Curves and Surfaces**", Ph.D Dissertation, Asian Institute of Technology School of Advanced Technologies, Bangkok, Thailand.
24. To, T. and Phien, H., 1994, "Linking Bezier and Generalized Ball Curves", **In Proc. Int. Computer Symposium**, Vol. 2, pp. 738-743, Taiwan.
25. Wang, G.J., 1987, "Ball Curve of High Degree and Its Geometric Properties", **Appl. Math: A journal of Chinese Universities**, Vol. 2, No. 1, pp. 126-140.
26. Wang, G.J. and Cheng, M., "New Algorithms for Evaluating Parametric Surface", **Progress in Natural Science**, 11, pp. 142-148.

APPENDIX A

EXAMPLES

Conversions

Bézier to DP Control Points

For degree 4,

$$\mathbf{d}_{4,0,0} = \mathbf{b}_{4,0,0},$$

$$\mathbf{d}_{3,1,0} = \frac{3}{4}\mathbf{b}_{2,2,0} + \frac{1}{4}\mathbf{b}_{3,1,0},$$

$$\mathbf{d}_{3,0,1} = \frac{3}{4}\mathbf{b}_{2,0,2} + \frac{1}{4}\mathbf{b}_{3,0,1},$$

$$\mathbf{d}_{2,2,0} = \mathbf{b}_{2,2,0},$$

$$\mathbf{d}_{2,1,1} = \frac{1}{4}\mathbf{b}_{2,0,2} + \frac{1}{2}\mathbf{b}_{2,1,1} + \frac{1}{4}\mathbf{b}_{2,2,0},$$

$$\mathbf{d}_{2,0,2} = \mathbf{b}_{2,0,2},$$

$$\mathbf{d}_{1,3,0} = \frac{3}{4}\mathbf{b}_{2,2,0} + \frac{1}{4}\mathbf{b}_{1,3,0},$$

$$\mathbf{d}_{1,2,1} = \frac{1}{4}\mathbf{b}_{2,2,0} + \frac{1}{2}\mathbf{b}_{1,2,1} + \frac{1}{4}\mathbf{b}_{0,2,2},$$

$$\mathbf{d}_{1,1,2} = \frac{1}{4}\mathbf{b}_{2,0,2} + \frac{1}{2}\mathbf{b}_{1,1,2} + \frac{1}{4}\mathbf{b}_{0,2,2},$$

$$\mathbf{d}_{1,0,3} = \frac{3}{4}\mathbf{b}_{2,0,2} + \frac{1}{4}\mathbf{b}_{1,0,3},$$

$$\mathbf{d}_{0,4,0} = \mathbf{b}_{0,4,0},$$

$$\mathbf{d}_{0,3,1} = \frac{3}{4}\mathbf{b}_{0,2,2} + \frac{1}{4}\mathbf{b}_{0,3,1},$$

$$\mathbf{d}_{0,2,2} = \mathbf{b}_{0,2,2},$$

$$\mathbf{d}_{0,1,3} = \frac{3}{4}\mathbf{b}_{0,2,2} + \frac{1}{4}\mathbf{b}_{0,1,3},$$

$$\mathbf{d}_{0,0,4} = \mathbf{b}_{0,0,4}.$$

For degree 5,

$$\mathbf{d}_{5,0,0} = \mathbf{b}_{5,0,0}$$

$$\mathbf{d}_{4,1,0} = \frac{1}{5}\mathbf{b}_{2,2,1} + \frac{1}{5}\mathbf{b}_{2,3,0} + \frac{2}{5}\mathbf{b}_{3,2,0} + \frac{1}{5}\mathbf{b}_{4,1,0}$$

$$\mathbf{d}_{4,0,1} = \frac{1}{5}\mathbf{b}_{2,0,3} + \frac{1}{5}\mathbf{b}_{2,1,2} + \frac{2}{5}\mathbf{b}_{3,0,2} + \frac{1}{5}\mathbf{b}_{4,0,1}$$

$$\mathbf{d}_{3,2,0} = \frac{3}{10}\mathbf{b}_{2,2,1} + \frac{3}{10}\mathbf{b}_{2,3,0} + \frac{2}{5}\mathbf{b}_{3,2,0}$$

$$\mathbf{d}_{3,1,1} = \frac{1}{10}\mathbf{b}_{2,0,3} + \frac{1}{10}\mathbf{b}_{2,1,2} + \frac{1}{10}\mathbf{b}_{2,2,1} + \frac{1}{10}\mathbf{b}_{2,3,0} + \frac{3}{20}\mathbf{b}_{3,0,2} + \frac{3}{10}\mathbf{b}_{3,1,1} + \frac{3}{20}\mathbf{b}_{3,2,0}$$

$$\mathbf{d}_{3,0,2} = \frac{3}{10}\mathbf{b}_{2,0,3} + \frac{3}{10}\mathbf{b}_{2,1,2} + \frac{2}{5}\mathbf{b}_{3,0,2}$$

$$\mathbf{d}_{2,3,0} = \frac{3}{10}\mathbf{b}_{2,2,1} + \frac{2}{5}\mathbf{b}_{2,3,0} + \frac{3}{10}\mathbf{b}_{3,2,0}$$

$$\mathbf{d}_{2,2,1} = \frac{1}{30}\mathbf{b}_{0,2,3} + \frac{1}{30}\mathbf{b}_{0,3,2} + \frac{1}{30}\mathbf{b}_{1,2,2} + \frac{1}{30}\mathbf{b}_{2,0,3} + \frac{1}{30}\mathbf{b}_{2,1,2} + \frac{8}{15}\mathbf{b}_{2,2,1} + \frac{2}{15}\mathbf{b}_{2,3,0} + \frac{1}{30}\mathbf{b}_{3,0,2} + \frac{2}{15}\mathbf{b}_{3,2,0}$$

$$\mathbf{d}_{2,1,2} = \frac{1}{30}\mathbf{b}_{0,2,3} + \frac{1}{30}\mathbf{b}_{0,3,2} + \frac{1}{30}\mathbf{b}_{1,2,2} + \frac{2}{15}\mathbf{b}_{2,0,3} + \frac{8}{15}\mathbf{b}_{2,1,2} + \frac{1}{30}\mathbf{b}_{2,2,1} + \frac{1}{30}\mathbf{b}_{2,3,0} + \frac{2}{15}\mathbf{b}_{3,0,2} + \frac{1}{30}\mathbf{b}_{3,2,0}$$

$$\mathbf{d}_{2,0,3} = \frac{2}{5}\mathbf{b}_{2,0,3} + \frac{3}{10}\mathbf{b}_{2,1,2} + \frac{3}{10}\mathbf{b}_{3,0,2}$$

$$\mathbf{d}_{1,4,0} = \frac{1}{5}\mathbf{b}_{1,4,0} + \frac{1}{5}\mathbf{b}_{2,2,1} + \frac{2}{5}\mathbf{b}_{2,3,0} + \frac{1}{5}\mathbf{b}_{3,2,0}$$

$$\mathbf{d}_{1,3,1} = \frac{1}{10}\mathbf{b}_{0,2,3} + \frac{3}{20}\mathbf{b}_{0,3,2} + \frac{1}{10}\mathbf{b}_{1,2,2} + \frac{3}{10}\mathbf{b}_{1,3,1} + \frac{1}{10}\mathbf{b}_{2,2,1} + \frac{3}{20}\mathbf{b}_{2,3,0} + \frac{1}{10}\mathbf{b}_{3,2,0}$$

$$\mathbf{d}_{1,2,2} = \frac{2}{15}\mathbf{b}_{0,2,3} + \frac{2}{15}\mathbf{b}_{0,3,2} + \frac{8}{15}\mathbf{b}_{1,2,2} + \frac{1}{30}\mathbf{b}_{2,0,3} + \frac{1}{30}\mathbf{b}_{2,1,2} + \frac{1}{30}\mathbf{b}_{2,2,1} + \frac{1}{30}\mathbf{b}_{2,3,0} + \frac{1}{30}\mathbf{b}_{3,0,2} + \frac{1}{30}\mathbf{b}_{3,2,0}$$

$$\mathbf{d}_{1,1,3} = \frac{3}{20}\mathbf{b}_{0,2,3} + \frac{1}{10}\mathbf{b}_{0,3,2} + \frac{3}{10}\mathbf{b}_{1,1,3} + \frac{1}{10}\mathbf{b}_{1,2,2} + \frac{3}{20}\mathbf{b}_{2,0,3} + \frac{1}{10}\mathbf{b}_{2,1,2} + \frac{1}{10}\mathbf{b}_{3,0,2}$$

$$\mathbf{d}_{1,0,4} = \frac{1}{5}\mathbf{b}_{1,0,4} + \frac{2}{5}\mathbf{b}_{2,0,3} + \frac{1}{5}\mathbf{b}_{2,1,2} + \frac{1}{5}\mathbf{b}_{3,0,2}$$

$$\begin{aligned}
\mathbf{d}_{0,5,0} &= \frac{3}{20}\mathbf{b}_{0,2,3} + \frac{1}{10}\mathbf{b}_{0,3,2} + \frac{3}{10}\mathbf{b}_{1,1,3} + \frac{1}{10}\mathbf{b}_{1,2,2} + \frac{3}{20}\mathbf{b}_{2,0,3} + \frac{1}{10}\mathbf{b}_{2,1,2} + \frac{1}{10}\mathbf{b}_{3,0,2} \\
\mathbf{d}_{0,4,1} &= \frac{1}{5}\mathbf{b}_{0,2,3} + \frac{2}{5}\mathbf{b}_{0,3,2} + \frac{1}{5}\mathbf{b}_{0,4,1} + \frac{1}{5}\mathbf{b}_{1,2,2} \\
\mathbf{d}_{0,3,2} &= \frac{3}{10}\mathbf{b}_{0,2,3} + \frac{2}{5}\mathbf{b}_{0,3,2} + \frac{3}{10}\mathbf{b}_{1,2,2} \\
\mathbf{d}_{0,2,3} &= \frac{2}{5}\mathbf{b}_{0,2,3} + \frac{3}{10}\mathbf{b}_{0,3,2} + \frac{3}{10}\mathbf{b}_{1,2,2} \\
\mathbf{d}_{0,1,4} &= \frac{1}{5}\mathbf{b}_{0,1,4} + \frac{2}{5}\mathbf{b}_{0,2,3} + \frac{1}{5}\mathbf{b}_{0,3,2} + \frac{1}{5}\mathbf{b}_{1,2,2} \\
\mathbf{d}_{0,0,5} &= \mathbf{b}_{0,0,5}
\end{aligned}$$

DP to Bézier Control Points

For degree 4,

$$\begin{aligned}
\mathbf{b}_{4,0,0} &= \mathbf{d}_{4,0,0}, \\
\mathbf{b}_{3,1,0} &= 4\mathbf{d}_{3,1,0} - 3\mathbf{d}_{2,2,0}, \\
\mathbf{b}_{3,0,1} &= 4\mathbf{d}_{3,0,1} - 3\mathbf{d}_{2,0,2}, \\
\mathbf{b}_{2,2,0} &= \mathbf{d}_{2,2,0}, \\
\mathbf{b}_{2,1,1} &= 2\mathbf{d}_{2,1,1} - \frac{1}{2}\mathbf{d}_{2,0,2} - \frac{1}{2}\mathbf{d}_{2,2,0}, \\
\mathbf{b}_{2,0,2} &= \mathbf{d}_{2,0,2}, \\
\mathbf{b}_{1,3,0} &= 4\mathbf{d}_{1,3,0} - 3\mathbf{d}_{2,2,0}, \\
\mathbf{b}_{1,2,1} &= 2\mathbf{d}_{1,2,1} - \frac{1}{2}\mathbf{d}_{2,2,0} - \frac{1}{2}\mathbf{d}_{0,2,2}, \\
\mathbf{b}_{1,1,2} &= 2\mathbf{d}_{1,1,2} - \frac{1}{2}\mathbf{d}_{2,0,2} - \frac{1}{2}\mathbf{d}_{0,2,2}, \\
\mathbf{b}_{1,0,3} &= 4\mathbf{d}_{1,0,3} - 3\mathbf{d}_{2,0,2}, \\
\mathbf{b}_{0,4,0} &= \mathbf{d}_{0,4,0}, \\
\mathbf{b}_{0,3,1} &= 4\mathbf{d}_{0,3,1} - 3\mathbf{d}_{0,2,2}, \\
\mathbf{b}_{0,2,2} &= \mathbf{d}_{0,2,2}, \\
\mathbf{b}_{0,1,3} &= 4\mathbf{d}_{0,1,3} - 3\mathbf{d}_{0,2,2}, \\
\mathbf{b}_{0,0,4} &= \mathbf{d}_{0,0,4}.
\end{aligned}$$

For degree 5,

$$\begin{aligned}
\mathbf{b}_{5,0,0} &= \mathbf{d}_{5,0,0} \\
\mathbf{b}_{4,1,0} &= -\frac{8}{261}\mathbf{d}_{0,2,3} - \frac{8}{261}\mathbf{d}_{0,3,2} - \frac{2}{261}\mathbf{d}_{1,2,2} - \frac{8}{261}\mathbf{d}_{2,0,3} - \frac{2}{261}\mathbf{d}_{2,1,2} + \frac{178}{261}\mathbf{d}_{2,2,1} + \frac{712}{261}\mathbf{d}_{2,3,0} - \\
&\quad \frac{8}{261}\mathbf{d}_{3,0,2} - \frac{1898}{261}\mathbf{d}_{3,2,0} + 5\mathbf{d}_{4,1,0} \\
\mathbf{b}_{4,0,1} &= -\frac{8}{261}\mathbf{d}_{0,2,3} - \frac{8}{261}\mathbf{d}_{0,3,2} - \frac{2}{261}\mathbf{d}_{1,2,2} + \frac{712}{261}\mathbf{d}_{2,0,3} + \frac{178}{261}\mathbf{d}_{2,1,2} - \frac{2}{261}\mathbf{d}_{2,2,1} - \frac{8}{261}\mathbf{d}_{2,3,0} - \\
&\quad \frac{1898}{261}\mathbf{d}_{3,2,0} - \frac{8}{261}\mathbf{d}_{3,2,0} + 5\mathbf{d}_{4,0,1} \\
\mathbf{b}_{3,2,0} &= \frac{4}{87}\mathbf{d}_{0,2,3} + \frac{4}{87}\mathbf{d}_{0,3,2} + \frac{1}{87}\mathbf{d}_{1,2,2} + \frac{4}{87}\mathbf{d}_{2,0,3} + \frac{1}{87}\mathbf{d}_{2,1,2} - \frac{89}{87}\mathbf{d}_{2,2,1} - \frac{356}{87}\mathbf{d}_{2,3,0} + \frac{4}{87}\mathbf{d}_{3,0,2} + \\
&\quad \frac{514}{87}\mathbf{d}_{3,2,0} \\
\mathbf{b}_{3,1,1} &= -\frac{4}{783}\mathbf{d}_{0,2,3} - \frac{4}{783}\mathbf{d}_{0,3,2} - \frac{1}{783}\mathbf{d}_{1,2,2} + \frac{176}{783}\mathbf{d}_{2,0,3} + \frac{44}{783}\mathbf{d}_{2,1,2} + \frac{44}{783}\mathbf{d}_{2,2,1} + \frac{176}{783}\mathbf{d}_{2,3,0} - \\
&\quad \frac{1129}{783}\mathbf{d}_{3,0,2} + \frac{10}{3}\mathbf{d}_{3,1,1} - \frac{1129}{783}\mathbf{d}_{3,2,0} \\
\mathbf{b}_{3,0,2} &= \frac{4}{87}\mathbf{d}_{0,2,3} + \frac{4}{87}\mathbf{d}_{0,3,2} + \frac{1}{87}\mathbf{d}_{1,2,2} - \frac{356}{87}\mathbf{d}_{2,0,3} - \frac{89}{87}\mathbf{d}_{2,1,2} + \frac{1}{87}\mathbf{d}_{2,2,1} + \frac{4}{87}\mathbf{d}_{2,3,0} + \frac{514}{87}\mathbf{d}_{3,0,2} + \\
&\quad \frac{4}{87}\mathbf{d}_{3,2,0} \\
\mathbf{b}_{2,3,0} &= \frac{4}{87}\mathbf{d}_{0,2,3} + \frac{4}{87}\mathbf{d}_{0,3,2} + \frac{1}{87}\mathbf{d}_{1,2,2} + \frac{4}{87}\mathbf{d}_{2,0,3} + \frac{1}{87}\mathbf{d}_{2,1,2} - \frac{89}{87}\mathbf{d}_{2,2,1} + \frac{514}{87}\mathbf{d}_{2,3,0} + \frac{4}{87}\mathbf{d}_{3,0,2} -
\end{aligned}$$



$$\frac{356}{87} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{2,2,1} = -\frac{28}{261} \mathbf{d}_{0,2,3} - \frac{28}{261} \mathbf{d}_{0,3,2} - \frac{7}{261} \mathbf{d}_{1,2,2} - \frac{28}{261} \mathbf{d}_{2,0,3} - \frac{7}{261} \mathbf{d}_{2,1,2} + \frac{623}{261} \mathbf{d}_{2,2,1} - \frac{118}{261} \mathbf{d}_{2,3,0} - \frac{28}{261} \mathbf{d}_{3,0,2} - \frac{118}{261} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{2,1,2} = -\frac{28}{261} \mathbf{d}_{0,2,3} - \frac{28}{261} \mathbf{d}_{0,3,2} - \frac{7}{261} \mathbf{d}_{1,2,2} - \frac{118}{261} \mathbf{d}_{2,0,3} + \frac{623}{261} \mathbf{d}_{2,1,2} - \frac{7}{261} \mathbf{d}_{2,2,1} - \frac{28}{261} \mathbf{d}_{2,3,0} - \frac{118}{261} \mathbf{d}_{3,0,2} - \frac{28}{261} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{2,0,3} = \frac{4}{87} \mathbf{d}_{0,2,3} + \frac{4}{87} \mathbf{d}_{0,3,2} + \frac{1}{87} \mathbf{d}_{1,2,2} + \frac{514}{87} \mathbf{d}_{2,0,3} - \frac{89}{87} \mathbf{d}_{2,1,2} + \frac{1}{87} \mathbf{d}_{2,2,1} + \frac{4}{87} \mathbf{d}_{2,3,0} - \frac{356}{87} \mathbf{d}_{3,0,2} + \frac{4}{87} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{1,4,0} = -\frac{8}{261} \mathbf{d}_{0,2,3} - \frac{8}{261} \mathbf{d}_{0,3,2} - \frac{2}{261} \mathbf{d}_{1,2,2} + 5 \mathbf{d}_{1,4,0} - \frac{8}{261} \mathbf{d}_{2,0,3} - \frac{2}{261} \mathbf{d}_{2,1,2} + \frac{178}{261} \mathbf{d}_{2,2,1} - \frac{1898}{261} \mathbf{d}_{2,3,0} - \frac{8}{261} \mathbf{d}_{3,0,2} + \frac{712}{261} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{1,3,1} = \frac{176}{783} \mathbf{d}_{0,2,3} - \frac{1129}{783} \mathbf{d}_{0,3,2} + \frac{44}{783} \mathbf{d}_{1,2,2} + \frac{10}{3} \mathbf{d}_{1,3,1} - \frac{4}{783} \mathbf{d}_{2,0,3} - \frac{1}{783} \mathbf{d}_{2,1,2} + \frac{44}{783} \mathbf{d}_{2,2,1} - \frac{1129}{783} \mathbf{d}_{2,3,0} - \frac{4}{783} \mathbf{d}_{3,0,2} + \frac{176}{783} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{1,2,2} = \frac{118}{261} \mathbf{d}_{0,2,3} - \frac{118}{261} \mathbf{d}_{0,3,2} + \frac{623}{261} \mathbf{d}_{1,2,2} - \frac{28}{261} \mathbf{d}_{2,0,3} - \frac{7}{261} \mathbf{d}_{2,1,2} - \frac{7}{261} \mathbf{d}_{2,2,1} - \frac{28}{261} \mathbf{d}_{2,3,0} - \frac{28}{261} \mathbf{d}_{3,0,2} - \frac{28}{261} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{1,1,3} = -\frac{1129}{783} \mathbf{d}_{0,2,3} + \frac{176}{783} \mathbf{d}_{0,3,2} + \frac{10}{3} \mathbf{d}_{1,1,3} + \frac{44}{783} \mathbf{d}_{1,2,2} - \frac{1129}{783} \mathbf{d}_{2,0,3} + \frac{44}{783} \mathbf{d}_{2,1,2} - \frac{1}{783} \mathbf{d}_{2,2,1} - \frac{4}{783} \mathbf{d}_{2,3,0} + \frac{176}{783} \mathbf{d}_{3,0,2} - \frac{4}{783} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{1,0,4} = -\frac{8}{261} \mathbf{d}_{0,2,3} - \frac{8}{261} \mathbf{d}_{0,3,2} + 5 \mathbf{d}_{1,0,4} - \frac{2}{261} \mathbf{d}_{1,2,2} - \frac{1898}{261} \mathbf{d}_{2,0,3} + \frac{178}{261} \mathbf{d}_{2,1,2} - \frac{2}{261} \mathbf{d}_{2,2,1} - \frac{8}{261} \mathbf{d}_{2,3,0} + \frac{712}{261} \mathbf{d}_{3,0,2} - \frac{8}{261} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{0,5,0} = \mathbf{d}_{0,5,0}$$

$$\mathbf{b}_{0,4,1} = \frac{712}{261} \mathbf{d}_{0,2,3} - \frac{1898}{261} \mathbf{d}_{0,3,2} + 5 \mathbf{d}_{0,4,1} + \frac{178}{261} \mathbf{d}_{1,2,2} - \frac{8}{261} \mathbf{d}_{2,0,3} - \frac{2}{261} \mathbf{d}_{2,1,2} - \frac{2}{261} \mathbf{d}_{2,2,1} - \frac{8}{261} \mathbf{d}_{2,3,0} - \frac{8}{261} \mathbf{d}_{3,0,2} - \frac{8}{261} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{0,3,2} = -\frac{356}{87} \mathbf{d}_{0,2,3} + \frac{514}{87} \mathbf{d}_{0,3,2} - \frac{89}{87} \mathbf{d}_{1,2,2} + \frac{4}{87} \mathbf{d}_{2,0,3} + \frac{1}{87} \mathbf{d}_{2,1,2} + \frac{1}{87} \mathbf{d}_{2,2,1} + \frac{4}{87} \mathbf{d}_{2,3,0} + \frac{4}{87} \mathbf{d}_{3,0,2} + \frac{4}{87} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{0,2,3} = \frac{514}{87} \mathbf{d}_{0,2,3} - \frac{356}{87} \mathbf{d}_{0,3,2} - \frac{89}{87} \mathbf{d}_{1,2,2} + \frac{4}{87} \mathbf{d}_{2,0,3} + \frac{1}{87} \mathbf{d}_{2,1,2} - \frac{1}{87} \mathbf{d}_{2,2,1} + \frac{4}{87} \mathbf{d}_{2,3,0} + \frac{4}{87} \mathbf{d}_{3,0,2} + \frac{4}{87} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{0,1,4} = 5 \mathbf{d}_{0,1,4} - \frac{1898}{261} \mathbf{d}_{0,2,3} + \frac{712}{261} \mathbf{d}_{0,3,2} + \frac{178}{261} \mathbf{d}_{1,2,2} - \frac{8}{261} \mathbf{d}_{2,0,3} - \frac{2}{261} \mathbf{d}_{2,1,2} - \frac{2}{261} \mathbf{d}_{2,2,1} - \frac{8}{261} \mathbf{d}_{2,3,0} - \frac{8}{261} \mathbf{d}_{3,0,2} - \frac{8}{261} \mathbf{d}_{3,2,0}$$

$$\mathbf{b}_{0,0,5} = \mathbf{d}_{0,0,5}$$

Degree Elevation

For degree 2,

$$\mathbf{d}_{2,0,0}^{(1)} = \mathbf{d}_{1,0,0},$$

$$\mathbf{d}_{1,1,0}^{(1)} = \frac{1}{2} \mathbf{d}_{1,0,0} + \frac{1}{2} \mathbf{d}_{0,1,0},$$

$$\mathbf{d}_{1,0,1}^{(1)} = \frac{1}{2} \mathbf{d}_{1,0,0} + \frac{1}{2} \mathbf{d}_{0,0,1},$$

$$\mathbf{d}_{0,2,0}^{(1)} = \mathbf{d}_{0,1,0},$$

$$\mathbf{d}_{0,1,1}^{(1)} = \frac{1}{2} \mathbf{d}_{0,1,0} + \frac{1}{2} \mathbf{d}_{0,0,1},$$

$$\mathbf{d}_{0,0,2}^{(1)} = \mathbf{d}_{0,0,1}.$$

For degree 3,

$$\mathbf{d}_{3,0,0}^{(1)} = \mathbf{d}_{2,0,0},$$

$$\mathbf{d}_{2,1,0}^{(1)} = \frac{1}{3}\mathbf{d}_{2,0,0} + \frac{2}{3}\mathbf{d}_{1,1,0},$$

$$\mathbf{d}_{2,0,1}^{(1)} = \frac{1}{3}\mathbf{d}_{2,0,0} + \frac{2}{3}\mathbf{d}_{1,0,1},$$

$$\mathbf{d}_{1,2,0}^{(1)} = \frac{1}{3}\mathbf{d}_{0,2,0} + \frac{2}{3}\mathbf{d}_{1,1,0},$$

$$\mathbf{d}_{1,1,1}^{(1)} = \frac{1}{3}\mathbf{d}_{1,1,0} + \frac{1}{3}\mathbf{d}_{0,1,0} + \frac{1}{3}\mathbf{d}_{0,0,1},$$

$$\mathbf{d}_{1,0,2}^{(1)} = \frac{1}{3}\mathbf{d}_{0,0,2} + \frac{2}{3}\mathbf{d}_{1,0,1},$$

$$\mathbf{d}_{0,3,0}^{(1)} = \mathbf{d}_{0,2,0},$$

$$\mathbf{d}_{0,2,1}^{(1)} = \frac{1}{3}\mathbf{d}_{0,2,0} + \frac{2}{3}\mathbf{d}_{0,1,1},$$

$$\mathbf{d}_{0,1,2}^{(1)} = \frac{1}{3}\mathbf{d}_{0,0,2} + \frac{2}{3}\mathbf{d}_{0,1,1},$$

$$\mathbf{d}_{0,0,3}^{(1)} = \mathbf{d}_{0,0,2}.$$

For degree 4,

$$\mathbf{d}_{4,0,0}^{(1)} = \mathbf{d}_{3,0,0},$$

$$\mathbf{d}_{3,1,0}^{(1)} = \mathbf{d}_{3,0,0} + \frac{3}{2}\mathbf{d}_{2,1,0} - \frac{3}{2}\mathbf{d}_{1,2,0},$$

$$\mathbf{d}_{3,0,1}^{(1)} = \mathbf{d}_{3,0,0} + \frac{3}{2}\mathbf{d}_{2,0,1} - \frac{3}{2}\mathbf{d}_{1,0,2},$$

$$\mathbf{d}_{2,2,0}^{(1)} = \frac{1}{2}\mathbf{d}_{2,1,0} + \frac{1}{2}\mathbf{d}_{1,2,0},$$

$$\mathbf{d}_{2,1,1}^{(1)} = \mathbf{d}_{1,1,1} + \frac{1}{4}\mathbf{d}_{2,1,0} + \frac{1}{4}\mathbf{d}_{2,0,1} - \frac{1}{4}\mathbf{d}_{1,2,0} - \frac{1}{4}\mathbf{d}_{1,0,2},$$

$$\mathbf{d}_{2,0,2}^{(1)} = \frac{1}{2}\mathbf{d}_{2,0,1} + \frac{1}{2}\mathbf{d}_{1,0,2},$$

$$\mathbf{d}_{1,3,0}^{(1)} = \mathbf{d}_{0,3,0} + \frac{3}{2}\mathbf{d}_{1,2,0} - \frac{3}{2}\mathbf{d}_{2,1,0},$$

$$\mathbf{d}_{1,2,1}^{(1)} = \mathbf{d}_{1,1,1} + \frac{1}{4}\mathbf{d}_{1,2,0} + \frac{1}{4}\mathbf{d}_{0,2,1} - \frac{1}{4}\mathbf{d}_{2,1,0} - \frac{1}{4}\mathbf{d}_{0,1,2},$$

$$\mathbf{d}_{1,1,2}^{(1)} = \mathbf{d}_{1,1,1} + \frac{1}{4}\mathbf{d}_{1,0,2} + \frac{1}{4}\mathbf{d}_{0,1,2} - \frac{1}{4}\mathbf{d}_{2,0,1} - \frac{1}{4}\mathbf{d}_{0,2,1},$$

$$\mathbf{d}_{1,0,3}^{(1)} = \mathbf{d}_{0,0,3} + \frac{3}{2}\mathbf{d}_{1,0,2} - \frac{3}{2}\mathbf{d}_{2,0,1},$$

$$\mathbf{d}_{0,4,0}^{(1)} = \mathbf{d}_{0,3,0},$$

$$\mathbf{d}_{0,3,1}^{(1)} = \mathbf{d}_{0,3,0} + \frac{3}{2}\mathbf{d}_{0,2,1} - \frac{3}{2}\mathbf{d}_{0,1,2},$$

$$\mathbf{d}_{0,2,2}^{(1)} = \frac{1}{2}\mathbf{d}_{0,2,1} + \frac{1}{2}\mathbf{d}_{0,1,2},$$

$$\mathbf{d}_{0,1,3}^{(1)} = \mathbf{d}_{0,0,3} + \frac{3}{2}\mathbf{d}_{0,1,2} - \frac{3}{2}\mathbf{d}_{0,2,1},$$

$$\mathbf{d}_{0,0,4}^{(1)} = \mathbf{d}_{0,0,3}.$$

CURRICULUM VITAE

NAME	Mr. Podcharid Krungkarnchana
DATE OF BIRTH	21 November 1985
EDUCATION RECORD	
HIGH SCHOOL	High School Graduation Satit Rajabhat School, Nakhon Prathom, 2003
BACHELOR'S DEGREE	Bachelor of Engineering (Computer Engineering), King Mongkut's University of Technology Thonburi, 2007
MASTER'S DEGREE	Master of Engineering (Computer Engineering), King Mongkut's University of Technology Thonburi, 2010
PUBLICATIONS	<p>Krungkarnchana, P. and Dejdumrong, N., 2009, "A New Model of Triangular DP Surfaces and Its Properties", The 6th International Conference on Computer Graphics, Imaging and Visualization (CGIV'09), Tianjin, China, August 11-14, 2009, pp. 245 - 249.</p> <p>Krungkarnchana, P. and Dejdumrong, N., 2010, "An Approach to the Determination of DP Triangular Surfaces with Its Quadratic Complexity", The 7th International Conference organized by Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology Association (ECTI-CON'2010), Chiang Mai, Thailand, May 19-22, 2010, pp. 1236 - 1240.</p>



King Mongkut's University of Technology Thonburi

Agreement on Intellectual Property Rights Transfer for Postgraduate Students

Date...January 07. 2011

Name Mr. Podcharid Middle Name.....Surname/Family Name

Krungkarnchana Student Number 51450013 who is a student of King's Mongkut's University of Technology Thonburi (KMUTT) in Graduate Diploma Master Degree

Doctoral Degree

Program Master of Science Field of Study Computer Engineering Faculty/School Engineering

Home Address 33/2 Nangsoifa Road Thapeeleang District Muang Province Suphanburi

Postal Code 72000 Country Thailand

I, as 'Transferer', hereby transfer the ownership of my thesis copyright to King's Mongkut's University of Technology Thonburi who has appointed Assoc. Prof. Dr. Piyabutr Wanichpongpan Associate Dean for Academic Affairs (Acting for Dean) to be 'Transferee' of copyright ownership under the 'Agreement' as follows.

1. I am the author of the thesis entitled A New Construction of DP Triangular Patches and Its Applications under the supervision of Asst. Prof. Dr. Natasha Dejdumrong who is my supervisor, in accordance with the Thai Copyright Act B.E. 2537. The thesis is a part of the curriculum of KMUTT.

2. I hereby transfer the copyright ownership of all my works in the thesis to KMUTT throughout the copyright protection period in accordance with the Thai Copyright Act B.E. 2537, effective on the approval date of thesis proposal consented by KMUTT.

3. To have the thesis distributed in any form of media, I shall in each and every case stipulate the thesis as the work of KMUTT.

4. For my own distribution of thesis or the reproduction, adjustment, or distribution of thesis by the third party in accordance with the Thai Copyright Act B.E. 2537 with remuneration in return, I am subject to obtain a prior written permission from KMUTT.

5. To use any information from my thesis to make an invention or create any intellectual property works within ten (10) years from the date of signing this Agreement, I am subject to obtain prior written permission from KMUTT, and KMUTT is entitled to have intellectual property rights on such inventions or intellectual property works, including entitling to take royalty from licensing together with the distribution of any benefit deriving partly or wholly from the works in the future, conforming with the Regulation of King Mongkut's Institute of Technology Thonburi Re the Administration of Benefits deriving from Intellectual Property B.E. 2538.

6. If the benefits arise from my thesis or my intellectual property works owned by KMUTT, I shall be entitled to gain the benefits according to the allocation rate stated in the

Regulation of King Mongkut's Institute of Technology Thonburi Re the Administration of Benefits deriving from Intellectual Property B.E. 2538.

Signature.....Podcharid K.....Transferor


(Mr. Podcharid Krungkarnchana)

Student

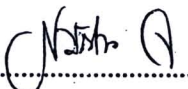
Signature..........Transferee

(Assoc. Prof. Dr. Piyabutr Wanichpongpan)

Associate Dean for Academic Affairs (Acting for Dean)

Signature..........Witness

(Assoc. Prof. Dr. Tiranee Achalakul)

Signature..........Witness

(Asst. Prof. Dr. Natasha Dejdumrong)

