Modified Form of Low-Reynolds-number Turbulence Model for Predicting Turbulent Heat Transfer

Kittisak Khuwaranyu[1], Sompong Putivisuttisak[2]

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Abstract

This research presents the Finite Volume Method (FVM) for predicting turbulent flow and heat transfer. For combing the Low-Reynolds-Number (LRN) k–ω model with a Length Scale Correction (LSC) term and High-Re model, the Baseline model was employed. The new model is so call "BLL model" for complex heat transfer and flow phenomena. The modified model is validated with benchmark problems (the fully developed channel flow and backward-facing step flow and heat transfer) before being applied to the problem of recirculating flow and heat transfer over repeated square ribs. Performance of the model is investigated and compared with available experimental data, Direct Numerical Simulation (DNS) data and numerical results using other turbulence models. It is seen that the model gives superior results especially for the near-wall flow patterns. The results demonstrate that the performance of the BLL model is much better than that of the High-Re model, LRN model, and it can be shown that this model is the appropriate choice for near-wall flow problem.

Keywords: LRN model, k–ω, Turbulent, Heat transfer, Finite volume Method

Introduction

Many form of turbulence models which are used for predicting turbulence effects of heat transfer and flow phenomena. The most popular form is the one proposed by Launder and Spalding[1] so call the standard k–ε model. The disadvantage of the standard k–ε model is the inability to predict accurate results for flow with adverse pressure gradients.

The LRN models shared a weak point with the LRN k-ε model which was the uncertainty of ε specification at the wall[2]. Pang and Davidson[3] implemented the LRN k-ω model by introducing a damping function into the turbulent kinetic energy term. Their model was compared with two LRN k-ε models and one LRN k-ω model.

Several concepts were also used to increase the model accuracy. Jia et al.[4] integrated the reformulated SSG model[5] based on the ω-equation and the SST model[6]. The obtained result was better than the previous one because the new model had good applicability for complex flow fields

This research presents a concept for a new turbulence model, which combines the Low-Reynolds-Number (LRN) k-ω model with a Length Scale Correction (LSC) term. For combing the LRN the Baseline model was employed. The new model is so call "BLL model" for complex heat transfer and flow phenomena. Here, the capability and performance of the BLL models for calculation of a thermal viscous flow will be made.

Theoretical formulations

The Reynolds-averaging principle is applied to the Navier-Stokes equations. After performing the averaging, the continuity and momentum equations can be shown as follows:

[1] *Corresponding author, Faculty of Engineering and Industrial Technology, Silpakorn University (Sanam Chandra Palace), Nakhon Phathom 73000, Thailand. Email: kittisak_mesu@yahoo.com.*

[2] *Faculty of Engineering, Chulalongkorn University, Bangkok, 10330*

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho \bar{u}_i)}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial(\rho \bar{u}_i)}{\partial t}+\frac{\partial(\rho \bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i}+\frac{\partial \bar{\sigma}_{ij}}{\partial x_j}-\frac{\partial(\rho \overline{u_i' u_j'})}{\partial x_j}, \quad (2)$$

where $\rho\overline{u_i'u_j'}$ are the Reynolds stresses ($R_{ij}=\rho\overline{u_i'u_j'}$).

Turbulence models

In the present work, a new model called BLL model which is based on LRN k–ε with LSC and transformed k–ε, is proposed. The standard k–ε model[7], LRN k–ε model[8] and High-Re k–ε model[9] are employed for results comparison.

The BLL model

The basic idea of the new model is based on a combination of an accurate formulation of the Wilcox LRN k–ε model and the concept of a Baseline model to reduce the sensivity to the freestream (in the outer part of the boundary-layer and in free-shear flows). In the near-wall region, the Length Scale Correction (LSC) term is employed. The equations of the proposed model are reformulated by multiplying the LRN k–ε model with LSC term by a function (1–F_b) and adding with the multiplication of the transformed High-Re k–ε equation and a function F_b. F_b is a blending function (a simple exponential function is used at the beginning) which ensures that the model behaves as a High-Re model away from the surface and as the LRN model in the near-wall region. For the dissipation rate, the cross diffusion term is removed. To compensate for the inferior performance of the transformed k–ε, the LSC term () is employed.

After rearrangement, the new model can be shown as follows:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho \bar{u}_j k)}{\partial x_j} = P_k-\beta^* f_k \rho\omega k+\frac{\partial}{\partial x_j}\left[(\mu+\mu_t\sigma_k)\frac{\partial k}{\partial x_j}\right], \quad (3)$$

$$\frac{\partial(\rho \omega)}{\partial t}+\frac{\partial(\rho \bar{u}_j \omega)}{\partial x_j} = \gamma f_\omega \frac{\omega}{k}P_k-\beta\rho\omega^2+\frac{\partial}{\partial x_j}\left[(\mu+\mu_t\sigma_\omega)\frac{\partial \omega}{\partial x_j}\right]-S_\omega\rho(1-F_b), \quad (4)$$

where $\mu_t=\rho f_\mu k/\omega$ is the turbulent viscosity and P_k is the production of the turbulent energy which can be expressed as:

$$P_k=\mu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial u_i}\right)\frac{\partial \bar{u}_i}{\partial x_j}.$$

The model constant is blended by the relation of the model constants in the LRN k–ω model and the transformed k–ε model. If represent constants in the LRN k–ω model $(\sigma_{k1}, f_{k1}, ...)$, ϕ_2, represent constants in the transformed k–ε turbulence rand represent the corresponding constants of the new model model with a Length relation can be written as:

$$\phi = (1-F_b)\phi_1+F_b\phi_2 \quad (5)$$

Two sets of model constants are given as follows:

Set 1 (LRN *k–ω*)[10]

$\beta^*=0.09$

$f_{k1}=\left[0.278+(\mathrm{Re}_T/8)^4\right]\left[1+(\mathrm{Re}_T/8)^4\right]^{-1}$,

$\sigma_{k1}=0.5$, $\gamma_1=0.56$,

$f_{\omega 1}=(0.1+\mathrm{Re}_T/2.7)\left[(1+\mathrm{Re}_T/2.7)f_\mu\right]^{-1}$,

$\beta_1=0.075$, $\sigma_{\omega 1}=2.0$ and

$f_\mu=(0.025+\mathrm{Re}_T/6)(1+\mathrm{Re}_T/6)^{-1}$.

Set 2 (High *k–ε*)[11]

$\beta^*=0.09$, $f_{k2}=1.0$, $\sigma_{k2}=1.0$, $\gamma_2=0.44$,

$f_{\omega 2}=1.0$, $\beta_2=0.0828$ and $\sigma_{\omega 2}=0.856$.

After rearrangement, the new model can be shown as follows:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho \bar{u}_j k)}{\partial x_j}$$

$$=P_k-\beta^* f_k \rho\omega k+\frac{\partial}{\partial x_j}\left[(\mu+\mu_t\sigma_k)\frac{\partial k}{\partial x_j}\right], \quad (6)$$

$$\frac{\partial(\rho\omega)}{\partial t}+\frac{\partial(\rho \bar{u}_j \omega)}{\partial x_j} = \gamma\, f_\omega \frac{\omega}{k}P_k-\beta\,\rho\omega^2$$

$$+\frac{\partial}{\partial x_j}\left[(\mu+\mu_t\sigma_\omega)\frac{\partial \omega}{\partial x_j}\right]-S_\omega\rho(1-F_b), \quad (7)$$

where $\mu_t=\rho f_\mu k/\omega$ is the turbulent viscosity and P_k is the production of the turbulent energy which can be expressed as:

$$P_k=\mu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial u_i}\right)\frac{\partial \bar{u}_i}{\partial x_j}.$$

The blending function (F_b) is selected to ensure asymptotic consistency with the near-wall behavior of the equation of motion. The value of function Fb will be designed to be zero in the near-wall region (activating the LRN model) and set to unity away from the surface (switching to the High-Re model) as shown in Fig. 1. For the present model, the blending function from the LRN two-equation model of Abe et al.8 has been tentatively adopted, as expressed in Eq. (8):

$$F_b=\left(1-e^{-(y^*/14)}\right)^2\left(1+5\,\mathrm{Re}_T^{-0.75}\,e^{-(\mathrm{Re}_T/200)^2}\right), \quad (8)$$

where $y^*=(\nu\omega k)^{0.25}y/\nu$ and $\mathrm{Re}_T=\rho k/\omega\mu$.

Length Scale Correction term (LSC term)

The LSC term has been well known for turbulent separated flows. Launder[12] applied the Yap correction[13] as an extra term to the ε-equation of LRN k-ε model. The Yap correction can be written as the ratio of the computational length scale to the local equilibrium length scale as follows:

$$\rho S_\varepsilon=0.83\rho\frac{\varepsilon^2}{k}\left(\frac{k^{3/2}}{\varepsilon\ell_e}-1\right)\left(\frac{k^{3/2}}{\varepsilon\ell_e}\right)^2, \quad (9)$$

where is the local equilibrium length scale ($\ell_e=\kappa y_n/C_\mu^{3/4}$) and y_n is the normal distance to the nearest wall.

Wilcox[9] has shown that an extra cross-diffusion term appears in the resultant ε-equation. This term, similar to the so-called "Yap correction", helps suppressing the rate of the near-wall turbulent length scale. In the present work, the turbulent length scale ($L_t=k^{1/2}/0.09\omega$) is applied to the Yap correction concept. A new length scale correction for ω-equation (LSC-ω) can be expressed as follows:

$$\rho S_\omega=0.075\frac{\rho k^{3/2}\omega}{\ell_e}\left(\frac{L_t}{\ell_e}-1\right)\left(\frac{L_t}{\ell_e}\right), \quad (10)$$

where ℓ_e is the near-wall equilibrium length scale ($\ell_e=Cy_n$), y_n yn is the distance from the wall, the turbulence scale constant (C) is equal to $\kappa/C_\mu^{0.75}=2.495$, and κ is the von Kárman constant.

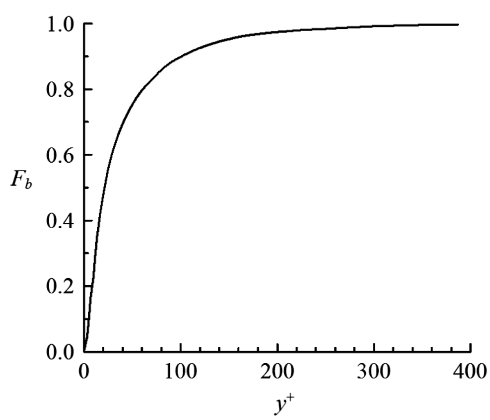

Figure 1 Blending function Fb versus y+

Energy equation

$$\rho\bar{u}_j\frac{\partial \bar{T}}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\frac{\mu}{\mathrm{Pr}}\frac{\partial \bar{T}}{\partial x_j}-\overline{u_j'T'}\right), \quad (11)$$

For including turbulent effect, the energy equation will have the new unknown terms, so call turbulent heat flux. The terms are expressed by the Boussinesq approximation as:

$$-\overline{u'_j T'} = \left(\frac{\mu_t}{\Pr_t}\right)\left(\frac{\partial \overline{T}}{\partial x_j}\right)$$

where μ_e is the effective diffusion coefficient, $\mu_e = \frac{\mu}{\Pr} + \frac{\mu_t}{\Pr_t}$

$\Pr_t$ is the turbulent Prandtl number,

$\Pr_t = \sigma_t = \nu_t / \alpha_t$,

α_t is the eddy diffusivity,

ν_t is the turbulent eddy viscosity.

Numerical procedures

The finite volume method (FVM) is used to solve the set of governing equations on a staggered grid. To obtain the solution that couples the pressure and velocity, the SIMPLE algorithm[14] is employed to calculate the pressure correction terms. Then, the resulting algebraic equations are iteratively solved with a line-by-line TDMA procedure. The convection terms are approximated by the second-order upwind scheme. The hybrid differencing scheme is employed in the turbulence-transport equations to ensure a stable solution procedure. The convergence criterion utilized in this work is that the maximum normalized sum of the absolute residual source for all the computed nodes is less than 10-6. For all the investigated cases, the number of sufficiently small grids is ensured from the grid-independency tests.

The inlet boundary values are prescribed for all variables. At the outlet, the streamwise gradients of the flow variables are set to zero. The wall boundary conditions are applied: u = v = 0 and k = 0. The use of LRN models requires a fine grid to minimize the dependence of the solution on the grid. The boundary condition of εat the first grid point in the near-wall region is given as10

$$\omega = \frac{6\nu}{c_{\omega 2} y_1^2} \quad \text{as } y_1 \to 0 . \tag{12}$$

Results and Discussions

Turbulent heat transfer and flow in smooth wall channel for Re_τ = 180

The selected first test case is a fully-developed channel heat transfer and flow at = 180, for which DNS data exist. The simulating condition, the wall boundary is applied uniform wall temperature, the Reynolds number, Re_τ is 180 and Pr is 0.71 .

The predicted results are compared with the DNS data from Kim and Moin[15] and the numerical results of other turbulence model, ANK model and Wilcox model.

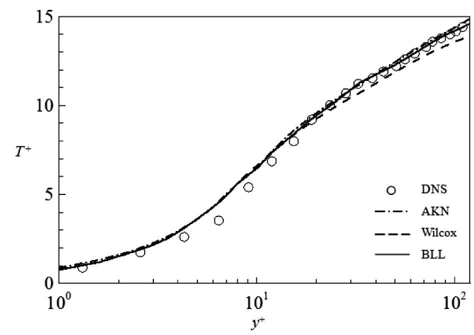

Figure 2 Comparison of the predicted temperature with the DNS, AKN model and Wilcox model

Figure 2 presents the dimensionless of temperature at different. The numerical results are in good agreement with the DNS data and the results of ANK model but we can see that the Wilcox model slightly underpredicts for the region > 20. The Wilcox model result, its is high sensivity in freestream that cause the under predicted result.

Turbulent heat transfer and flow over Backward facing step

The proposed model predicts the simple flow in smooth channel wall. The next suitable test case is the turbulent flow over Backward-Facing Step (BFS). The flow over BFS is widely use in model test because this flow are complex flow phenomenon such as sudden expansion flow, recirculating and reattachment. In this research, we have applied the model to selected case of BFS flow, for which simulating parameters are available from Avancha and Plether[16] for 5540.

In the test case, the boundary conditions and properties are, the Reynolds number, , based on the backward facing step height and free stream velocity is 5540. Where , Uref is free stream velocity at inlet and h is the step height, the configuration is shown in Figure 3. The test section is 41 mm height (h), Expansion ratio (ER) is 1.5, the channel length behind BFS and inlet section height are 20h and 3h, respectively. The magnitude of 1000 W/m2 uniform heat flux applies at bottom wall behind the step.

For the heat transfer investigation, the Staton number, ratio of the convection heat transfer and heat flux, can be describe as:

$$\mathrm{St} = \frac{h}{\rho u C_p} = \frac{q''}{\rho U C_p \left[T_w(x) - T_b(x)\right]}, \quad (13)$$

Where h is the convection coefficient of fluid and $T_w(x)$ and $T_b(x)$ are the wall temperature and bulb temperature at different location of x, respectively.

Figure 4 presents the distribution of the Staton number along bottom wall behind the step. It can be seen that the results from the k–ω model and the BLL model is better in agreement with the LES data than those of the standard k–ε model. But three models give the Stanton number distribution in similar curve line, it can be explain that heat transfer capacity is low at circulation region and increase over the region behind the backward-facing step.

Figure 5 shows the mean temperature, ((T-Tref) / Tref), at the location x/h = 1.0, 2.0, 3.0, 4.0 and 5.0 . The BLL model can be predict in good agreement with the LES data and the k–ε model give a slightly under prediction of temperature. Moreover, the Standard k–ε model obtains underprediction of mean temperature at all of investigated section, x/h.

Conclusion

In this work, the developed model proposed for improving and increasing prediction accuracy. The results demonstrate that the standard k–ε model give good predicting in simple flow. For complex heat transfer and flow, the AKN model and the BLL model produce similar results. However, the results obtain that the performance of the BLL model is much better that that of the High-Re model and LRN model. For improving the model, the complex heat transfer and flow phenomena need to apply for the BLL model.

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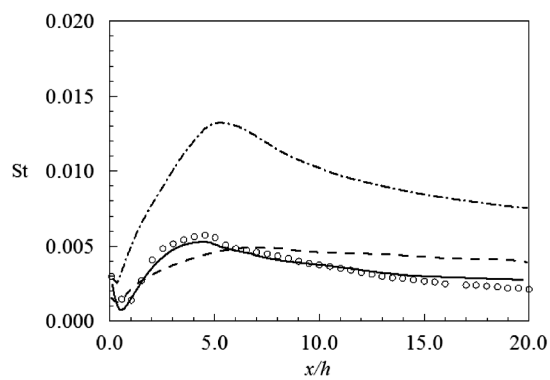
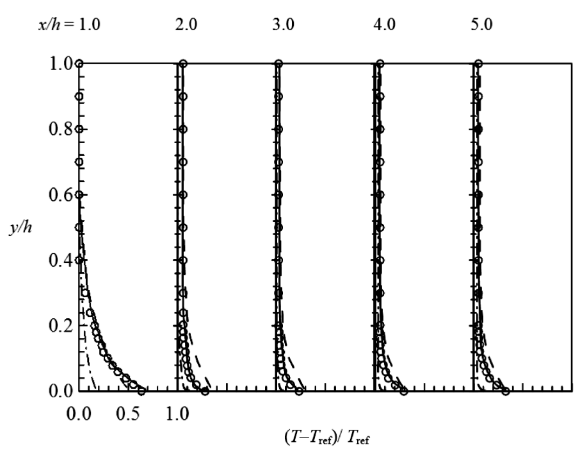


(○ LES, --- $k-\omega$, -·- Standard $k-\varepsilon$, — BLL model)

Figure 3 Mean Staton number



(○ LES, --- $k-\omega$, -·- Standard $k-\varepsilon$, — BLL model)

Figure 4 Mean temperature distribution