

CHAPTER IV

CONCLUSIONS

There are our summary results : Let W be the number of isolated copies of a fixed connected G graph in a random graph $\mathbb{G}(n, p)$.

1. The distribution of W can be approximated by Poisson distribution with parameter

$$\lambda = E(W) = \binom{n}{v_G} \frac{v_G!}{\text{Aut}(G)} p^{e_G} q^{\binom{v_G}{2} - e_G + v_G(n - v_G)}.$$

2. If $A \subseteq \mathbb{N}$ and $n > 2v_G$, then

$$|P(W \in A) - \text{Poi}_\lambda(A)| \leq C_{\lambda, A} \frac{(np)^{v_G}}{q^{2v_G^2} e^{np}} \left[\frac{1 + np}{np} \right]$$

where $C_{\lambda, A} = \frac{v_G^2}{\text{Aut}(G)} \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A + 1} \right\}$,

$$\Delta(\lambda) = \begin{cases} e^\lambda + \lambda - 1 & \text{if } \lambda^{-1}(e^\lambda - 1) \leq M_A, \\ 2(e^\lambda - 1) & \text{if } \lambda^{-1}(e^\lambda - 1) > M_A, \end{cases}$$

and

$$M_A = \begin{cases} \max\{w \mid C_w \subseteq A\} & \text{if } 0 \in A, \\ \min\{w \mid w \in A\} & \text{if } 0 \notin A, \end{cases}$$

when $C_w = \{0, 1, \dots, w\}$.

3. Let $p = \frac{1}{n^\gamma}$ for some $\gamma \in \mathbb{R}^+ \setminus \{1\}$ and let $A \subseteq \mathbb{N}$. We obtain that

- 1) if $\gamma > 1$ then

$$|P(W \in A) - \text{Poi}_\lambda(A)| \leq \frac{C(\lambda, A, v_G)}{n^{(\gamma-1)v_G}},$$

where $C(\lambda, A, v_G) = \frac{2v_G^2}{\text{Aut}(G)q^{2v_G^2}} \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A + 1} \right\}$.

- 2) if $0 < \gamma < 1$ then

$$|P(W \in A) - \text{Poi}_\lambda(A)| \leq \frac{C(\lambda, A, v_G)}{n^{(1-\gamma)v_G}},$$

where $C(\lambda, A, v_G) = \frac{2v_G^2(2v_G)!}{\text{Aut}(G)q^{2v_G^2}} \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A + 1} \right\}$.