บทที่ 5

Conclusion

We consider DBI dark energy motivated from open string constructions as [33, 34] where the solitary $D3$-brane moving through a particular warped compactification of type IIB. Coupling between scalar field and dark matter creating the fifth force is reduced by chameleon mechanism [35]. The chameleon interaction can be suppressed in laboratory experiments, due to mass dependent on local matter density environment. The DBI scalar field action is

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\Big[-T\left(\Gamma - 1\right) - V\Big]$$

We have finally found the DBI equation of motion,

$$-\frac{T'}{2\Gamma}\left(\Gamma-1\right)^2 - V' + \frac{\Box^2\phi}{\Gamma} - \frac{1}{\Gamma^2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\Gamma) = -\frac{\mathcal{L}'_m}{\sqrt{-g}}$$

and the full DBI field equation of motion with chameleon mechanism is found to be

$$-\frac{T'}{2\Gamma}(\Gamma-1)^2 + \frac{\Box^2\phi}{\Gamma} - \frac{g^{\mu\nu}}{\Gamma^2}\partial_\mu\phi\partial_\nu\Gamma = V' + \frac{\beta\rho}{M_p}(1-3\omega)e^{(1-3\omega)\beta\phi/M_p}$$

The static case solution is found using exponential integral function for

- (1) small field velocities

  As $\Gamma = 1$ (approaching canonical case), the solution is

$$\begin{aligned}\phi(r) \;\simeq\; & \phi_\infty \\ & + \frac{A}{r}\, e^{-m_\infty(r-R)}(1+\epsilon) \\ & - \frac{3\epsilon A^2}{4}\frac{m_\infty^2\phi_\infty}{r^2 T}\, e^{-2m_\infty(r-R)} \\ & - \frac{3\epsilon A}{8}\frac{m_\infty^3\phi_\infty}{rT}\left[e^{m_\infty(r+2R)}\mathrm{Ei}(-3m_\infty r) + e^{-m_\infty(r-2R)}\mathrm{Ei}(-m_\infty r)\right] \\ & + \ldots\end{aligned}$$

- (2) relativistic case

  As $\Gamma \gg 1$ then $\phi_r^2 \gg T$ and we can reasonably assume $\phi_r \gg 1$ with $\phi_{rr} \sim 0$, then the solution is becomes

$$\phi(r) \simeq \phi_\infty + \frac{2\sqrt{T}}{rm_\infty^2}$$

When assuming realistic situation $V(\phi)$ is a runaway potential. The tension $T(\phi)$ is chosen to be

$$T(\phi) = T_0(1+\frac{\phi^2}{M^2})^2$$

For a spherical compact body, the DBI chameleon will lead infinite series in $1/r^{l+2}$ correction to Newton's law which a modified gravitational potential

$$\Phi(r) = \Phi_N(r) + \beta\frac{\phi(r)}{M_P}$$

The Eddington parameters involving the $1/r$ and $1/r^2$ corrections to the Newton potential is found to be

$$\phi_0 = \phi_\infty - \frac{X}{r}, \qquad X \sim \frac{G_N m_0 \phi_\infty}{\Phi_N}$$

and the thin-shell constraint is $\phi_\infty \leq 10^{-26} M_P$.