



## Simulation of Flexural Behavior of Damaged Prestressed Concrete Beam by Finite Element Method

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### ABSTRACT

Finite element model is applied to simulate the flexural behavior of bonded post tension prestressed concrete (PC) T beam. Flexural behavior of PC beam is characterized by load-deformation relationship in both linear elastic uncrack and nonlinear after cracking stage. Finite element analysis software, ANSYS, is utilized the simulation of flexural behaviors both loading and unloading path. Nonlinearity of materials are considered and taken into account. The developed model is verified by experimental results. The study finds good agreement between analytical and experimental results. Degradation of PC girder due to overload beyond cracking can be expressed by the loss of flexural stiffness and permanent deformation after unloading due to accumulated cracks. The verified finite element model (FEM) can be parametrically used to predict flexural behavior of PC beam in various levels of overloading.

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## 1. Introduction

Flexural behaviors of PC beam in terms of load-deformation relationship, cracking, failure mode, stiffness, strength and ductility can be performed by experiment. Test results give the most clearly understanding of flexural behaviors of prestressed concrete (PC) beam. However, tests are time consuming, expensive, and test results are generally limited to surface measurements. (Kim, *et al.*, 2010). Finite element model (FEM) can be applied in prediction of flexural response of PC girder under applied load in both linear elastic and nonlinear region. If the model is verified by experimental results, then parametric study can be performed in computer. In this work, a reliable three dimensional finite element model is developed to simulate flexural behavior of bonded post-tension T beams. The model is verified by

\*Corresponding author (Sayan Sirimontree). Tel/Fax: +66-2-564-3004 ext 3112 E-mail address: [ssayan@engr.tu.ac.th](mailto:ssayan@engr.tu.ac.th). ©2015. International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies. Volume 6 No.5 ISSN 2228-9860 eISSN 1906-9642. Online Available at <http://TUENGR.COM/V06/225.pdf>.

experimental results performed by Piyatrapoomi (2011). Degradation behaviors of PC beam due to overload in post cracking stage can be described by the decrease of flexural stiffness indicated by slope of load-deformation relationship and the increase of permanent deformation caused by crack which is accumulated in each level of applied load beyond cracking.

## 2. Literature Review

Arduini and Nanni (1997) used experimental data obtained from strengthened, precracked, reinforced concrete (RC) specimens together with the results of material characterization. The strengthening applied carbon fiber-reinforced plastic (CFRP) sheets to the concrete surface. Two specimens were investigated, under applied load (simulating the total service load) and external prestressing with adhesion of CFRP reinforcement. The work also built analytical model to simulate the load-deflection behavior and the failure mode of the precracked RC specimens. Ductile to brittle failure mechanisms were simulated and verified.

Légeron et al. (2005) presented a simplified finite-element analysis program based on multilayer elements and damage mechanics modeling of concrete behavior. A model was used to predict the behavior of three different structures: overreinforced normal-strength concrete and high-strength concrete (HSC) beams tested monotonically, HSC columns tested under constant axial load and cyclic flexure, and bridge piers subjected to earthquake-type loading by the pseudodynamic test method. The predictions were in excellent agreement with experimental results.

Tavárez (2001) conducted a simulation to study behaviors of composite grid reinforced concrete beams using explicit finite element methods.

Kwak & Filippou (1990) dealt with the finite element analysis of the monotonic behavior of reinforced concrete beams, slabs and beam-column joint subassemblages. A smeared finite element model is used based on an improved cracking criterion, deriving from fracture mechanics principles. Correlation studies between analytical and experimental results and several parameter studies are conducted to verify of the models.

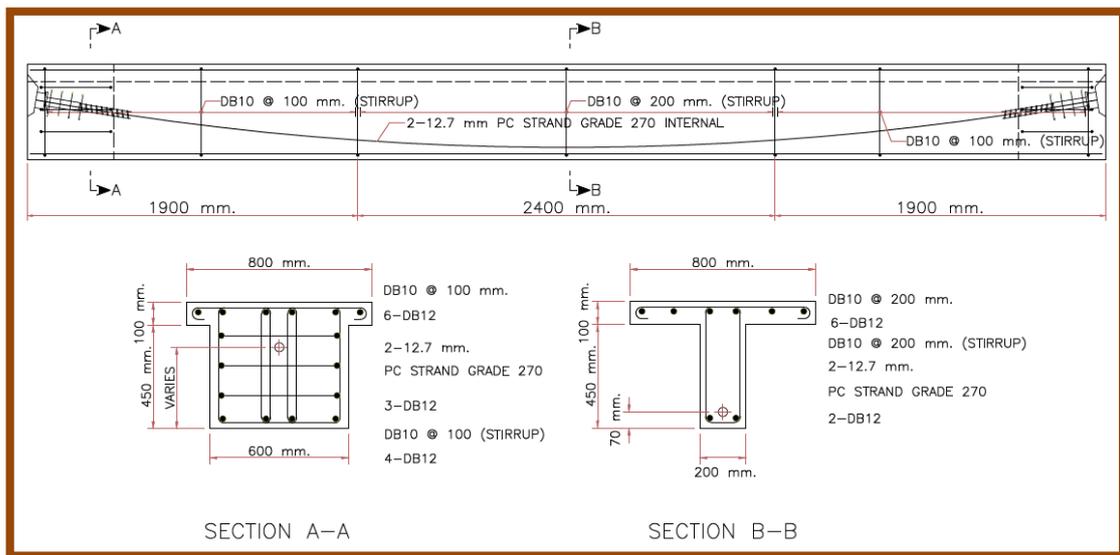
Deng *et al.* (2001) presents a procedure to improve the accuracy of the classical grillage method, a more refine FEM, for the nonlinear analysis of concrete girder bridges. The studied procedure used equivalent element plastic hinge lengths to account for the actual mesh size instead of a mesh-independent global plastic hinge length.

This study focuses on finite element model applied to simulate the flexural behaviors of bonded post tension prestressed concrete (PC) T beam. Load-deformation relationship characterizes flexural behaviors, both linear elastic uncrack and nonlinear after cracking stages.

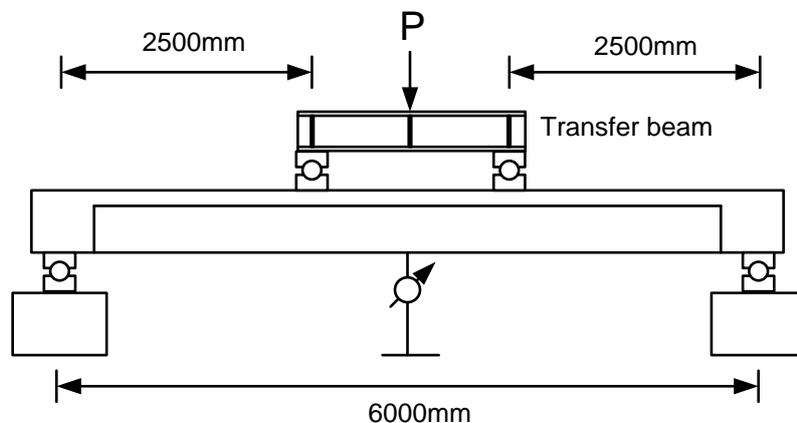
Nonlinearities of materials are taken into account. The developed model is then verified with experimental results.

### 3. Details of Post-tension Beam Specimen

Figure 1 gives the details of bonded post-tension prestressed concrete T beam used in experimental studies performed by Piyatrapoomi (2011). This T beam is used to verify the FEM model of this study. Parabolic shape of tendon composed of steel tube containing 2-12.7 mm diameter PC strand is used in PC beam test specimen. PC strand is stressed to 75% of specified breaking strength ( $0.75f_{pu}$ ). Cement grouting is performed after the stressing of PC strand. Two point loads are setup through the use of transfer beam as shown by Figure 2. A static load test, load and unload, is performed in testing process at each step up to failure. Response of the test beam under applied load, deflection and strain are detected and recorded by data logger.



**Figure 1:** Details of post-tension concrete T beams.



**Figure 2:** Test set up of PC beam specimen by Piyatrapoomi (2011).

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## 4. Nonlinear Finite Element Model

Analytical tool used in FEM analysis of PC beam in this work is ANSYS. A three dimensional finite element mesh is used to represent the PC beam by taking the advance of symmetry; only one haft of the beam in longitudinal direction has been used. A smear crack model is applied to concrete while a discrete model is applied to ordinary reinforcement and pre-stressing strand. A boundary condition needs to be applied at point of symmetry and where the supports and loading exist.

### 4.1 Material Model

#### 4.1.1 Material model for concrete

The stress-strain relationship for concrete is obtained by equations 1 to 3, numerical expressions (Desayi and Krishnan, 1964). The multi-linear compressive uniaxial stress-strain calculated by these equations is shown by Figure 3. The value used for the uniaxial tensile cracking stress of concrete is 4.2 MPa.

$$f = \frac{E_c \varepsilon}{1 + \frac{\varepsilon}{\varepsilon_o}}; \quad \varepsilon_1 \leq \varepsilon \leq \varepsilon_o \quad (1),$$

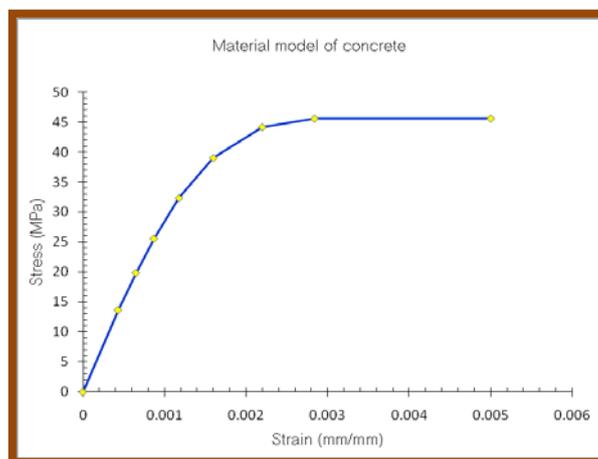
$$\varepsilon_o = \frac{2f'_c}{E_c} \quad (2),$$

$$E_c = \frac{f}{\varepsilon} \quad ; \quad 0 \leq \varepsilon \leq \varepsilon_1 \quad (3)$$

Where  $f$  is stress at any strain  $\varepsilon$ ,  $\varepsilon_o$  is strain at the ultimate compressive strength  $f'_c$ ,  $\varepsilon_1$  is strain corresponding to  $0.3f'_c$ , and  $E_c$  is elastic modulus of concrete.

**Table 1:** Properties of concrete.

Material	Compressive strength (MPa.)	Modulus of elasticity (MPa.)	Poisson's ratio
Concrete	45.6	31,814	0.2



**Figure 3:** Simplified Compressive Uniaxial Stress-Strain Curve for Concrete.

#### 4.1.2 Material model for Pre-stressing Strand

A Ramberg-Osgood function (Ramberg and Osgood, 1943) was used as material model of PC strand as shown by Equation 4. A multi-linear isotropic stress-strain curve of strand is shown in Figure 4.

**Table 2:** Properties of Grade 270 7-wire low relaxation strand.

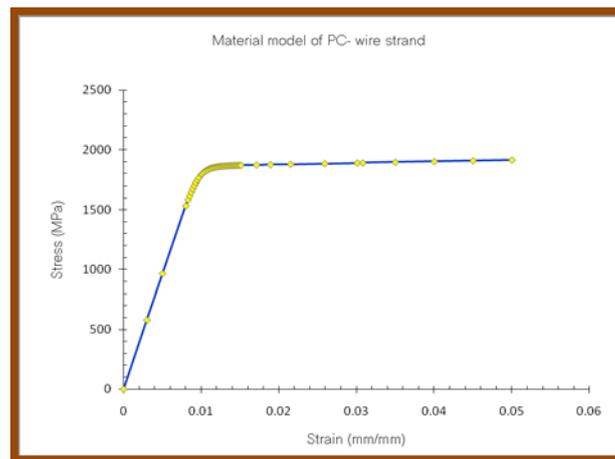
Material	Breaking strength (MPa.)	Modulus of elasticity (MPa.)	Poisson 's ratio
Pre-stressing strand 12.7mm.diameter	1,914	193,178	0.3

$$f_p = 200 \times 10^3 \varepsilon_{pf} \left\{ 0.025 + \frac{0.975}{[1 + (118 \varepsilon_{pf})^{10}]^{0.1}} \right\} \leq f_{pu} \quad (\text{MPa}) \quad (4)$$

$f_p$  = stress in PC strand

$\varepsilon_{pf}$  = strain in PC strand

$f_{pu}$  = breaking tensile strength of PC strand



**Figure 4:** Stress-strain relation for pre-stressing strand.

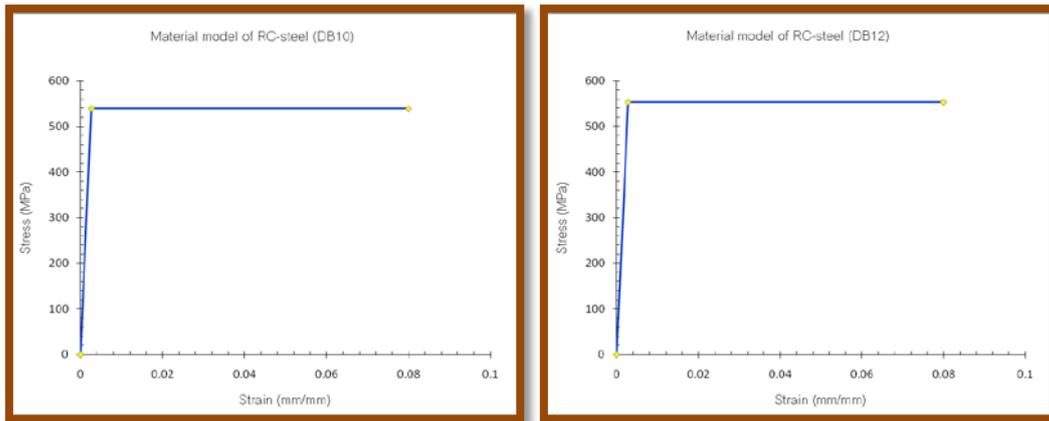
#### 4.1.3 Material model for Ordinary Reinforcing steel

The stress-strain relationship for ordinary reinforcing steel is assumed to be a bilinear relationship, elastic perfectly plastic, as shown in Figure 5.

**Table 3:** Property of non-prestress reinforcement grade SD40.

Material	Cross-Sectional area (mm <sup>2</sup> )	Yield strength (MPa.)	Ultimate strength (MPa.)	Modulus of elasticity (MPa.)	Poisson's ratio
DB10	78.54	539	666	200,042	0.3
DB12	113	552	682	200,042	0.3

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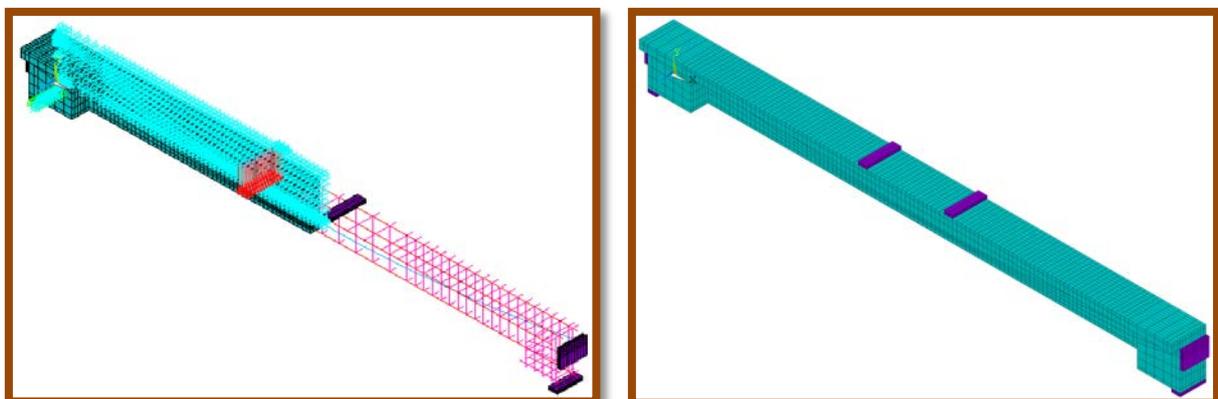
**Figure 5:** Stress-Strain relation for ordinary reinforcing steels.

#### 4.1.4 Material model for Steel Plates

The stress-strain relationship of steel plate is assumed to be a bilinear relationship, elastic perfectly plastic. The value used for elastic modulus is 200,000 MPa., yield stress 294.2 MPa. and Poisson's ratio 0.3.

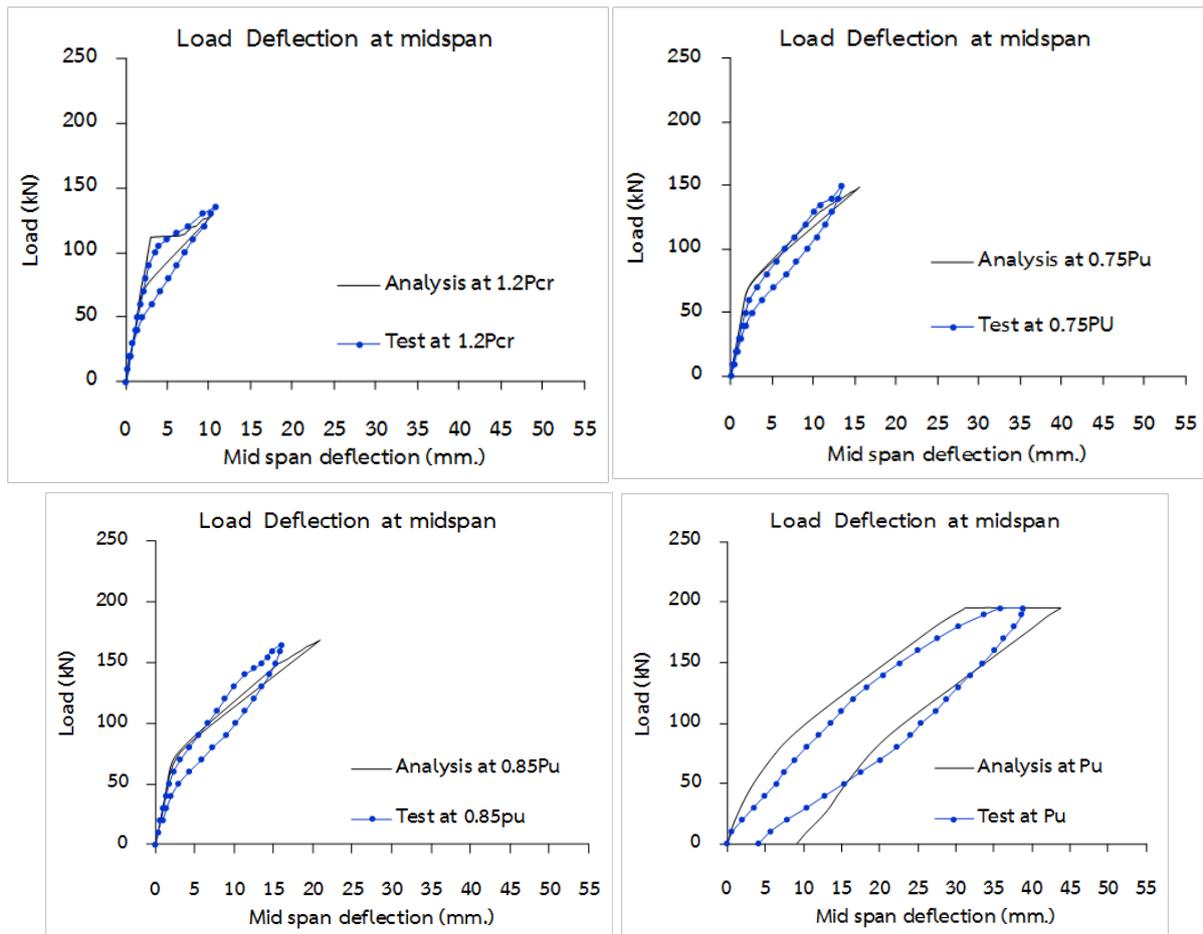
#### 4.2 Element Types

There are three elements types used in FEM model. Solid 65 is used to represent the concrete. The element has eight nodes with three degree of freedom at each node-translation in the nodal x, y, and z directions. The element is capable of plastic deformation, cracking in three orthogonal directions and crushing. Solid 45 is used to represent steel plate placed at both end of beam in order to avoid unrealistic cracks due to stress concentrations and at the supports in the beam model. The element is defined with eight nodes having three degrees of freedom at each node-translation in the nodal x, y, and z directions. Finally link8 is used to represent the ordinary reinforcing steel and pre-stressing strand. Two nodes are required for this element each node-translation in the nodal x, y, and z directions. The finite element mesh, boundary condition and loading arrangement are shown in Figure 6.



Finite Element Mesh, Boundary conditions and load arrangement (selected concrete elements removed, to illustrate internal reinforcement).

**Figure 6:** Post-tension PC Beam-Finite Element Mesh.



**Figure 7:** Comparisons between test and analytical results.

## 5. Analytical Results and Discussions

### 5.1 Model Verification

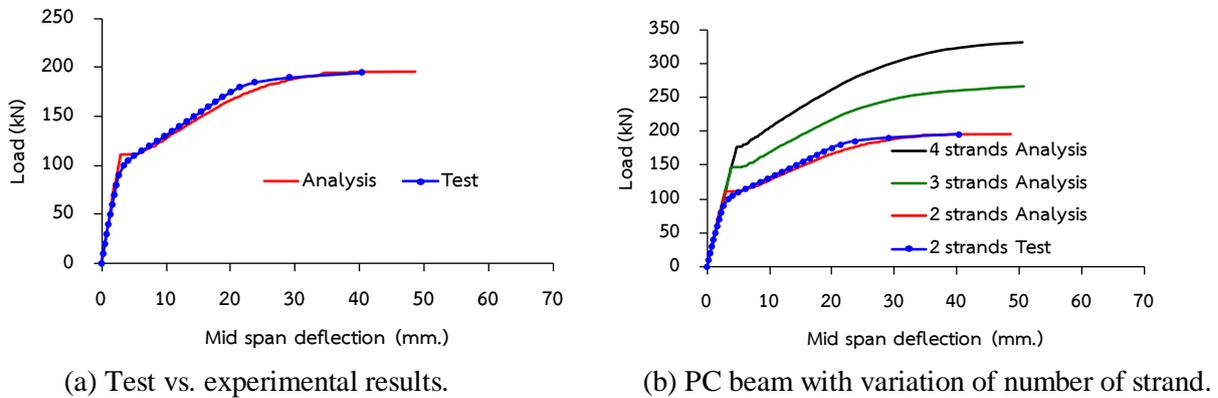
The results from analytical model utilized by ANSYS are compared to experimental results, load-mid span deflection relationship in loading and unloading cycle, as seen by Figure 7. It can be observed that good agreement between test and analytical results. Flexural stiffness of the test beam is found to be decrease while the permanent deformation increases due to the accumulation of damage when the applied load is higher than cracking load. Figure 8(a) show load-mid span deflection relationship at peak load of each cycle comparing to analytical results which is performed monotonically up to failure. It can be said that the developed model is reliable for prediction of flexural behavior of PC beam. Load-mid span deflection of PC beam with variation of internal strand is shown by Figure 8(b).

### 5.2 Degradation of PC beam due to overloading

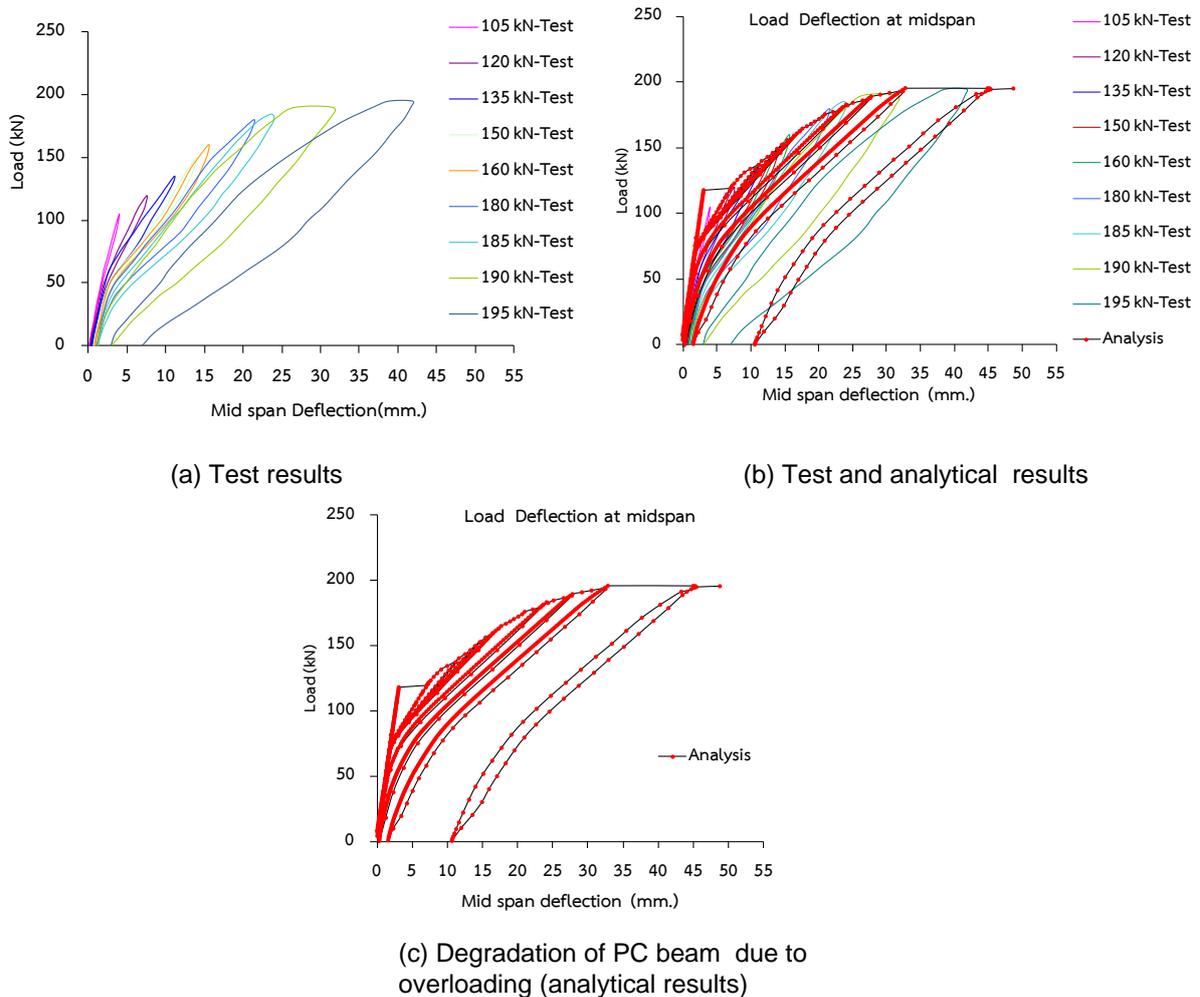
Figure 9(a) shows degradation behaviors expressed by load-mid span deflection of the test beam due to overloading beyond cracking stage from test data. The analytical results of

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degradations of the test beam are compared to test results in Figure 9(b). Good correlation between two results leads to the reliable prediction of degradation of PC beam due to overloading as can be seen by Figure 9(c). The degradation at each level of load beyond cracking is useful in the design of beam repair or strengthening.



**Figure 8:** Load-mid span deflection at peak load of each loading cycle of test beam.



**Figure 9:** Degradation of PC beam due to overloading.

It can be seen from Figure 9 that significantly decrease of flexural stiffness and increase of permanent deformation are found when the overload is greater than 85% of ultimate load ( $0.85P_u$ ). The rebound of mid span deflection is significantly large when the applied load less than  $0.85P_u$ . This shows that strengthening can effectively be performed by means of external post-tension to recover the flexural stiffness and reduced permanent deflection, which will be further investigated.

## 6. Conclusion

Nonlinear finite element model developed in this works verified by experimental results can be used to predict flexural behaviors of bonded PC beam. Load and unloading behaviors can be performed by the model. Degradation of PC beam due to overloading in terms of load deformation relationship, reduction of flexural stiffness, and increased permanent deformation, can be constructed using FEM. The PC beam degradation data is useful to aid the design of beam repair or strengthening.

## 7. References

- Arduini, M., & Nanni, A. (1997). Behavior of precracked RC beams strengthened with carbon FRP sheets. *Journal of Composites for Construction*, 1(2), 63-70.
- Deng, L., Ghosn, M., Znidaric, A., & Casas, J. R. (2001). Nonlinear flexural behavior of prestressed concrete girder bridges. *Journal of Bridge Engineering*, 6(4), 276-284.
- Desayi, P. and Krishnan, S., (1964). Equation for the Stress-Strain Curve of Concrete. *Journal of the American Concrete Institute*, Vol. 61 : pp. 345-350
- Fanning, P. (2001). Nonlinear Models of Reinforced and Post-tensioned Concrete. *Electronic Journal of Structural Engineering*, 2, 111-119.
- Kachlakev, D. I. Miller, T., Yim, S., Chansawat, k., Potisuk, T., 2001. Finite Element Modeling of Reinforced Concrete Structures Strengthened with FRP Laminates Final Report SPR 316, Oregon Department of Transportation.
- Kanisorn, (2011). Effects of Structural Damage and External prestressing on Flexural Rigidity and Natural Frequencies of Post-Tensioned Prestressed concrete Beams. Master's thesis, Thammasat University, Faculty Civil Engineering, Thailand.
- Kwak, H. G., & Filippou, F. C. (1990). *Finite element analysis of reinforced concrete structures under monotonic loads* (pp. 33-39). Berkeley, CA, USA: Department of Civil Engineering, University of California.
- Légeron, F., Paultre, P., & Mazars, J. (2005). Damage mechanics modeling of nonlinear seismic behavior of concrete structures. *Journal of Structural Engineering*, 131(6), 946-955.
- Michael P. Collins/Denis. Mitchell. (1997). *Prestressed Concrete Structures*, Canada by Copywell Ontario.Piyatrapoomi,

Tavárez, F. A. (2001). *Simulation of behavior of composite grid reinforced concrete beams using explicit finite element methods* (Master Thesis, University of Wisconsin-Madison).

Thananun Phuwadolpaisarn, Sayan Sirimontree, and Boonsap Witchayangkoon. (2013). Relations between Structural Damage and Level of External Prestressing Force on the Flexural Behavior of Post-Tensioned Prestressed Concrete. *International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies*, 4(4) 241-251.

U. Kim, P.R. Chakrabarti & J.H. (2010). *Challenges, Opportunities and Solutions in Structural Engineering and Construction – Ghafoori (ed.)*. Taylor & Francis, London.

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