CHAPTER 4 COMPARISONS AMONG THREE TYPES OF TRIANGULAR DP SURFACES

In this chapter, we compare three types of triangular DP surfaces from different perspectives, i.e., the basis functions, computational complexity, linear independence property and shape preserving property.

4.1 Basis Functions

The polynomials are the key features of this comparison. The polynomials of triangular DP surfaces should require fewer coefficient values, as is true for the concepts of the DP curve's polynomials. Moreover, the summation of all new proposed polynomials in each degree should sum to 1.

Table 4.1 Comparison among polynomials for three types of triangular DP surfaces of degree 4

Points	DP (2008)	Our First DP	Our Second DP
$4,0,0$	$\frac{1}{3}u^4$	u^4	u^4
$3,1,0$	$\frac{1}{3}u^3v$	u^3v	u^3v
$3,0,1$	$\frac{1}{3}u^3w$	u^3w	u^3w
$2,2,0$	$\frac{1}{3}(u^2v+uv^2)$	u^2v+uv^2	$3uv(u+v)$
$2,1,1$	$\frac{1}{3}-\frac{1}{3}(u^2+v^2+w^2)$	$\frac{1}{3}((u+v)w+(u+w)v+(v+w)u)$	$6u^2vw$
$2,0,2$	$\frac{1}{3}(u^2w+uw^2)$	u^2w+uw^2	$3uw(u+w)$
$1,3,0$	$\frac{1}{3}uv^3$	uv^3	uv^3
$1,2,1$	$\frac{1}{3}-\frac{1}{3}(u^2+v^2+w^2)$	$\frac{1}{3}((u+v)w+(u+w)v+(v+w)u)$	$6uv^2w$
$1,1,2$	$\frac{1}{3}-\frac{1}{3}(u^2+v^2+w^2)$	$\frac{1}{3}((u+v)w+(u+w)v+(v+w)u)$	$6uvw^2$
$1,0,3$	$\frac{1}{3}uw^3$	uw^3	uw^3
$0,4,0$	$\frac{1}{3}v^4$	v^4	v^4
$0,3,1$	$\frac{1}{3}v^3w$	v^3w	v^3w
$0,2,2$	$\frac{1}{3}(v^2w+vw^2)$	v^2w+vw^2	$3vw(v+w)$
$0,1,3$	$\frac{1}{3}vw^3$	vw^3	vw^3
$0,0,4$	$\frac{1}{3}w^4$	w^4	w^4

From the observation of the polynomials in each model, it is obvious that only our second proposed triangular DP surface satisfies the desired characteristics of DP curves.

4.2 Computational Complexity

Our two models of triangular DP surface are quadratic in computational complexity, $O(n^2)$. Both surfaces were proven to be calculated faster than either the triangular DP surface (2008) or Bézier Surface, which is $O(n^3)$, as shown in Table 4.2.

Table 4.2 Number of interpolations for three types of triangular DP surfaces

Degree (n)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
DP (2008)	1	4	7	14	24	40	61	90	126	172	227	294	372
Our First DP	1	4	7	11	14	20	24	30	36	43	49	58	65
Our Second DP	1	4	10	14	23	32	42	54	69	82	100	118	137

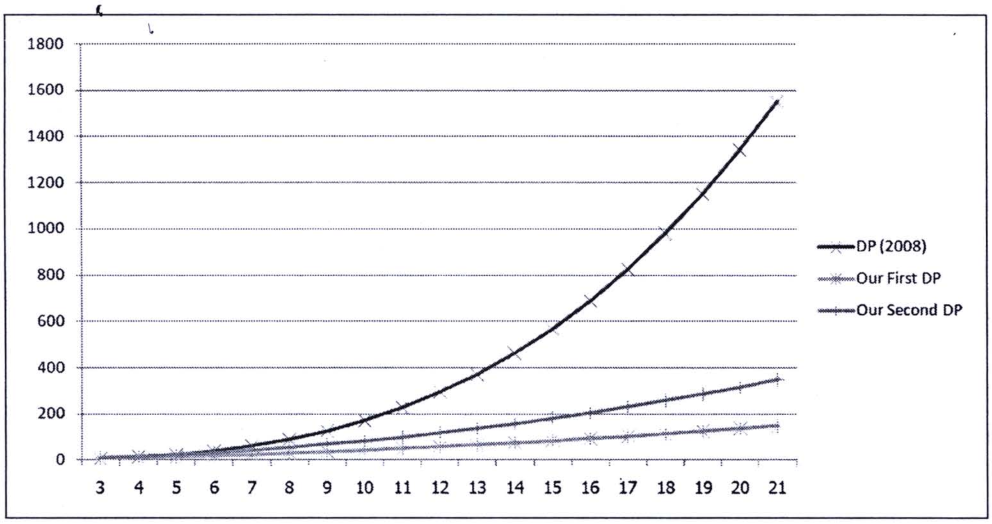

Figure 4.1 The Number of Interpolations for Three Types of Triangular DP Surfaces

It is obvious that our two models use less computational time than the triangular DP surface (2008). Although our first model of triangular DP surface is the fastest algorithm, it does not satisfy the linearly independent property.

4.3 Convex Hull Property

Convex hull property is an important property. It cans guarantee that the surface will lie inside the convex hull of its control net. However, triangular DP surface (2008) [6] did not satisfy the affine and convexity conditions in geometric modeling since

$$\sum_{i+j+k=n} D^n_{i,j,k}(u,v,w) \neq 1, n \geq 3. \qquad (4.1)$$

Thus, triangular DP surface (2008) lacks the convexity property. However, our two models possess the convexity property.

4.4 Orthogonal Property

Only our second model of triangular DP surface possesses both the convexity property and quadratic complexity $O(n^2)$. The triangular DP surface (2008) lacks of the convexity properly and our first model of triangular DP surface is not linearly independent, $|M_D| = 0$, M_D^{-1} does not exist for degree $n \geq 5$.

4.5 Illustration of DP Triangular Patches

The parametric 3D plot will show us about shape preserving property. If the surface nearly lies on every control point, the algorithm has shape preserving property.

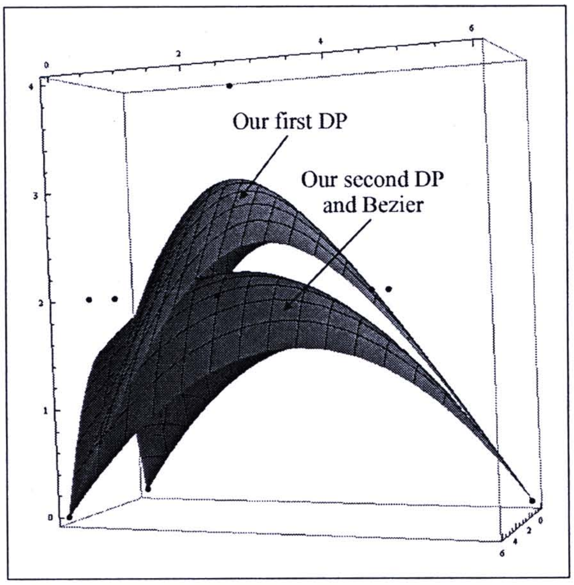

Figure 4.2 3D Parametric Plot for Triangular DP Surfaces of Degree 3

The parametric 3D plot of triangular DP surfaces for degree 3 is shown in Figure 4.2. At degree 3, our second model use the same algorithm with Bézier . Consequently, the result will be the same. Our first model seems applied from the real characteristics of DP curve. Beginning with degree 3, Chen's model can not be shown by parametric 3D plot because it lacks the convexity property.

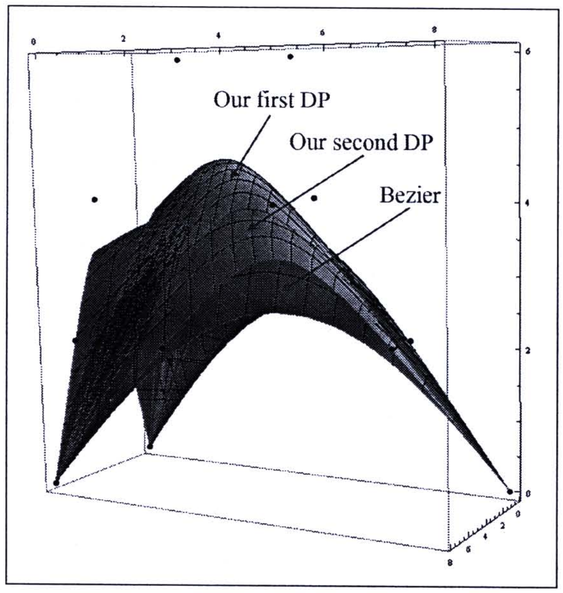

Figure 4.3 3D Parametric Plot for Triangular DP Surfaces of Degree 4

The parametric 3D plot of triangular DP surfaces for degree 4 is shown in Figure 4.3. At degree 4, both of our models exhibit the real characteristics of DP curve. Chen's model can not be shown by parametric 3D plot in this degree.

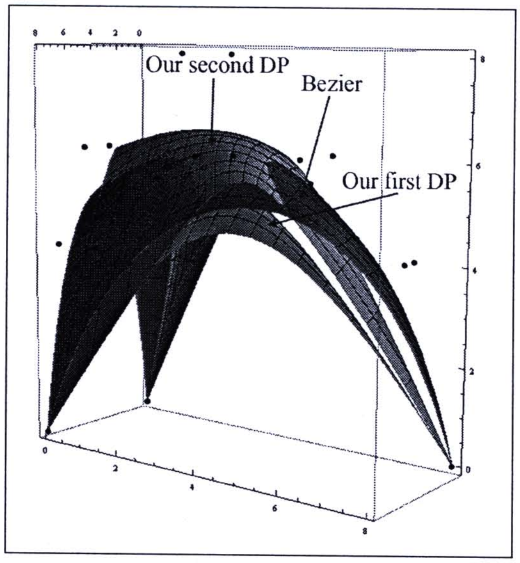

Figure 4.4 3D Parametric Plot for Triangular DP Surfaces of Degree 5

The parametric 3D plot of triangular DP surfaces for degree 5 is shown in Figure 4.4. At degree 5, only the second model is applied from the DP curve. Unfortunately, the first model seems to be not exactly applied from the DP curve, unlike the previous degree. Chen's model also could not be shown by parametric 3D plot in this degree.

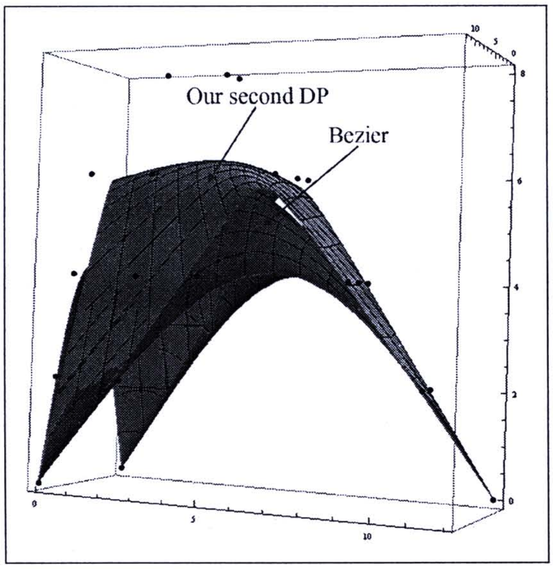

Figure 4.5 3D Parametric Plot for Triangular DP Surfaces of Degree 6

The parametric 3D plot of triangular DP surfaces for degree 6 is shown in Figure 4.5. At degree 6, only the second model can be shown by parametric 3D plot. The first model can not be shown by parametric 3D plot because it lacks of linear independence property. Chen's model also can not be shown by parametric 3D plot in this degree.

It is obvious that our second triangular DP surface is directly applied from the DP curve. Nevertheless, beginning with degree 3, triangular DP surface (2008) can not be shown by parametric 3D plot because it lacks the convexity property. Beginning with degree 6, our first model also can not be shown by parametric 3D plot because it lacks the linear independence property.