

บทที่ 4

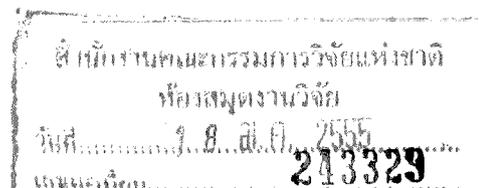
DBI Chameleons with runaway potential and string-inspired tension

We consider the dynamics of a scalar field coupled to matter according to

$$S = \int d^4x \left[\frac{R}{2\kappa_4^2} - T(\phi) \sqrt{1 + \frac{\partial_\mu \phi \partial^\mu \phi}{T(\phi)}} + V(\phi) - T(\phi) \right] + S_m(\psi_m, A^2(\phi)g_{\mu\nu}) \quad (4.1)$$

where $A(\phi) = e^{\beta\phi/m_{\text{Pl}}}$ and $V(\phi)$ is a runaway potential. The tension $T(\phi)$ is chosen to be

$$T(\phi) = T_0 \left(1 + \frac{\phi^2}{M^2}\right)^2 \quad (4.2)$$



When $\phi \ll M$, the tension is constant while $T(\phi) \sim \phi^4$ in an AdS region $\phi \gg M$.

The equations of motion for such an action read

$$\square\phi - \frac{1}{\Gamma}(\partial\phi)^2 = T'\Gamma(\Gamma - 1) + V'\Gamma - \frac{1}{2}T'(\Gamma^2 - 1) + \beta\frac{\rho}{m_{\text{Pl}}}\Gamma \quad (4.3)$$

in the presence of non-relativistic matter of density ρ .

4.1 Cosmological evolution

At large scale, the cosmological evolution of the field is governed by

$$\ddot{\phi} + \Gamma^2 3H\dot{\phi} + \Gamma^3 (V' - T') + T' - \frac{3}{2}\frac{T'}{T}\dot{\phi}^2 + \Gamma^3\frac{\beta\rho}{m_{\text{Pl}}} = 0 \quad (4.4)$$

4.2 Static radial case

We consider static configurations in spherical coordinates for which the radial equation becomes

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} \left[1 + T^{-1} \left(\frac{d\phi}{dr} \right)^2 \right] = \left[1 + \frac{3}{2}T^{-1} \left(\frac{d\phi}{dr} \right)^2 \right] \left(V' + \beta\frac{\rho}{m_{\text{Pl}}} \right) \quad (4.5)$$

Let us now consider the case of a small perturbation by the non-linear terms $T^{-1}(\frac{d\phi_0}{dr})^2$ with respect to a spherical profile obtained when the kinetic terms are canonical.

Expanding

$$\phi = \phi_0 + \delta\phi \quad (4.6)$$

we find that

$$\frac{d^2}{dr^2}\delta\phi + \frac{2}{r}\frac{d}{dr}\delta\phi - m^2\delta\phi = T_0^{-1} \left(\frac{d\phi_0}{dr} \right)^2 \left(\frac{3}{2}V'_0 - \frac{2}{r}\frac{d\phi_0}{dr} \right) \quad (4.7)$$

where

$$m^2 = V''(\phi_0) + \beta_2 \frac{\rho}{M_P^2} \quad (4.8)$$

and

$$V'_0 = V'(\phi_0) + \beta \frac{\rho}{M_P} \quad (4.9)$$

. We will analyse this equation in the case of a spherical body large enough to have a thin shell.

4.3 Spherical Bodies

We consider a compact body with a density ϕ_c embedded in a fluid of density ρ_∞ . We also assume that the object has a thin-shell obtain when $|\phi_\infty - \phi_c| \ll \Phi_N$ where Φ_N is Newton's potential at the surface of the body of radius R . As $\phi_c \ll \phi_\infty$ for dense bodies, this is tantamount to a condition on ϕ_∞ .

4.3.1 Multipoles

The solution of the DBI equation (4.7) outside the body is given by

$$\delta\phi(r) = - \int d^3r' \frac{e^{-m_\infty|r-r'|}}{4\pi|r-r'|} F(r') \quad (4.10)$$

where

$$F(r') = T_0^{-1} \left(\frac{d\phi_0}{dr} \right)^2 \left(\frac{3}{2} V'_0 - \frac{2}{r} \frac{d\phi_0}{dr} \right) \quad (4.11)$$

We consider distances which are far less than the inverse mass m_∞^{-1} and use the fact that $d\phi_0/dr$ is zero outside a shell of width ΔR .

$$\delta\phi(r) = - \int_{\Delta R} d^3r' \frac{F(r')}{4\pi|r-r'|} \quad (4.12)$$

using the multipolar expansion of $1/|r - r'|$ we find

$$\delta\phi(r) = - \sum_{l=0}^{\infty} \frac{A_l}{r^{l+1}} \quad (4.13)$$

where

$$A_l = \frac{1}{4\pi} \int_{\Delta R} d^3r r^l F(r) P_l(\cos\theta) \quad (4.14)$$

in terms of the Legendre Polynomials. Notice that the corrections to the thin shell solution has multipoles of arbitrary order despite being in a spherical situation. In particular this will lead to the presence of an infinite series in $1/r^{l+2}$ correcting Newton's law.

4.3.2 The Eddington Parameters

The force induced by a spherical body is modified by the scalar fields. This is due to the fact that test particles feel the potential

$$\Phi(r) = \Phi_N(r) + \beta \frac{\phi(r)}{M_P} \quad (4.15)$$

In the chameleon case, the metric outside a body reads

$$ds^2 = e^{2\beta\phi/m_{Pl}} [(-1 + 2\Phi_N(r)) dt^2 + (1 + 2\phi_N(r)) (dr^2 + r^2 d\Omega^2)] \quad (4.16)$$

This can be written (as $\phi/M_P \ll 1$)

$$ds^2 = \left(-1 + 2\Phi - 4\beta \frac{\phi}{M_P} \right) dt^2 + (1 + 2\Phi) (dr^2 + r^2 d\Omega^2) \quad (4.17)$$

The scalar field modifies the trajectories of test particles. For large bodies such as the planets in the solar system, the post-Newtonian formalism can be applied and

corrections to Newton's law in $1/r^2$ and $1/r^3$ have been parameterised. The Eddington parameters involve the $1/r$ and $1/r^2$ corrections to the Newton potential. Denoting by

$$\phi_0 = \phi_\infty - \frac{X}{r} \quad (4.18)$$

the background solution outside the spherical body where

$$X \sim \frac{G_N m_0 \phi_\infty}{\Phi_N} \quad (4.19)$$

we truncate

$$\phi(r) = -\frac{a_1}{r} - \frac{a_2}{r^2} + O(1/r^3) \quad (4.20)$$

where

$$\begin{aligned} a_1 &= X + A_0 \\ a_2 &= A_1 \end{aligned} \quad (4.21)$$

We find then the modification of the metric

$$\begin{aligned} g_{00} &= -1 + \frac{2G_N m_0}{r} + \frac{2\beta a_1}{M_P r} + \frac{2\beta a_2}{M_P r^2}, \\ g_{rr} &= 1 + \frac{2G_N m_0}{r} - \frac{2\beta a_1}{M_P r} - \frac{2\beta a_2}{M_P r^2} \end{aligned} \quad (4.22)$$

This has to be compared to the post Newtonian parametrisation

$$\begin{aligned} g_{00} &= -1 + \frac{2G_N m}{r} + \frac{\Gamma_{\text{edd}} - \beta_{\text{edd}}}{2} \frac{G_N^2 m^2}{r^2}, \\ g_{rr} &= -1 + \frac{2G_N \Gamma_{\text{edd}} m}{r} \end{aligned} \quad (4.23)$$

where m_0 is the bare mass of the spherical body. Redefining the mass to take into account the energy density carried by the scalar field

$$m = m_0 + 2\beta \frac{a_1}{M_P G_N} \quad (4.24)$$

we find that the Eddington parameters are given by

$$\Gamma_{\text{edd}} = 1 - \frac{2\beta a_1}{G_N M_P m} \quad (4.25)$$

and

$$\beta_{\text{edd}} = 1 - \frac{2\beta a_1}{G_N M_P m} - \frac{4\beta a_2}{M_P G_N^2 m^2} \quad (4.26)$$

From spherical symmetry we find that

$$A_1 \propto \int_0^\pi \sin \theta \cos \theta = 0 \quad (4.27)$$

implying that

$$a_2 = 0$$

and the Eddington parameters depend only on $a_1 = X + A_0$. In particular, the most stringent constraints follows from the Cassini bound on

$$|\Gamma_{\text{edd}} - 1| \leq 10^{-5} \quad (4.28)$$

In the thin shell case, the correction due to ϕ_0 satisfies this constraint. Perturbatively we must impose that $A_0 \ll X$. Let us now calculate A_0 and impose that the deviation from Newton's law is small. In the thin-shell between $(R - \Delta R)$ and R we have

$$\frac{d\phi_0}{dr} = \frac{\beta \rho_c}{3M_P} r - \frac{\beta \rho_c}{3M_P r^2} R^3 \quad (4.29)$$

The existence of a thin shell imposes that

$$\phi_\infty \leq 6\beta \Phi_N M_P \quad (4.30)$$

Imposing that the earth and the sun have both a thin shell to prevent changes of the trajectories of the planets and the moon implies that

$$\phi_\infty \leq 10^{-8} M_P \quad (4.31)$$

where $\Phi_N \sim 10^{-9}$ at the surface of the sun. This gives a bound

$$\phi(r) \leq 10^{-8} M_P \quad (4.32)$$

inside the thin shell of the sun. For the earth this is

$$\phi(r) \leq 10^{-12} M_P \quad (4.33)$$

In the following we shall focus on scales $M \gg 10^{-8} M_P$ then $T = M^4$. As result we find

$$F(r) = \frac{1}{6} \frac{\beta^3 \rho_c^3}{M^4 M_P^3} \left(r - \frac{R^3}{r^2} \right)^2 \quad (4.34)$$

and therefore

$$A_0 \approx \frac{3}{2} \frac{\beta^3 \rho_c^3}{M^4 M_P^3} R^2 (\Delta R)^3 \quad (4.35)$$

where the width of the shell is

$$\frac{\Delta R}{R} = \frac{\phi_\infty - \phi_c}{6\beta\Phi_N} \quad (4.36)$$

This can be written as

$$A_0 = 351\beta^3\Phi_N^3 \frac{M_P^4}{T_0} \frac{1}{R M_P} \left(\frac{\Delta R}{R} \right)^3 \quad (4.37)$$

while

$$X = 6\beta \frac{\Delta R}{R} \quad (4.38)$$

Hence $A_0 \ll X$.

4.3.3 Laboratory Tests

Laboratory tests allow us to test the higher order corrections to Newton's law. Defining

$$\alpha_l = \frac{1}{2} \int_0^\pi \sin \theta P_l(\cos \theta) \quad (4.39)$$

we find that

$$A_l = \alpha_l R^l A_0 \quad (4.40)$$

The correction to Newton's potential $\delta\phi_l(r)$ induced by the higher order terms reads

$$\frac{\delta\phi_l(r)}{\Phi_N(r)} = 351 \beta^4 \alpha_l \Phi_N^2 \frac{M_P^4}{T_0} \left(\frac{1}{RM_P}\right)^2 \left(\frac{\Delta R}{R}\right)^3 \left(\frac{R}{r}\right)^l \quad (4.41)$$

The Eotwash group has parameterised these deviations as

$$\frac{\delta\phi_l(r)}{\Phi_N(r)} = \beta_l \left(\frac{1\text{mm}}{r}\right)^l \quad (4.42)$$

leading to

$$\beta_l = \frac{9\beta\alpha_l M_P^4}{\Phi_N T_0} \left(\frac{1}{RM_P}\right)^2 \left(\frac{R}{1\text{mm}}\right)^3 \frac{(\phi_\infty - \phi_c)^3}{M_P^3} \quad (4.43)$$

For test bodies of radius $R = 1$ cm and weight 40g, Newton's potential is

$$\Phi_N \sim 10^{-27} \quad (4.44)$$

we find

$$\beta_l \sim 10^{60+l} \frac{M_P^4 \phi_\infty^3}{T_0 M_P^3} \quad (4.45)$$

for $\alpha_l = O(1)$ and $\beta = O(1)$. Using

$$\beta_l \leq 10^{-3} \quad (4.46)$$

for $l = 1 \dots 7$, this leads to

$$\phi_\infty \leq 10^{-23} \left(\frac{M_P^4}{T_0}\right)^{1/3} M_P \quad (4.47)$$

The presence of a thin shell requires

$$\phi_\infty \leq 10^{-26} M_P \quad (4.48)$$

implying that

$$\frac{T_0}{M_P^4} \geq 10^{-9} \quad (4.49)$$

Hence laboratory tests impose a drastic constraint on the tension at the tip of the throat.

