บทที่ 2

DBI action

Let us begin with the following action for a BPS $D3$-brane localised in a warped compacification of type IIB string theory. We will also assume the existence of a matter sector coupled to the world-volume theory of the brane. The resulting action can then be written in the form

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left(T(\phi)W(\phi)\sqrt{1-\frac{2X}{T}} - T(\phi) + V(\phi)\right) \tag{2.1}$$

This is the generalised Dirac-Born-Infeld (DBI) action where

$$\begin{aligned} X &\equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -\frac{1}{2}(\nabla\phi)^2 \\ \Gamma &\equiv \sqrt{1+\frac{(\nabla\phi)^2}{T}} \end{aligned}$$

and here

$$W = 1 \tag{2.2}$$

for a usual DBI action. One can find that

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\Big[-T\left(\Gamma-1\right) - V\Big] \tag{2.3}$$

where

$$T = \frac{1}{f(\phi)} \tag{2.4}$$

and $f(\phi)$ is warped factor. For AdS throat,

$$f(\phi) = \frac{\lambda}{\phi^4}\,. \tag{2.5}$$

Combining Einstein-Hilbert, matter field and DBI scalar field,

$$S = \int \mathrm{d}^4x \left\{ \mathcal{L}_{\mathrm{EH}} + \mathcal{L}_m + \sqrt{-g}\Big[-T\,(\Gamma - 1) - V \Big] \right\}\,. \tag{2.6}$$

This can read

$$S = \int d^4x \sqrt{-g} \left(\frac{M_p^2}{2}\mathcal{R} - T(\phi)(\Gamma - 1) - V(\phi) + \frac{1}{\sqrt{-g}}\mathcal{L}_m(\psi_m, \tilde{g}_{\mu\nu}) \right) \tag{2.7}$$

where $\mathcal{R}$ is the Ricci-scalar, ψ_m is a matter field and we have defined Jordan frame metric

$$\tilde{g}_{\mu\nu} = e^{2\beta\phi/M_p} g_{\mu\nu}. \tag{2.8}$$

The kinetic term for the scalar field ϕ is encoded in the DBI-part of the action $(-T\,(\Gamma - 1))$. Variation of the action with respect to the scalar field results in the Euler-Lagrange equation,

$$\frac{\delta \mathcal{L}_\phi}{\delta \phi} = \frac{\partial \mathcal{L}_\phi}{\partial \phi} - \nabla_\mu \left[\frac{\partial \mathcal{L}_\phi}{\partial(\nabla_\mu \phi)} \right] = 0 \tag{2.9}$$

We can also include matter term into the Euler-Lagrange equation as well. After long and very detail calculation, we have found that

$$\frac{\partial \mathcal{L}_{\phi+m}}{\partial \phi} = \sqrt{-g}\left[-\frac{T'}{2\Gamma}(\Gamma - 1)(\Gamma - 1) - V' \right] + \mathcal{L}'_m \tag{2.10}$$

and

$$\frac{\partial \mathcal{L}_{\phi+m}}{\partial(\nabla_\mu \phi)} = -\sqrt{-g}\frac{1}{\Gamma}\partial^\nu \phi \tag{2.11}$$

with

$$\nabla_\mu \left(\frac{\partial \mathcal{L}_{\phi+m}}{\partial(\nabla_\mu \phi)}\right) = -\frac{\sqrt{-g}}{\Gamma}\left[\Box^2\phi - \frac{1}{\Gamma}g^{\rho\nu}(\partial_\rho\phi)(\partial_\nu\Gamma)\right] \tag{2.12}$$

where

$$\Box^2\phi \equiv \frac{1}{\sqrt{-g}}\,\partial_\nu\left[\sqrt{-g}\,g^{\mu\nu}\partial_\nu\phi\right] \tag{2.13}$$

and

$$' \equiv \frac{\mathrm{d}}{\mathrm{d}\phi} \tag{2.14}$$

We finally obtain the DBI equation of motion,

$$-\frac{T'}{2\Gamma}(\Gamma-1)^2 - V' + \frac{\Box^2\phi}{\Gamma} - \frac{1}{\Gamma^2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\Gamma) = -\frac{\mathcal{L}'_m}{\sqrt{-g}} \tag{2.15}$$

The last term is matter Lagrangian density when the scalar field couples to the metric via equation (2.8)-the chameleon mechanism, the matter Lagrangian read

$$\mathcal{L}'_m = -\sqrt{-g}\frac{\beta}{M_p}\rho(1-3w)e^{\beta(1-3w)\phi/M_p} \tag{2.16}$$

where w is equation of state parameter of barotropic fluid, i.e. matter and radiation. The full DBI field equation of motion with chameleon mechanism is hence

$$-\frac{T'}{2\Gamma}(\Gamma-1)^2 + \frac{\Box^2\phi}{\Gamma} - \frac{g^{\mu\nu}}{\Gamma^2}\partial_\mu\phi\partial_\nu\Gamma = V' + \frac{\beta\rho}{M_p}(1-3\omega)e^{(1-3\omega)\beta\phi/M_p} \tag{2.17}$$

where the final term on the right hand side is proportional to the energy density of the matter sector. One finds this expression by noting that variation of the matter Lagrangian yields a term proportional to $\tilde{T}^{\mu\nu}\tilde{g}_{\mu\nu}$ which for an isotropic fluid will be of the form $-(1-3\omega)\tilde{\rho}$ in the Jordan frame. In Einstein frame we see that ρ is conformally related to $\tilde{\rho}$

$$\rho = \tilde{\rho} e^{3(1+\omega)\beta\phi/M_p}. \tag{2.18}$$

The entire right hand side can therefore be regarded as an effective potential for the scalar field which we define as

$$V_{\text{eff}} = V_\phi + \frac{\beta\rho}{M_p}(1-3\omega)e^{(1-3\omega)\beta\phi/M_p}. \tag{2.19}$$

The notation we use is that $\partial_X = X_\phi$ for a quantity X, and $\Box^2$ has the usual form in a curved geometry.