CHAPTER 1 INTRODUCTION

1.1 Statement of the Problem

In general, there are two major varieties of surfaces: *rectangular* and *triangular* surfaces. Most of the early Computer-Aided Design (CAD) efforts were developed in the automobile industry. Rectangular patches are suitably used for car body design such as roof, hood, and doors. These parts fundamentally have a rectangular geometry. It seems natural to apply rectangular patches for other parts, for example the design of the interior car body panels. However, such structures do not possess a rectangular structure, and rectangular patches are not a good choice for modeling these complex geometries. Since triangular patches are better suited to model complex geometries than rectangular patches, it seems obvious that triangular patches should be added into CAD systems.

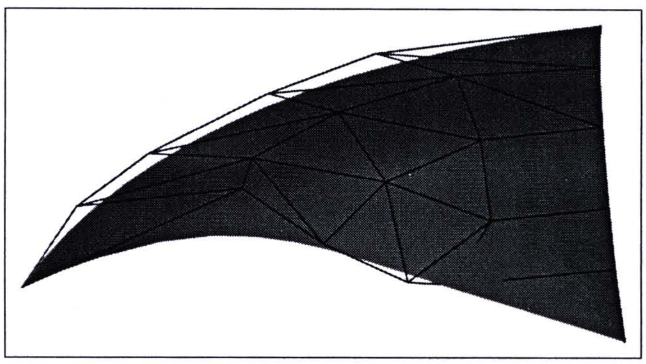

Figure 1.1 Example of Triangular Meshes

A triangle mesh is a type of polygon mesh in computer graphics. It includes a set of triangles that are connected by their edges or corners. Hardware devices can operate on triangles that are grouped into meshes more efficiently than on a similar number of triangles that are presented individually. When using individual triangles, the hardware device has to operate on three vertices for every triangle. In a large geometry, there could be many triangles meeting at a single vertex. A mesh makes it possible to process those vertices just once Thus it is possible to do a fraction of the work and achieve the same effect.

Triangular Bézier surface has been specifically used in several Computer-Aided Design and Computer-Aided Manufacturing (CAD/CAM) and Computer-Aided Geometric Design (CAGD) applications, especially in subtle geometric and shape design. However, this type of surface has cubic computational complexity. Some impressive work has paid attentions to the reduction of the evaluation time for the rectangular surfaces [6, 11, 20, 21]. The techniques simply convert a Bézier control net into a new form of several curves with linear complexity, i.e., Said-Ball, Wang-Ball, and DP curves. Although these techniques can reduce the evaluation time from cubic to quadratic computation, unfortunately, they can not be readily applied for the case of triangular patches. Triangular surfaces must be expressed in the forms of bivariate polynomials while curves and rectangular surfaces are simply defined by univariate functions.

There have been several attempts to propose models of triangular surfaces in order to decrease the surface construction time [6, 15, 17] as well as proposing new unified approaches of triangulation. Triangular Said-Ball surface was first introduced in 1991 by Goodman and Said. This surface has not much used because it still requires cubic computational time, $O(n^3)$. That is the same as that of Bézier triangular patch. Later, two different types of triangular surfaces were established by Hu et. al. [17] in 1998 and by Chen [6] in 2008. Nevertheless, the computational time of these two types of surfaces cannot be reduced to quadratic complexity, $O(n^2)$. Moreover, the work of Chen [6] lacks a partition of unity that is one of the most important characteristics in curve approximation modeling.

1.2 Objectives of the Study

The main objective of this study is to construct a new model of triangular DP surfaces and to document its important properties. In the future, this model may be used in CAD/CAM and CAGD applications for faster triangular surface construction. This main objective can be divided into two sub-objectives as follows:

1. Define a new model of triangular DP surfaces with quadratic time computation.
2. Propose an efficient algorithm for evaluating DP triangular surfaces that is better than the algorithm proposed by Chen [6] in 2008.

1.3 Scope of the Study

The following list is the scope of the study in order to complete the work.

1. Define a new DP bivariate basis functions for a model of triangular DP surfaces.
2. Provide recursive formulae for evaluating a point on a DP triangular patch.
3. Prove the convexity of this model, i.e., prove for affine and convex combination of this new DP bivariate basis.
4. Provide some examples of the conversions from Bézier into DP triangular surfaces and vice versa.

5. Offer somes examples of the relationships of degree elevation.

1.4 Organization of the Thesis

This thesis consists of five chapters. Statement of the problem, objectives and scope of the study are explicitly stated in Chapter 1. Next, Chapter 2 reviews a few essential theories related to the study. The theories comprise algorithm concepts for curve and triangular surface modeling, the degree elevation and the degree reduction for DP curves. In Chapter 3, a new triangular DP basis, its recurrence algorithm, recursive formulae, degree elevation, conversion between Bézier and DP triangular surfaces and an evaluation algorithm with quadratic complexity, $O(n^2)$, are proposed. Chapter 4 Compares the three types of triangular DP surfaces. Finally, discusses the conclusion, limitations and future work recommendations Chapter 5.